

Exercise 6.8

1. Find the sum of first 15 terms of the geometric sequence $1, \frac{1}{3}, \frac{1}{9}, \dots$

Solution:

Let $1, \frac{1}{3}, \frac{1}{9}, \dots$ is a G.P.

And

$$a = 1, r = \frac{1}{3} = \frac{1}{3}, n = 15$$

$$S_n = \frac{a_1(1-r^n)}{1-r} \quad [\because r < 1]$$

$$S_{15} = \frac{1 \left[1 - \left(\frac{1}{3} \right)^{15} \right]}{1 - \frac{1}{3}}$$

$$= \frac{\left[1 - \frac{1}{3^{15}} \right]}{\frac{3-1}{3}}$$

$$= \frac{3}{2} \left[\frac{3^{15} - 1}{3^{15}} \right]$$

$$= \frac{3^{15} - 1}{2(3^{14})} = \frac{14348907 - 1}{2(4782969)} = \frac{14348906}{9565938}$$

$$S_{15} = \frac{7174453}{4782969}$$

hence, $S_{15} = \frac{7174453}{4782969}$

2. Sum of the n terms, the series

(i) $.2 + .22 + .222 + \dots$ (ii) $3 + 33 + 333 + \dots$

Solution:

i. $.2 + .22 + .222 + \dots$ to n term

$$\begin{aligned}
&= \frac{2}{10} + \frac{22}{100} + \frac{222}{1000} + \dots \dots \dots \\
&= 2 \left[\frac{2}{10} + \frac{22}{100} + \frac{222}{1000} + \dots \dots \dots \right] \\
&= \frac{2}{9} \left[\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \dots \dots \dots \right] \\
&= \frac{2}{9} \left[1 - \frac{1}{10} + 1 - \frac{1}{100} + 1 - \frac{1}{1000} + \dots \dots \dots \right] \\
&= \frac{2}{9} \left[1 + 1 + 1 + \dots \dots n \text{ term} \left(\frac{1}{10} + \frac{11}{100} + \frac{111}{1000} + \dots n \text{ term} \right) \right] \\
&= \frac{2}{9} \left[n \left(\frac{1}{10} + \frac{11}{100} + \frac{111}{1000} + \dots n \text{ term} \right) \right]
\end{aligned}$$

sum of the terms

$$S = \frac{2}{9} [n + s_1]$$

Where

$$S_1 = \frac{1}{10} + \frac{11}{100} + \frac{111}{1000} + \dots n \text{ term}$$

$$a = \frac{1}{10}, r = \frac{\frac{1}{100}}{\frac{1}{10}} = \frac{1}{10}, n = n$$

$$S_1 = \frac{a_1(1-r^n)}{1-r} \quad [\because r < 1]$$

$$\begin{aligned}
S_n &= \frac{\frac{1}{10} \left[1 - \left(\frac{1}{10} \right)^n \right]}{1 - \frac{1}{10}} \\
&= \frac{\frac{1}{10} \left[1 - \frac{1}{10^n} \right]}{\frac{10-1}{10}}
\end{aligned}$$

$$= \frac{1}{9} \left[1 - \frac{1}{10^n} \right]$$

$$= \frac{1}{9} \left[\frac{10^n - 1}{10^n} \right]$$

$$S_1 = \frac{1}{9} \left[\frac{10^n - 1}{10^n} \right]$$

$$\text{hence, } S = \frac{2}{9} \left[n + \frac{1}{9} \left[\frac{10^n - 1}{10^n} \right] \right]$$

(ii) $3 + 33 + 333 + \dots$ to n term

$$= 3[1 + 11 + 111 + \dots \dots \dots \text{to } n \text{ term}]$$

$$= \frac{3}{9} [9 + 99 + 999 + \dots \dots \text{to } n \text{ term}]$$

$$= \frac{3}{9} [10 - 1 + 100 - 1 + 1000 - 1 + \dots \dots \text{to } n \text{ term}]$$

$$= \frac{3}{9} [10 + 100 + 1000 + \dots \dots \text{to } n \text{ term} - [1 + 1 + 1 + \dots \dots n \text{ term}]]$$

