Exercise 6.8

1. Find the sum of first 15 terms of the geometric sequence 1, $\frac{1}{3}, \frac{1}{9}, \dots \dots$

Solution:

Let **1**, $\frac{1}{3}, \frac{1}{9}, \dots \dots$ Is a G.P.

And

$$a = 1, r = \frac{1}{3} = \frac{1}{3}, n = 15$$

$$S_n = \frac{a_1(1-r^n)}{1-r} \qquad [\because r > 1]$$

$$S_{15} = \frac{1\left[1-\left(\frac{1}{3}\right)^{1/5}\right]}{1-\frac{1}{3}}$$

$$= \frac{\left[1-\frac{1}{3^{15}}\right]}{\frac{3-1}{3}}$$

$$= \frac{3}{2}\left[\frac{3^{15}-1}{3^{15}}\right]$$

$$= \frac{3^{15}-1}{2(3^{14})} = \frac{14348907-1}{2(4785969)} = \frac{14348906}{9565938}$$

$$S_{15} = \frac{7174453}{4782969}$$

hence, $S_{15} = \frac{7174453}{4782969}$

2. Sum of the n terms, the series

(i) $.2 + .22 + .222 + \cdots$ **(ii)** $3 + 33 + 333 + \cdots$

Solution:

i. $.2 + .22 + .222 + \cdots$ **to n term**

$$= \frac{2}{10} + \frac{22}{100} + \frac{222}{1000} + \cdots \ldots \ldots$$

$$= 2\left[\frac{2}{10} + \frac{22}{100} + \frac{222}{1000} + \cdots \ldots \ldots .\right]$$

$$= \frac{2}{9}\left[\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \cdots \ldots \ldots .\right]$$

$$= \frac{2}{9}\left[1 - \frac{1}{10} + 1\frac{1}{100} + 1\frac{1}{1000} + \cdots \ldots \ldots .\right]$$

$$= \frac{2}{9}\left[1 + 1 + 11 + \cdots \ldots . n\ term \left(\frac{1}{10} + \frac{11}{100} + \frac{111}{1000} + \cdots . n\ term\right)\right]$$

$$= \frac{2}{9}\left[n \left(\frac{1}{10} + \frac{11}{100} + \frac{111}{1000} + \cdots . n\ term\right)\right]$$

sum of the terms

$$S = \frac{2}{9}[n + s_1]$$

Where

$$S_1 = \frac{1}{10} + \frac{11}{100} + \frac{111}{1000} + \cdots . n\ term$$

$$a = \frac{1}{10}, r = \frac{\frac{1}{100}}{\frac{1}{10}} = \frac{1}{10},\ n = n$$

$$S_1 = \frac{a_1(1-r^n)}{1-r} \qquad [\because r < 1]$$

$$S_n = \frac{\frac{1}{10}\left[1 \quad \frac{(1)^n}{10}\right]}{1 \quad \frac{1}{10}}$$

$$= \frac{\frac{1}{10}\left[1 - \frac{1}{10^n}\right]}{\frac{10-1}{10}}$$

$$= \frac{1}{9}\left[1 - \frac{1}{10^n}\right]$$

$$= \frac{1}{9}\left[\frac{10^n \quad 1}{10^n}\right]$$

$$S_1 = \frac{1}{9}\left[\frac{10^n \quad 1}{10^n}\right]$$

hence, $S = \frac{2}{9}\left[n + \frac{1}{9}\left[\frac{10^n\ \ 1}{10^n}\right]\right]$

(ii) $3 + 33 + 333 + \cdots$ **to n term**

$= 3[\ 1 + 11 + 111 + \cdots \ldots \ldots . to\ n\ term]$

$= \frac{3}{9}\ [\ 9 + 99 + 999 + \cdots \ldots to\ n\ term]$

$= \frac{3}{9}\ [\ 10 - 1 + 100 - 1 + 1000 - 1 + \cdots . . to\ n\ term]$

$= \frac{3}{9}\ [\ 10 + 100 + 1000 + \cdots \ldots to\ n\ term - [\ 1 + 1 + 1 + \cdots \ldots n\ term]$

