EXERCISE 5.2

Resolve the following into Partial Fraction:

1. $\frac{2x^2-3x+4}{(x-1)^3}$

2. $\frac{5x^2-2x+3}{(x+3)^3}$

3. $\frac{4x}{(x+1)^2(x-1)}$

4. $\frac{9}{(x+2)^2(x-1)}$

5. $\frac{1}{(x-3)^2(x+1)}$

6. $\frac{x^2}{(x-2)^2(x-1)^3}$

7. $\frac{1}{(x-1)^2(x+1)}$

8. $\frac{x^2}{(x-1)^2(x+1)}$

9. $\frac{x-1}{(x-2)(x+1)^2}$

10. $\frac{4x^3}{(x^2-1)(x+1)^2}$

1. $\frac{2x^2-3x+4}{(x-1)^3}$

Solution:

Let $\frac{2x^2-3x+4}{(x-1)^3}$ **=** $\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{(x-1)^3}$....(1)

By multiplying $(x-1)^3$ **both sides, we get**

$$2x^2-3x+4=A(x-1)^2+B(x-1)+C......(2)$$
$$x=1$$
$$2(1)^2-3(1)+4=A(1-1)^2+B(1-D+C)$$
$$2-3+4=C$$
$$3=C$$
$$C=3$$
$$2x^2-3x+4=A(x^2-2x+1)+B(x-1)+C$$
$$2x^2-3x+4=Ax^2-2Ax+A+Bx-B+C$$
$$2x^2-3x+4=Ax^2+(-2A+B)x+(A-B+C)$$

By equating the co-efficient of x2, x and constant

$$A=2$$
$$-2A+B=-3$$
$$-2(2)+(B)=-3$$
$$-4+B=-3$$
$$B=-3+4$$
$$B=1$$

So, A=2, B=1 and C=3

Therefore, by putting their value in eq (1)

$$\frac{2x^2-3x+4}{(x-1)^3}=\frac{2}{x-1}+\frac{1}{(x-1)^2}+\frac{3}{(x-1)^3}$$

Hence; $$\frac{2x^2-3x+4}{(x-1)^3}=\frac{2}{x-1}+\frac{1}{(x-1)^2}+\frac{3}{(x-1)^3}$$

2. $\frac{5x^2-2x+3}{(x+3)^3}$

Solution

Let $\frac{5x^2-2x+3}{(x+3)^3}$ **=** $\frac{A}{x+2}+\frac{B}{(x+2)^2}+\frac{C}{(x-2)^3}$.......(1)

By multiplying $(x+2)^3$ **both sides, we get**

$$5x^2-2x+3=A(x+2)^2+B(x+2)+C......(2)$$
$$x=-2$$
$$5(-2)^2-2(-2)+3=A(-2+2)^2+B(-2+2)+C$$
$$20+4+3=A(0)+B(0)+C$$
$$C=27$$

And

$$5x^2-2x+3=A(x^2+4x+4)+Bx+2B+C$$
$$5x^2-2x+3=Ax^2+4Ax+4A+Bx+2B+C$$
$$5x^2-2x+3=Ax^2+(4A+B)+4A+2B+C$$

by equating the co-efficient of x², x and constant

$$A=5$$
$$4A+B=-2$$
$$4(5)+B=-2$$
$$20+B=-2$$
$$B=-2-20$$
$$B=-22$$
$$so;$$
$$A=5;B=-22;C=27$$

Therefore, by putting these values in eq (1)

$$\frac{5x^2-2x+3}{(x+2)^3}=\frac{5}{x+2}-\frac{22}{(x+2)^2}+\frac{27}{(x+2)^3}$$

3. $\frac{4x}{(x+1)^2(x-1)}$

Solution:

Let $\frac{4x}{(x+1)^2(x-1)}$ **=** $\frac{A}{x+1}+\frac{B}{(x+1)^2}+\frac{C}{x-1}$.....(1)

By multiplying $(x+1)^2(x-1)$ **both sides, we get**

$4x = A(x+1)^2(x-1)+B(x-2)+C(x+1)^2$.....(2)