Sum of terms

$$S = \frac{3}{9} [s_1 - n]$$

$$\text{Where } s_1 = 10 + 100 + 1000 + \dots \dots$$

$$a_1 = 10, r = \frac{100}{10} = 10, n = n$$

So,

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad [\because r > 1]$$

$$s_1 = \frac{10(10^n - 1)}{10 - 1}$$

$$= \frac{10}{9} [10^n - 1]$$

$$s_1 = \frac{10}{9} [10^n - 1]$$

$$\text{Hence, } S = \frac{1}{3} \left[\frac{10}{9} [10^n - 1] - n \right]$$

3. Sum to n the series

$$\text{(I) } 1 + (a + b) + (a^2 + ab + b^2) + (a^3 + a^2b + ab^2 + b^3) + \dots$$

$$(II) \quad r + 1(1+k)r^2 + (1+k+k^2)r^3 + \dots$$

Solution:

$$\begin{aligned} i. \quad & 1 + (a+b) + (a^2 + ab + b^2) + (a^3 + a^2b + ab^2 + b^3) + \dots \\ &= \frac{1}{a-b} \left[(a-b)1 + (a-b)(a+b) + (a-b)(a^2 + ab + b^2) \right. \\ &\quad \left. + (a-b)(a^3 + a^2b + ab^2 + b^3) + \dots \dots n \text{ term} \right] \\ &= \frac{1}{a-b} [(a-b) + (a^2 + b^2) + (a^2 - b^2) + (a^4 - \\ &\quad b^4) + \dots] \\ &= \frac{1}{a-b} [a - b + a^2 + b^2 + a^2 - b^2 + a^4 - b^4 + \dots \dots n \text{ term}] \\ &= \frac{1}{a-b} [(a + a^2 + a^3 + \dots \dots n \text{ term}) - (b + b^2 + b^3 \\ &\quad + \dots \dots n \text{ term})] \end{aligned}$$

Sum of terms

$$S = \frac{1}{(a-b)} [S_1 - S_2]$$

Where

$$S_1 = a + a^2 + a^3 + \dots \dots n \text{ term}$$

$$\text{and } S_2 = b + b^2 + b^3 + \dots \dots n \text{ term}$$

$$S_1 = a + a^2 + a^3 + \dots \dots n \text{ term is G.P.}$$

$$\text{Then } S_n = \frac{a(r^n - 1)}{r - 1}$$

$$\begin{aligned} S_1 &= \frac{a(r^n - 1)}{r - 1} \quad [\because r > 1] \\ &= \frac{a(a^n - 1)}{a - 1} \end{aligned}$$

And

$$\begin{aligned} S_2 &= \frac{b(b^n - 1)}{b - 1} \\ &= \frac{b(b^n - 1)}{b - 1} \end{aligned}$$

$$S = \frac{1}{a-b} \left[\frac{a(a^n - 1)}{a - 1} - \frac{b(b^n - 1)}{b - 1} \right]$$

Hence,

$$S = \frac{1}{a-b} \left[\frac{a(a^n-1)}{a-1} - \frac{b(b^n-1)}{b-1} \right]$$

$$\text{ii. } r + 1(1+k)r^2 + (1+k+k^2)r^3 + \dots \dots \dots$$

$$r + 1(1+k)r^2 + (1+k+k^2)r^3 + \dots \dots \dots$$

Solution:

$$\frac{1}{1-K} [(1-K)r + (1-K)(1+K)r^2 + (1-K)(1+K+K^2)r^3$$

$$+ \dots \dots \dots n \text{ term}]$$

$$\frac{1}{1-K} [(1-K)r + (1-K^2)r^3 + (1-K)(1+K+K^2)r^3 + \dots \dots \dots]$$

$$\frac{1}{1-K} [1 - rk + r^2 + r^2K^2 + r^2 - r^3K^3 + \dots \dots \dots n \text{ term}]$$

$$\frac{1}{1-K} [(r + r^2 + r^3 + \dots \dots \dots n \text{ term}) - (rK + r^2K^2 + r^3K^3$$

$$+ \dots \dots \dots n \text{ term})]$$

Then Sum of terms

$$S = \frac{1}{1-K} [s_1 - s_2]$$

Where

$$S_1 = r + r^2 + r^3 + \dots \dots \dots n \text{ term}$$

$$= \frac{r(r^n-1)}{r-1} \quad [\because r > 1]$$

$$S_2 = rK + r^2K^2 + r^3K^3 + \dots \dots \dots n \text{ term}$$

$$= \frac{rK(rK^n-1)}{rK-1} \quad [\because r > 1]$$

$$= \frac{rK(rK^n-1)}{rK-1}$$

So,

$$S = \frac{1}{1-K} \left[\frac{r(r^n-1)}{r-1} + \frac{rK(rK^n-1)}{rK-1} \right]$$

Hence,
$$S = \frac{1}{1-K} \left[\frac{r(r^n-1)}{r-1} + \frac{rK(rK^n-1)}{rK-1} \right]$$

4. Sum the series $2 + (1+i) + \left(\frac{1}{1+i}\right) + \dots$ To 8th term.

Solution:

$2 + (1+i) + \left(\frac{1}{1+i}\right) + \dots$ To 8th term.