Sum of terms

$S = \frac{3}{9}[s_1 - n]$

Where $s_1 = 10 + 100 + 1000 + \cdots \ldots$

$a_1 = 10, r = \frac{100}{10} = 10, n = n$

So,

$s_n = \frac{a(r^n - 1)}{r - 1}$ $[\because r > 1]$

$s_1 = \frac{10(10^n - 1)}{10 - 1}$

$= \frac{10}{9}[10^n - 1]$

$s_1 = \frac{10}{9}[10^n - 1]$

Hence, $S = \frac{1}{3}\left[\frac{10}{9}[10^n - 1] - n\right]$

3. Sum to n the series

(I) $\boldsymbol{1 + (a + b) + (a^2 + ab + b^2) + (a^3 + a^2b + ab^2 + b^3) + \ldots\ldots\ldots\ldots}$

(II) $\boldsymbol{r + 1(1+k)r^2 + (1+k+k^2)r^3 + \ldots\ldots\ldots\ldots}$

Solution:

i. $\boldsymbol{1 + (a+b) + (a^2+ab+b^2) + (a^3+a^2b+ab^2+b^3) + \ldots\ldots\ldots\ldots}$

$$= \frac{1}{a-b}\begin{bmatrix}(a-b)1 + (a-b)(a+b) + (a-b)(a^2+ab+b^2) \\ +(a-b)(a^3+a^2b+ab^2+b^3) + \cdots \ldots \ldots n\ term\end{bmatrix}$$

$$= \frac{1}{a-b}[(a-b) + (a^2+b^2) + (a^2-b^2) + (a^4 - b^4) + \ldots\ldots\ldots\ldots$$

$$= \frac{1}{a-b}[a - b + a^2 + b^2 + a^2 - b^2 + a^4 - b^4 + \ldots\ldots\ldots\ n \text{ term}]$$

$$= \frac{1}{a-b}[(a + a^2 + a^3 + \cdots \ldots\ldots n\ term) - (b + b^2 + b^3 + \cdots \ldots n\ term)]$$

Sum of terms

$$S = \frac{1}{(a-b)}[\,s_1 - s_2]$$

Where

$$S_1 = a + a^2 + a^3 + \cdots \ldots\ldots n\ term$$

and $S_2 = b + b^2 + b^3 + \cdots \ldots n\ term$

$S_1 = a + a^2 + a^3 + \cdots \ldots\ldots n\ term$ is G.P.

Then $s_n = \frac{a(r^n-1)}{r-1}$

$$s_1 = \frac{a(r^n-1)}{r-1} \quad [\therefore r > 1]$$

$$= \frac{a(a^n-1)}{a-1}$$

And

$$s_2 = \frac{b(b^n-1)}{r-1}$$

$$= \frac{b(b^n-1)}{b-1}$$

$$S = \frac{1}{a-b}\left[\frac{a(a^n-1)}{a-1} - \frac{b(b^n-1)}{b-1}\right]$$

Hence,

$$S = \frac{1}{a-b}\left[\frac{a(a^n-1)}{a-1} - \frac{b(b^n-1)}{b-1}\right]$$

ii. $\boldsymbol{r + 1(1+k)r^2 + (1+k+k^2)r^3 + \ldots\ldots\ldots\ldots}$

$\boldsymbol{r + 1(1+k)r^2 + (1+k+k^2)r^3 + \ldots\ldots\ldots\ldots}$

Solution:

$$\frac{1}{1-K}[(1-K)r + (1-K)(1+K)r^2(1-K)(1+K+K^2)r^3 + \cdots \ldots\ldots n\ term]$$

$$\frac{1}{1-K}[(1-K)r + (1-K^2)r^3 + (1-K)(1+K+K^2)r^3 + \ldots\ldots\ldots\ldots]$$

$$\frac{1}{1-K}[1 - rk + r^2 + r^2K^2 + r^2 - r^3K^3 + \ldots\ldots\ldots\ n\ \text{term}]$$

$$\frac{1}{1-K}[(r + r^2 + r^3 + \cdots \ldots\ldots n\ term) - (rK + r^2K^2 + r^3K^3 + \cdots \ldots n\ term)]$$