$x = 1$

$4(1) = A(1+1)(1-1)+B(1-1)+C(1+1)^2$

$4 = A(0)+B(0)+C(2)^2$

$4 = 4C$

$C = \frac{4}{4} = 1$

and

$x = -1$

$4(-1) = A(-1+1)(-1-1)+B(-1-1)+C(-1+1)^2$

$-4 = A(0)(-2)+B(-2)+C(0)^2$

$-4 = -2B$

$B = -\frac{-4}{-2} = 2$

$4x = A(x^2-1)+Bx+B+C(x^2+2x+1)$

$4x = Ax^2-A+Bx+B+Cx^2+2Cx+C$

$4x = (A+C)x^2+(B+2C)x+(-A+B+C)$

By equating the co-efficient of x2, x and constant

$A+C=0$

$A+1=0$

$A=0-1$

$A=-1$

So,

A=-1; B=2 and C=1

Therefore, by putting these values in eq (1)

$$\frac{4x}{(x+1)^2(x-1)} = \frac{-1}{x+1} + \frac{2}{(x+1)^2} + \frac{1}{x-1}$$

4. $\frac{9}{(x+2)^2(x-1)}$

Solution

Let $\frac{9}{(x+2)^2(x-1)} = \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{x-1}$ ......(1)

By multiplying $(x+2)^2(x-1)$ **both sides, we get**

$9 = A(x+2)(x-1) + B(x-1) + C(x+2)^2$.....(2)

$x = -2$

$9 = A(-2+2)(-2-1) + B(-2-1) + C(-2+2)^2$

$9 = A(0)(-3) + B(-3) + C(0)^2$

$9 = -3B$

$B = -3$

And

x=1

$9 = A(1+2)(1-1) + B(1-1)^2 + C(1+2)^2$

$9 = A(3)(0) + B(0)^2 + C(3)^2$

$9 = 9C$

$C = 1$

$9 = A(x^2 + x - 2) + B(x-1) + B(x-1) + C(x^2 + 4x + 4)$

$9 = Ax^2 + Ax - 2A + Bx - B$

$9 = (A+C)x^2 + (A+B+4C)x + (-2A - B + 4C)$

By equation the co-efficient of x2, x and constant

A+C=0

A+1=0

A=-1

S0, A=-1; B=-3 and C=1

Therefore, by putting these values in equation

$$\frac{9}{(x+2)^2(x-1)}=\frac{-1}{x+2}+\frac{-3}{(x+2)^2}+\frac{1}{x-1}$$

OR

$$\frac{9}{(x+2)^2(x-1)}=\frac{1}{x-1}-\frac{1}{x+2}+\frac{3}{(x-2)^2}$$

5. $\dfrac{1}{(x-3)^2(x-1)}$

Solution:

$$\frac{1}{(x-3)^2(x-1)} = \frac{A}{(x-3)}+\frac{B}{(x-3)^2}+\frac{C}{x-1}.....(1)$$

By multiplying $(x-3)^2(x-1)$ **both sides, we get**

$$1=A(x-3)(x+1)+B(x+1)+C(x-3)^2......(2)$$

$$x=1$$

$$1=a(-2)(0)+B(0)+C(-2)^2$$

$$1=0+0+16C$$

$$C=\frac{1}{16}$$

And put x=3 in eq (2)

$$1=A(3-3)(3+1)+C(3-3)^2$$

$$1=A(0)(4)+B(4)+C(0)^2$$

$$B=\frac{1}{4}$$

$$1=A(x^2+2x-3)+B(x+1)+C(x^2-6x+9)$$

$$1=Ax^2+2Ax-3A+Bx+B+Cx^2-6Cx+9C$$

$$1=(A+C)x^2+(2A+B-6C)x+(-3A+B+9C)$$

By equating the co-efficient of x2, x and constant

A+C=0

$C = -A = \frac{1}{16}$

so,

$A = -\frac{1}{16}; B = \frac{1}{4}; C = \frac{1}{16}$

Therefore, by putting these values in eq (1)

$$\frac{1}{(x-3)^2(x+1)} = -\frac{1}{16(x-3)} + \frac{1}{4(x-3)^2} + \frac{1}{16(x+1)}$$