$a = 2, r = \frac{1+i}{2}, n = 8$

$$r = \frac{1+i}{2} \times \frac{1+i}{1+i} = \frac{1+i^2}{2(1+i)} = \frac{1-(-1)}{2(1+i)} = \frac{1+1}{2(1+i)} = \frac{2}{2(1+i)} = \frac{1}{1+i}$$

So,

$$S_n = \frac{a_1(r^n-1)}{r-1}$$

$$S_8 = \frac{2 \left[1 - \left(\frac{1}{1+i} \right)^8 \right]}{1 - \frac{1}{1+i}} \quad [\because r < 1]$$

$$= \frac{2 \left[1 - \frac{1}{(1+i)^8} \right]}{\frac{+i}{1+i}}$$

$$= \frac{2(1+i)}{i} \left[1 - \frac{1}{(1+i)^8} \right]$$

$$= \frac{2(1+i)}{i} \left[\frac{(1+i)^8 - 1}{(1+i)^8} \right]$$

$$= -2 \left[\frac{(1+i)^8 - 1}{(1+i)^8} \right] (1+i)$$

$$= -2 \left[\frac{[(1+i)^2]^4 - 1}{[(1+i)^2]^8} \right] (1+i) \times \frac{1-i}{1-i}$$

$$= -2 \left[\frac{[1+i^2+2i]^4 - 1}{(1+i^2+2i)^4} \right] \frac{(1-i^2)}{1-i}$$

$$= -2 \left[\frac{[1+i^2+2i]^4 - 1}{(1+i^2+2i)^4} \right] \left[\frac{1-(-1)}{1-i} \right]$$

$$= -2 \left[\frac{(2i)^4 - 1}{(2i)^4} \right] \left(\frac{1+1}{1-i} \right)$$

$$= -2 \left[\frac{16i^4 - 1}{16i^4} \right] \left(\frac{2}{1-i} \right)$$

$$\begin{aligned}
 &= -2 \left[\frac{16-1}{16} \right] \frac{2}{1-i} \\
 &= -2 \left[\frac{15}{16} \right] \frac{2}{1-i} = \frac{15}{4(1+i)} \times \frac{1+i}{1-i} = \frac{15(1+i)}{4(1+i^2)} \\
 &= \frac{15(1+i)}{4(1-(-1)^2)} \\
 &= \frac{15(1+i)}{4(1+1)} \\
 &= \frac{15(1+i)}{4 \times 2}
 \end{aligned}$$

$$S_8 = \frac{15(1+i)}{8}$$

Hence, $S_8 = \frac{15}{8}(1+i)$

5. Find the sum of the following infinite geometric series:

(i) $\frac{1}{5} + \frac{1}{25} + \frac{1}{125} + \dots$ (ii) $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$ (iii) $\frac{9}{4} + \frac{2}{3} + 1 + \frac{2}{3} + \dots$
 (iv) $2 + 5 + 0.5 + \dots$ (v) $4 + 2\sqrt{2} + 2 + \sqrt{2} + 1 + \dots$ (vi) $0.1 + 0.005 + 0.025 + \dots$

Solution:

i. $\frac{1}{5} + \frac{1}{25} + \frac{1}{125} + \dots$

Here, $a = \frac{1}{5}, \frac{\frac{1}{5}}{\frac{1}{5}} = \frac{1}{5}$

$$S_8 = \frac{a}{1-r} = \frac{\frac{1}{5}}{1-\frac{1}{5}} = \frac{\frac{1}{5}}{\frac{4}{5}} = \frac{1}{4}$$

Hence, $S_8 = \frac{1}{4}$

Solution:

(ii) $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$

Here, $a = \frac{1}{2}, \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$

$$S_8 = \frac{a}{1-r} = \frac{\frac{1}{2}}{1-\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$$

$$\text{Hence, } S_8 = 1$$

$$(iii) \quad \frac{9}{4} + \frac{2}{3} + 1 + \frac{2}{3} + \dots$$

Solution:

$$\text{Here, } a = \frac{9}{4}, \frac{\frac{2}{3}}{\frac{9}{4}} = \frac{2}{3}$$

$$S_8 = \frac{a}{1-r} = \frac{\frac{9}{4}}{1-\frac{2}{3}} = \frac{9}{4} \cdot \frac{3}{3-2} = \frac{27}{4(1)} = \frac{27}{4}$$

$$\text{Hence, } S_8 = \frac{27}{4}$$

6. Find vulgar fraction equivalent to the following recurring decimals.

- (i) 1.34 (ii) 0.7 (iii) 0.259 (iv) 1.53
(v) 0.159 (vi) 1.147

Solution:

(i) 1.34

$$\begin{aligned} 1.34 &= 1.3434343434 \dots \dots \dots \\ &= 1 + 0.34343434 \dots \dots \dots \\ &= 1 + 0.34 + 0.0034 + 0.000034 + \dots \dots \dots \infty \\ &= 1 + \frac{34}{100} + \frac{34}{10000} + \frac{34}{1000000} + \dots \dots \dots \infty \\ &= 1 + \left[\frac{34}{100} + \frac{34}{10000} + \frac{34}{1000000} + \dots \dots \dots \infty \right] \end{aligned}$$

The sum of series

$$\begin{aligned} S_{\infty} &= 1 + \frac{\frac{34}{100}}{1-\frac{1}{100}} \\ &= 1 + \frac{34}{100-1} \end{aligned}$$

$$= 1 + \frac{34}{99}$$

$$S_{\infty} = \frac{99+34}{99} = \frac{133}{99}$$

Hence, fraction = $\frac{133}{99}$

(ii) 0.7

Solution:

$$\begin{aligned} 0.7 &= 0.777777 \dots \dots \dots \\ &= 0.7 + 0.07 + 0.007 + \dots \dots \dots \infty \\ &= \frac{7}{10} + \frac{7}{100} + \frac{7}{1000} + \dots \dots \dots \infty \end{aligned}$$

The sum of series

$$S_{\infty} = 1 + \frac{\frac{7}{10}}{1 - \frac{1}{10}}$$

$$S_{\infty} = \frac{7}{10-1} = \frac{7}{9}$$

Hence, fraction = $\frac{7}{9}$

(iii) 0.259

Solution:

$$\begin{aligned} 0.259 &= 0.259259259 \dots \dots \dots \\ &= 0.259 + 0.000259 + 0.000000259 + \dots \dots \dots \infty \\ &= \frac{259}{1000} + \frac{259}{100000} + \frac{259}{10000000} + \dots \dots \dots \infty \end{aligned}$$

The sum of series

$$\begin{aligned} S_{\infty} &= 1 + \frac{\frac{259}{1000}}{1 - \frac{1}{1000}} \\ S_{\infty} &= \frac{259}{1000-1} = \frac{259}{999} \end{aligned}$$

Hence, fraction = $\frac{259}{999}$

(iv) 1.53

Solution:

$$\begin{aligned} 1.53 &= 1.53535353 \dots \dots \dots \\ &= 1 + 0.53 + 0.0053 + 0.000053 + \dots \dots \dots \infty \\ &= 1 + \frac{53}{100} + \frac{53}{10000} + \frac{53}{1000000} + \dots \dots \dots \infty \\ &= 1 + \left[\frac{53}{100} + \frac{53}{10000} + \frac{53}{1000000} + \dots \dots \dots \infty \right] \end{aligned}$$

The sum of series

$$\begin{aligned} S_{\infty} &= 1 + \frac{\frac{53}{100}}{1 - \frac{1}{100}} \\ &= 1 + \frac{53}{100 - 1} \\ &= 1 + \frac{53}{99} \\ S_{\infty} &= \frac{99 + 53}{99} = \frac{152}{99} \\ S_{\infty} &= \frac{152}{99} \end{aligned}$$

Hence, fraction = $\frac{152}{99}$

(iv) 0.159

Solution:

$$\begin{aligned} 0.159 &= 0.159159159 \dots \dots \dots \\ &= 0.159 + 0.000159 + 0.000000159 + \dots \dots \dots \infty \\ &= \frac{159}{1000} + \frac{159}{1000000} + \frac{159}{1000000000} + \dots \dots \dots \infty \end{aligned}$$