Then Sum of terms

$$S = \frac{1}{1-K}[s_1 - s_2]$$

Where

$$S_1 = r + r^2 + r^3 + \cdots \ldots\ldots n\ term$$

$$= \frac{r(r^n-1)}{r-1} \quad [\because r > 1]$$

$$S_2 = rK + r^2K^2 + r^3K^3 + \cdots \ldots n\ term$$

$$= \frac{rK(rK^n-1)}{rK-1} \qquad [\because r > 1]$$

$$= \frac{rK(rK^n-1)}{rK-1}$$

So,

$$S = \frac{1}{1-K}\left[\frac{r(r^n-1)}{r-1} + \frac{rK(rK^n-1)}{rK-1}\right]$$

Hence, $S = \frac{1}{1-K}\left[\frac{r(r^n-1)}{r-1} + \frac{rK(rK^n-1)}{rK-1}\right]$

4. Sum the series $2 + (1+i) + \left(\frac{1}{1}\right) + \ldots\ldots\ldots$ To 8th term.

Solution:

$2 + (1+i) + \left(\frac{1}{1}\right) + \ldots\ldots\ldots$ **To 8th term.**

$a = 2, r = \frac{1+i}{2}, n = 8$

$r = \frac{1+i}{2} \times \frac{1+i}{1+i} = \frac{1+i^2}{2(1+i)} = \frac{1-(-1)}{2(1+i)} = \frac{1+1}{2(1+i)} = \frac{2}{2(1+i)} = \frac{1}{2(1+i)}$

So,

$S_n = \frac{a_1(r^n-1)}{r-1}$

$S_8 = \frac{2\left[1-\left(\frac{1}{1+i}\right)^n\right]}{1-\frac{1}{1+i}}$ $[\therefore r < 1]$

$= \frac{2\left[1-\frac{}{(\ +i)^8}\right]}{\frac{+i}{+i}}$

$= \frac{2(1+i)}{i}\left[1 - \frac{1}{(1+i)^8}\right]$

$= \frac{2(1+i)}{i}\left[\frac{(1+i)^8-1}{(1+i)^8}\right]$

$= -2\left[\frac{(1+i)^8-1}{(1+i)^8}\right](1+i)$

$= -2\left[\frac{[(1+i)^2]^8-1}{[(1+i)^2]^8}\right](1+i) \times \frac{1-i}{1-i}$

$= -2\left[\frac{[1+i^2+2i]^4-1}{(1+i^2+2i)^4}\right]\frac{(1-i^2)}{1-i}$

$= -2\left[\frac{[1+i^2+2i]^4-1}{(1+i^2+2i)^4}\right]\left[\frac{1-(-1)}{1-i}\right]$

$= -2\left[\frac{(2i)^4-1}{(2i)^4}\right]\left(\frac{1+1}{1-i}\right)$

$= -2\left[\frac{16i^4-1}{16i^4}\right]\left(\frac{2}{1-i}\right)$

$= -2\left[\frac{16-1}{16}\right]\frac{2}{1-i}$

$= -2\left[\frac{15}{6}\right]\frac{2}{1-i} = \frac{15}{4(1+i)} \times \frac{1+1}{1-i} = \frac{15(1+i)}{4(1+i^2)}$

$= \frac{15(1+i)}{4(1-(-1)^2)}$

$= \frac{15(1+i)}{4(1+1)}$

$= \frac{15(1+i)}{4\times2}$

$s_8 \quad = \frac{15(1+i)}{8}$

Hence, $\quad s_8 \quad = \frac{15}{8}(1+i)$

5. Find the sum of the following infinite geometric series:

(i) $\frac{1}{5}+\frac{1}{25}+\frac{1}{125}+\ldots\ldots\ldots$ **(ii)** $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\ldots\ldots$ **(iii)** $\frac{9}{4}+\frac{2}{3}+1+\frac{2}{3}+\ldots..$

(i) $2+5+0.5+\ldots\ldots..$ **(v)** $4+2\sqrt{2}+2+\sqrt{2}+1+\ldots\ldots.$ **(vi)** $0.1+0.005+0.025+\ldots\ldots..$

Solution:

I. $\frac{1}{5}+\frac{1}{25}+\frac{1}{125}+\ldots\ldots\ldots$

Here, $\quad a=\frac{1}{5}, \frac{\frac{1}{5}}{\frac{1}{5}}=\frac{1}{5}$

$S_8 = \frac{a}{1-r} = \frac{\frac{1}{5}}{1-\frac{1}{5}} = \frac{\frac{1}{5}}{\frac{4}{5}} = \frac{1}{4}$

Hence, $S_8 = \frac{1}{4}$

Solution:

(ii) $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\ldots..$

Here, $\quad a=\frac{1}{2}, \frac{\frac{1}{4}}{\frac{1}{2}}=\frac{1}{2}$

$S_8 = \frac{a}{1-r} = \frac{\frac{1}{2}}{1-\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$

Hence, $S_8 = 1$

(iii) $\frac{9}{4} + \frac{2}{3} + 1 + \frac{2}{3} + \ldots\ldots$

Solution:

Here, $a = \frac{9}{4}, \frac{\frac{2}{3}}{\frac{9}{4}} = \frac{2}{3}$

$S_8 = \frac{a}{1-r} = \frac{\frac{9}{4}}{1-\frac{2}{3}} = \frac{9}{4}\frac{3}{(3-2)} = \frac{27}{4(1)} = \frac{27}{4}$

Hence, $S_8 = \frac{27}{4}$

6. Find vulgar fraction equivalent to the following recurring decimals.

(i) 1.34 **(ii) 0.7** **(iii) 0.259** **(iv) 1.53**
(v) 0.159 **(vi) 1.147**

Solution:

(i) 1.34

$1.34 = 1.3434343434 \ldots\ldots\ldots$
$= 1 + 0.34343434 \ldots\ldots$
$= 1 + 0.34 + 0.0034 + 0.000034 + \cdots \ldots\ldots \infty$
$= 1 + \frac{34}{100} + \frac{34}{10000} + \frac{34}{100000} + \ldots\ldots\ldots\infty$
$= 1 + \left[\frac{34}{100} + \frac{34}{10000} + \frac{34}{100000} + \cdots \ldots\ldots \infty\right]$

The sum of series

$S\infty = 1 + \frac{\frac{34}{100}}{1-\frac{1}{100}}$

$= 1 + \frac{34}{100-1}$

$$= 1 + \frac{34}{99}$$

$$S\infty = \frac{99+34}{99} = \frac{133}{99}$$

Hence, fraction $= \frac{133}{99}$

(ii) 0.7

Solution:

$$0.7 = 0.777777 \ldots \ldots \ldots$$
$$= 0.7 + 0.07 + 0.007 + \cdots \ldots . . \infty$$
$$= \frac{7}{10} + \frac{7}{100} + \frac{7}{1000} + \ldots \ldots \ldots . \infty$$

The sum of series

$$S\infty = 1 + \frac{\frac{7}{10}}{1-\frac{1}{10}}$$

$$S\infty = \frac{7}{10-1} = \frac{7}{9}$$

Hence, fraction $= \frac{7}{9}$

(iii) 0.259

Solution:

$$0.259 = 0.259259259 \ldots \ldots \ldots$$
$$= 0.259 + 0.000259 + 0.000000259 + \cdots \ldots . . \infty$$
$$= \frac{259}{1000} + \frac{259}{100000} + \frac{259}{10000000} + \ldots \ldots \ldots . \infty$$

The sum of series

$$S\infty = 1 + \frac{\frac{259}{1000}}{1 - \frac{1}{1000}}$$
$$S\infty = \frac{259}{1000-1} = \frac{259}{999}$$

Hence, fraction $= \frac{259}{999}$

(iv) 1.53

Solution:

$$1.53 = 1.53535353 \ldots \ldots \ldots$$
$$= 1 + 0.53 + 0.0053 + 0.000053 + \cdots \ldots . . \infty$$
$$= 1 + \frac{53}{100} + \frac{53}{10000} + \frac{53}{100000} + \ldots\ldots\ldots\ldots \infty$$
$$= 1 + \left[\frac{53}{100} + \frac{53}{10000} + \frac{53}{100000} + \cdots \ldots \ldots . . \infty\right]$$