The sum of series

$$S_{\infty} = 1 + \frac{\frac{159}{1000}}{1 - \frac{1}{1000}}$$

$$S_{\infty} = \frac{159}{1000-1} = \frac{159}{999}$$

$$S_{\infty} = \frac{159}{999}$$

Hence, fraction = $\frac{159}{999}$

(vi) 1.147

Solution:

$$\begin{aligned} 1.147 &= 1.147147147 \dots \dots \dots \\ &= 1.1 + 0.0047 + 0.00047 + 0.00000047 + \dots \dots \dots \infty \\ &= 1.1 + \frac{47}{1000} + \frac{47}{100000} + \frac{47}{10000000} + \dots \dots \dots \infty \end{aligned}$$

The sum of series

$$\begin{aligned} S_{\infty} &= 1.1 + \frac{\frac{47}{1000}}{1 - \frac{1}{100}} \\ &= 1.1 + \frac{\frac{47}{1000}}{\frac{100-1}{100}} \\ &= 1.1 + \frac{\frac{47}{1000}}{\frac{99}{100}} \\ &= 1.1 + \frac{47}{1000} \times \frac{100}{99} \\ &= \frac{11}{10} = \frac{47}{990} \\ S_{\infty} &= \frac{1089+47}{990} = \frac{1136}{990} \\ S_{\infty} &= \frac{1136}{990} = \frac{568}{495} \end{aligned}$$

Hence, fraction = $\frac{568}{495}$

7. Find the sum of the infinity of the series; $r + (1+k)r^2 + (1+k+k^2)r^3 + \dots \dots \dots \infty$ and k being proper fractions.

Solution:

$$\begin{aligned}
 \text{Let } & r + (1+k)r^2 + (1+k+k^2)r^3 + \dots \infty \\
 &= \frac{1}{1-K} [(1-K)r + (1-K)(1+K)r^2 + (1-K)(1+K+K^2)r^3 \\
 &\quad + \dots \dots n \text{ term}] \\
 &= \frac{1}{1-K} [(1-K)r + (1-K^2)r^3 + (1-K^2)r^3 + \dots \dots] \\
 &= \frac{1}{1-K} [r - rk + r^2 + r^2K^2 + r^2 - r^3K^3 + \dots \dots n \text{ term}] \\
 &= \frac{1}{1-K} [(r + r^2 + r^3 + \dots \dots n \text{ term}) - (rK + r^2K^2 + r^3K^3 \\
 &\quad + \dots \dots n \text{ term})]
 \end{aligned}$$

$$\begin{aligned}
 \text{So, } &= \frac{1}{1-k} \left[\frac{r}{1-k} - \frac{kr}{1-kr} \right] \\
 &= \frac{1}{1-k} \left[\frac{r(1-kr) - kr(1-r)}{(1-k)(1-kr)} \right] \\
 &= \frac{1}{1-k} \left[\frac{r - kr^2 - kr + kr^2}{(1-k)(1-kr)} \right] \\
 &= \frac{1}{1-k} \left[\frac{r - kr}{(1-k)(1-kr)} \right] \\
 &= \frac{1}{1-k} \left[\frac{r(1-k)}{(1-k)(1-kr)} \right] \\
 &= \frac{r}{(1-k)(1-kr)}
 \end{aligned}$$

$$\text{Hence, } S_{\infty} = \frac{r}{(1-k)(1-kr)}$$

8. If $y = \frac{x}{2} + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \dots$ and if $0 < x < 2$, then prove that $x = \frac{2y}{1+y}$

Solution:

$$\begin{aligned}
 y &= \frac{x}{2} + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \dots \infty \\
 a &= \frac{x}{2}, r = \frac{\frac{1}{4}x^2}{\frac{1}{2}x} = \frac{1}{2}x \\
 S_{\infty} &= \frac{a}{1-r}
 \end{aligned}$$

$$= \frac{\frac{x}{2}}{1 - \frac{1}{2}x}$$

$$= \frac{\frac{x}{2}}{\frac{2-x}{2}}$$

$$y = \frac{x}{2-x}$$

$$(2-x)y = x$$

$$2y = x + xy$$

$$2y = x(1+y)$$

$$x = \frac{2y}{1+y}$$

Hence, proved $x = \frac{2y}{1+y}$

9. If $y = \frac{2}{3}x + \frac{4}{9}x^2 + \frac{8}{27}x^3 + \dots$ and if $0 < x < \frac{2}{3}$, then show that $x = \frac{3y}{2(1+y)}$

Solution:

$$y = \frac{2}{3}x + \frac{4}{9}x^2 + \frac{8}{27}x^3 + \dots \infty$$

$$a = \frac{2}{3}x, r = \frac{\frac{4}{9}x^2}{\frac{2}{3}x} = \frac{2}{3}x$$

$$S_{\infty} = \frac{\frac{2}{3}x}{1 - \frac{2}{3}x}$$

$$y = \frac{2x}{3-2x}$$

$$y(3-2x)y = 2x$$

$$3y - 2xy = 2x$$

$$3y = 2x + 2xy$$

$$3y = 2x(1+y)$$

$$2x = \frac{3y}{1+y}$$

$$x = \frac{3y}{2(1+y)}$$

Hence, proved $x = \frac{3y}{2(1+y)}$

10. A ball is dropped from the height of 27 meters and it rebounds two-third of the distance it falls. If it continues to fall in the same way what distance will it travel before coming to rest?

Solution:

Let height = 27

$$r = \frac{2}{3}$$

The ball cover distance

$$27, 2\left(27 \times \frac{2}{3}\right), 2\left(27 \times \frac{2}{3} \times \frac{2}{3}\right), \dots \dots \dots$$

So,

$$S_{\infty} = 27 + 2[18 + 12 + 8 \dots \dots \dots]$$

$$= 27 + 2\left[\frac{18}{1 - \frac{2}{3}}\right]$$

$$= 27 + 2\left[\frac{3 \times 18}{3 - 2}\right]$$

$$= 27 + 2\left[\frac{54}{1}\right]$$

$$= 27 + 2(54)$$

$$= 27 + 108 = 135$$

$$S_{\infty} = 135 \text{ meter}$$

Hence, the ball covers the distance of 135 meters.



11. What distance will a ball travel before coming to rest if it is dropped from a height of 75 meters and after each fall it rebounds $\frac{2}{5}$ of the distance it fell?

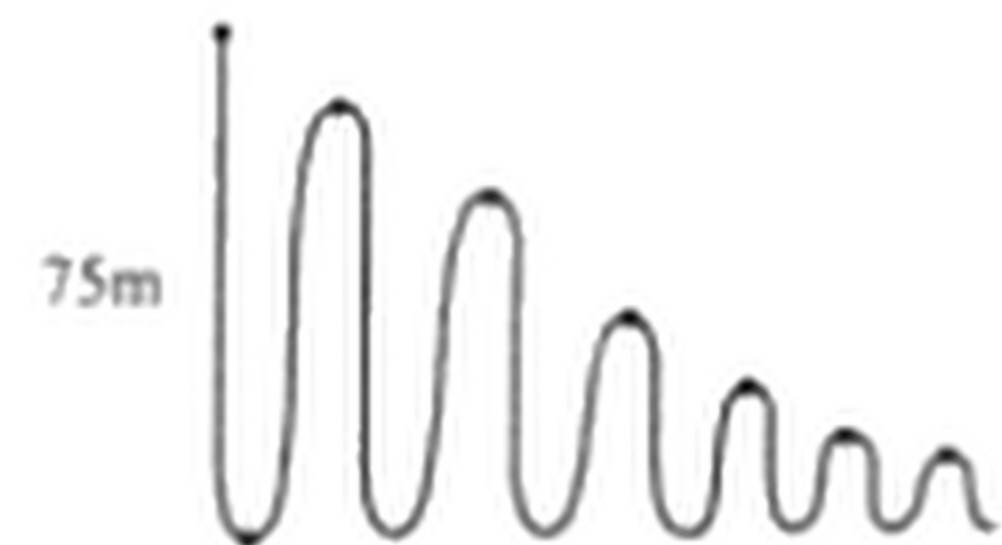
Solution:

Let height = 75m

$$r = \frac{2}{5}$$

The ball cover distance

$$75, 2\left(75 \times \frac{2}{5}\right), 2\left(75 \times \frac{2}{5} \times \frac{2}{5}\right), 2\left(75 \times \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5}\right) \dots \dots \dots \infty$$



$$75 + 60 + 24 + \frac{24}{5} + \dots \dots \dots \infty$$

So,

$$\begin{aligned} S_{\infty} &= 75 + \left[60 + 24 + \frac{24}{5} + \dots \dots \dots \right] \\ &= 75 + \left[\frac{60}{1 - \frac{2}{5}} \right] \\ &= 75 + \left[\frac{60 \times 5}{5 - 2} \right] \\ &= 75 + \left[\frac{300}{3} \right] \\ &= 75 + 100 \end{aligned}$$

$$S_{\infty} = 175 \text{ meter}$$

Hence, the ball covers the distance of 175 meters.