The sum of series

$$S\infty = 1 + \frac{\frac{53}{100}}{1 - \frac{1}{100}}$$

$$= 1 + \frac{53}{100 - \frac{}{00}}$$

$$= 1 + \frac{53}{99}$$

$$S\infty = \frac{99+53}{99} = \frac{152}{99}$$

$$S\infty = \frac{152}{99}$$

Hence, fraction $= \frac{152}{99}$

(iv) 0.159

Solution:

$$0.159 = 0.159159159 \ldots \ldots \ldots$$
$$= 0.159 + 0.000159 + 0.000000159 + \cdots \ldots . . \infty$$
$$= \frac{159}{1000} + \frac{159}{100000} + \frac{159}{10000000} + \ldots\ldots\ldots\ldots \infty$$

The sum of series

$$S\infty = 1 + \frac{\frac{159}{1000}}{1 - \frac{1}{1000}}$$

$$S\infty = \frac{159}{1000-1} = \frac{159}{999}$$
$$S\infty = \frac{159}{999}$$

Hence, fraction $= \frac{159}{999}$

(vi) 1.147

Solution:

$$1.147 = 1.147147147 \ldots \ldots \ldots$$
$$= 1.1 + 0.0047 + 0.00047 + 0.00000047 + \cdots \ldots . . \infty$$
$$= 1.1 + \frac{47}{1000} + \frac{47}{100000} + \frac{47}{10000000} + \ldots \ldots \ldots . \infty$$

The sum of series

$$S\infty = 1.1 + \frac{\frac{47}{1000}}{1-\frac{1}{100}}$$

$$= 1.1 + \frac{\frac{47}{1000}}{\frac{100\ \ 1}{100}}$$

$$= 1.1 + \frac{\frac{47}{1000}}{\frac{99}{100}}$$

$$= 1.1 + \frac{47}{1000} \times \frac{100}{99}$$

$$= \frac{11}{10} = \frac{47}{990}$$

$$S\infty = \frac{1089+47}{990} = \frac{1136}{990}$$

$$S\infty = \frac{1136}{990} = \frac{568}{495}$$

Hence, fraction $= \frac{568}{495}$

7. Find the sum of the infinity of the series; $r + (1 + k)r^2 + (1 + k + k^2)r^3 + \ldots\ldots..\infty$ and k being proper fractions.

Solution:

Let $r + (1+k)r^2 + (1+k+k^2)r^3 + \ldots\ldots\infty$

$$= \frac{1}{1-K}[(1-K)r + (1-K)(1+K)r^2(1-K)(1+K+K^2)r^3 + \cdots \ldots n\ term]$$

$$= \frac{1}{1-K}[(1-K)r + (1-K^2)r^3 + (1-K^2)r^3 + \ldots\ldots\ldots\ldots]$$

$$= \frac{1}{1-K}[r - rk + r^2 + r^2K^2 + r^2 - r^3K^3 + \ldots\ldots\ldots\ n\ \text{term}]$$

$$= \frac{1}{1-K}[(r + r^2 + r^3 + \cdots \ldots n\ term) - (rK + r^2K^2 + r^3K^3 + \cdots \ldots n\ term)]$$

So, $= \frac{1}{1-k}\left[\frac{r}{1-k} - \frac{kr}{1-kr}\right]$

$$= \frac{1}{1-k}\left[\frac{r(1-kr)-kr(1-r)}{(1-k)(1-kr)}\right]$$

$$= \frac{1}{1-k}\left[\frac{r-kr^2-kr+kr^2}{(1-k)(1-kr)}\right]$$

$$= \frac{1}{1-k}\left[\frac{r-kr}{(1-k)(1-kr)}\right]$$

$$= \frac{1}{1-k}\left[\frac{r(1-k)}{(1-k)(1-kr)}\right]$$

$$= \frac{r}{(1-k)(1-kr)}$$

Hence, $S\infty = \frac{r}{(1-k)(1-kr)}$

8. If $y = \frac{x}{2} + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \ldots\ldots\ldots$ and if $0 < x > 2$, then prove that $x = \frac{2y}{1+y}$