12. If $y = 1 + 2x + 4x^2 + 8x^3 + \dots \dots \dots$

- (i) show that $x = \frac{y-1}{2y}$
- (ii) find the interval in which the series is convergent.

Solution:

$$y = 1 + 2x + 4x^2 + 8x^3 + \dots \dots \dots$$

$$\text{Here, } a = 1, r = \frac{2x}{1} = 2x$$

$$S_{\infty} = \frac{a}{1-r}$$

$$y = \frac{1}{1-2x}$$

$$y(1-2x) = 1$$

$$y - 2xy = 1$$

$$y - 1 = 2xy$$

$$\frac{y-1}{2y} = x$$

$$x = \frac{y-1}{2y}$$

Hence proved $x = \frac{3y}{2(1+y)}$

iii. if the series is convergent.

$$|r| = \frac{2x}{1}$$

$$|2x| < 1$$

$$i.e. |x| < \frac{1}{2} \quad -\frac{1}{2} < x < \frac{1}{2}$$

13. if $y = 1 + \frac{x}{2} + \frac{x^2}{4} + \dots$

(i) show that $x = \frac{y-1}{2y}$

(ii) find the interval in which the series is convergent.

Solution:

$$y = 1 + \frac{x}{2} + \frac{x^2}{4} + \dots \infty$$

Here, $a = 1, r = \frac{\frac{x}{2}}{1} = \frac{x}{2}$

$$S_{\infty} = \frac{a}{1-r} = \frac{1}{1-\frac{x}{2}} = \frac{2}{2-x}$$

$$y = \frac{2}{2-x}$$

$$y(2-x) = 2$$

$$2y - xy = 2$$

$$2y - 2 = xy$$

$$2(y-1) = xy$$

$$x = \frac{2(y-1)}{y}$$

Hence proved

$$x = 2 \left(\frac{y-1}{y} \right)$$

iv. if the series is convergent.

$$|r| = \frac{x}{2} < 1$$

$$|x| < 2$$

$$i.e - 2 < x < 2$$

14. The sum of infinite geometric series is 9 and the sum of square of its terms is $\frac{81}{5}$. find the series.

Solution:

$$S_{\infty} = 9$$

And

$$a + a^2r^2 + a^2r^3 + \dots = \frac{81}{5} \quad \dots\dots\dots(ii)$$

So,

$$\frac{a}{1-r} = 9$$

$$a = 9(1-r) = 9 - 9r$$

$$a = 9 - 9r \quad \dots\dots\dots(iii)$$

And

From eq (ii)

$$\frac{a^2}{1-r^2} = \frac{81}{5}$$

$$\frac{a^2}{1 - \left(\frac{a^2r^2}{a^2} \right)} = \frac{81}{5}$$

$$\frac{a^2}{1-r^2} = \frac{81}{5}$$

From eq (iii) put the value of a.

$$\frac{(9-9r^2)}{1-r^2} = \frac{81}{5}$$

$$\frac{9^2(1-r^2)}{1-r^2} = \frac{81}{5}$$

$$\frac{81(1-r^2)}{1-r^2} = \frac{81}{5}$$

$$\frac{1-r}{1+r} = \frac{1}{5}$$

$$5(1-r) = 1+r$$

$$5-5r = 1+r$$

$$5-1 = r+5r$$

$$4 = 6r$$

$$r = \frac{4}{6} = \frac{2}{3}$$

Put the value of 'r' in eq (iii)

$$a = 9 - 9\left(\frac{2}{3}\right)$$

$$= 9 - 6 = 3$$

$$a = 3$$

So term

$$3, 3\left(\frac{2}{3}\right), 3\left(\frac{2}{3}\right)^2, 3\left(\frac{2}{3}\right)^3, \dots \dots \dots$$

$$3, 2, \frac{4}{3}, \frac{8}{9}, \dots \dots \dots$$

$$\text{Hence series} = 3, 2, \frac{4}{3}, \frac{8}{9}, \dots \dots \dots \infty$$