Solution:

$$y = \frac{x}{2} + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \ldots\ldots\ldots\infty$$

$$a = \frac{x}{2}, r = \frac{\frac{1}{4}x^2}{\frac{1}{2}x} = \frac{1}{2}x$$

$$S\infty = \frac{a}{1-r}$$

$$= \frac{\frac{x}{2}}{1-\frac{1}{2}x}$$

$$= \frac{\frac{x}{2}}{\frac{2-x}{2}}$$

$$y = \frac{x}{2-x}$$

$$(2-x)y = x$$

$$2y = x + xy$$

$$2y = x(1+y)$$

$$x = \frac{2y}{1+y}$$

Hence, proved $x = \frac{2y}{1+y}$

9. If $y = \frac{2}{3}x + \frac{4}{9}x^2 + \frac{8}{27}x^3 + \ldots\ldots$ and if $0 < x > \frac{2}{3}$, then show that $x = \frac{3y}{2(1+y)}$

Solution:

$$y = \frac{2}{3}x + \frac{4}{9}x^2 + \frac{8}{27}x^3 + \ldots\ldots\ldots\infty$$

$$a = \frac{2}{3}x, r = \frac{\frac{4}{9}x^2}{\frac{2}{3}x} = \frac{2}{3}x$$

$$S\infty == \frac{\frac{2}{3}x}{1-\frac{2}{3}x}$$

$$y = \frac{2x}{3-2x}$$

$$y(3-2x)y = 2x$$

$$3y - 2xy = 2x$$

$$3y = 2x + 2xy$$

$$3y = 2x(1+y)$$

$$2x = \frac{3y}{1+y}$$

$$x = \frac{3y}{2(1+y)}$$

Hence, proved $x = \frac{3y}{2(1+y)}$

10. A ball is dropped from the height of 27 meters and it rebounds two-third of the distance it falls. If it continues to fall in the same way what distance will it travel before coming to rest?

Solution:

Let height = 27

$r = \frac{2}{3}$

The ball cover distance

$27, 2\left(27 \times \frac{2}{3}\right), 2\left(27 \times \frac{2}{3} \times \frac{2}{3}\right), \ldots \ldots \ldots$

So,

$$S\infty = 27 + 2[18 + 12 + 8 \ldots \ldots \ldots ..]$$

$$= 27 + 2\left[\frac{18}{1-\frac{2}{3}}\right]$$

$$= 27 + 2\left[\frac{3 \times 18}{3-2}\right]$$

$$= 27 + 2\left[\frac{54}{1}\right]$$

$$= 27 + 2(54)$$

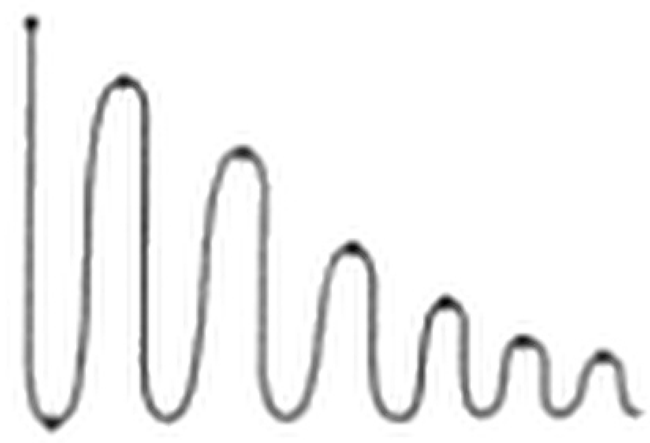
$$= 27 + 108 = 135$$

$$S\infty = 135\, meter$$

Hence, the ball covers the distance of 135 meters.

11. What distance will a ball travel before coming to rest if it is dropped from a height of 75 meters and after each fall it rebounds $\frac{2}{5}$ of the distance it fell?

Solution:



Let height = $75m$

$r = \frac{2}{5}$

The ball cover distance

$75, 2\left(75 \times \frac{2}{3}\right), 2\left(75 \times \frac{2}{3} \times \frac{2}{3}\right), 2\left(75 \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}\right) \ldots \ldots \ldots \infty$

$75 + 60 + 24 + \frac{24}{5}$..........$\infty$

So,

$$S\infty = 75 + \left[60 + 24 + \frac{24}{5} \dots\dots\dots\right]$$

$$= 27 + \left[\frac{60}{1-\frac{24}{60}}\right]$$

$$= 75 + \left[\frac{60 \times 60}{60 - 24}\right]$$

$$= 75 + \left[\frac{3600}{36}\right]$$

$$= 75 + 100$$

$$S\infty = 175\, meter$$

Hence, the ball covers the distance of 175 meters.

12. If $y = 1 + 2x + 4x^2 + 8x^2 + \dots\dots\dots\dots$

(i) show that $x = \frac{y-1}{2y}$

(ii) find the interval in which the series is convergent.

Solution:

$$y = 1 + 2x + 4x^2 + 8x^2 + \dots\dots\dots\dots$$

Here, $a = 1, r = \frac{2x}{1} = 2x$

$$S\infty = \frac{a}{1-r}$$

$$y = \frac{1}{1-2x}$$

$$y(1 - 2x) = 1$$

$$y - 2xy = 1$$

$$y - 1 = 2xy$$

$$\frac{y-1}{2y} = x$$

$x = \frac{y-1}{2y}$

Hence proved $x = \frac{3y}{2(1+y)}$

iii. if the series is convergent.

$|r| = \frac{2x}{1}$

$|2x| < 1$

$i.e\ |x| < \frac{1}{2}$ $-\frac{1}{2} < x < \frac{1}{2}$

13. if $y = 1 + \frac{x}{2} + \frac{x^2}{4} + \dots\dots\dots$

(i) show that $x = \frac{y-1}{2y}$

(ii) find the interval in which the series is convergent.

Solution:

$y = 1 + \frac{x}{2} + \frac{x^2}{4} + \dots\dots\dots\infty$

Here, $a = 1, r = \frac{\frac{x}{2}}{1} = 2x$

$S\infty = \frac{a}{1-r} = \frac{1}{1-\frac{x}{2}} = \frac{2}{2-x}$

$y = \frac{2}{2-x}$

$y(2-x) = 2$

$2y - xy = 2$

$2y - 2 = xy$

$2(y+1) = xy$

$x = \frac{2(y-1)}{y}$

Hence proved

$$x = 2\left(\frac{y-1}{y}\right)$$

iv. if the series is convergent.

$$|r| = \frac{x}{2} < 1$$

$$|x| < 2$$

$$i.e -2 < x < 2$$

14. The sum of infinite geometric series is 9 and the sum of square of its terms is $\frac{81}{5}$. find the series.

Solution:

$$S\infty = 9$$

And

$$a + a^2r^2 + a^2r^3 + \ldots\ldots\ldots = \frac{8}{5} \qquad \ldots\ldots\ldots\ldots(ii)$$

So,

$$\frac{a}{1-r} = 9$$

$$a = 9(1-r) = 9 - 9r$$

$$a = 9 - 9r \qquad \ldots\ldots\ldots\ldots\ (iii)$$

And

From eq (ii)

$$\frac{a^2}{1-r^2} = \frac{81}{5}$$

$$\frac{a^2}{1-\left(\frac{a^2r^2}{a^2}\right)} = \frac{81}{5}$$

$$\frac{a^2}{1-r^2} = \frac{81}{5}$$

From eq (iii) put the value of a.

$$\frac{(9-9r^2)}{1-r^2} = \frac{81}{5}$$

$$\frac{9^2(1-r^2)}{1-r^2} = \frac{81}{5}$$

$$\frac{81(1-r^2)}{1-r^2} = \frac{81}{5}$$

$$\frac{1-r}{1+r} = \frac{1}{5}$$

$$5(1-r) = 1+r$$

$$5-5r = 1+r$$

$$5-1 = r+5r$$

$$4 = 6r$$

$$r = \frac{4}{6} = \frac{2}{3}$$

Put the value of 'r' in eq (iii)

$$a = 9-9\left(\frac{2}{3}\right)$$

$$= 9-6 = 3$$

$$a = 3$$

So term

$$3, 3\left(\frac{2}{3}\right), 3\left(\frac{2}{3}\right)^2, 3\left(\frac{2}{3}\right)^3, \ldots\ldots\ldots$$

$$3, 2, \frac{4}{3}, \frac{8}{9}, \ldots\ldots\ldots..$$

Hence series $= 3, 2, \frac{4}{3}, \frac{8}{9}, \ldots\ldots\ldots..\infty$