6. $\frac{x^2}{(x-2)(x-1)^2}$

Solution

Let $\frac{x^2}{(x-2)(x-1)^2} = \frac{A}{(x-2)} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2} \text{....(1)}$

By multiplying $(x-2)(x-1)^2$ **both sides we get**

$x^2 = A(x-1)^2 + B(x-2)(x-1)$

$x = 1$

$(1)^2 = A(x-1)^2 + B(x-2) + C(1-2)$

$1 = A(0)^2 + B(-1)(0) + C(-1)$

$1 = 0 + 0 - C$

$C = -1$

$x = 2$

$(2)^2 = A(2-1)^2 + B(2-2)(2-1) + C(2-2)$

$4 = A(1)^2 + B(0)(1) + C(0)$

$4 = A + 0 + 0$

$A = 4$

$x^2 = A(x^2 - 2x + 1) + B(x^2 - 3x + 2) + C(x-2)$

$x^2 = Ax^2 - 2Ax + A + Bx^2 - 3xB + 2B + Cx - 2C$

$x^2 = (A+B)x^2 + (-2A - 3B + C)x + (A + 2B - 2C)$

By equating the co-efficient of x2, x and constant

A+B=1

4+B=1

B=1-4=-3

B=-3

So,

A=4 ; B=-3 ; C=-1

Therefore, by putting these values in eq (1)

$$\frac{x^2}{(x-2)(x-1)^2} = \frac{4}{x-2} + \frac{-3}{x-1} + \frac{-1}{(x-1)^2}$$

Hence;

$$\frac{x^2}{(x-2)(x-1)^2} = \frac{4}{x-2} - \frac{3}{x-1} - \frac{1}{(x-1)^2}$$

Q7. $\frac{1}{(x-1)^2(x+1)}$

Solution

Let $\frac{1}{(x-1)^2(x+1)} = \frac{A}{x-1} + \frac{B}{x-1} + \frac{C}{x+1}$ ....(i)

By multiplying $(x-1)^2(x+1)$ **both sides we get.**

$$1 = A(x-1)(x+1) + B(x+1) + C(x-1)^2 \text{ ....}(ii)$$

Put x = 1 in eq. (ii)

$$1 = A(1-1)(1+1) + B(1+1) + C(1-1)^2$$
$$1 = A(0) + B(2) + C(0)$$
$$1 = 2B$$
$$B = \frac{1}{2}$$

And put x = -1 in eq, (ii)

$$1 = A(-1-1)(-1+1) + B(-1+1) + C(-1-1)^2$$
$$1 = A(0) + B(0) + C(-2)^2$$
$$1 = C(4)$$

$$C = \frac{1}{4}$$

By equating the coefficient of x^2 , x and constant

$$A + c = 0$$
$$A = 0 - C = -\frac{1}{4}$$
$$A = -\frac{1}{4}$$

So

$$A = -\frac{1}{4} \quad ; \quad B = \frac{1}{2} \quad ; \quad C = \frac{1}{4}$$

Therefore, by putting these values in eq. (i)

$$\frac{1}{(x-1)^2(x+1)} = \frac{-1/4}{x-1} + \frac{1/2}{x-1} + \frac{1/4}{x+1}$$

Hence

$$\frac{1}{(x-1)^2(x+1)} = -\frac{1}{4(x-1)} + \frac{1}{2(x-1)} + \frac{1}{4(x+1)}$$

Q8. $\frac{x^2}{(x-1)^3(x+1)}$

Let $\frac{x^2}{(x-1)^3(x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} + \frac{D}{(x+1)}$ ________ (i)

By multiplying $(x-1)^3(x+1)$ both sides we get

$$x^2 = A(x-1)^2(x+1) + B(x-1)(x+1) + C(x+1) + D(x-1)^3 \quad \_\_\_\_\_\_\_\_\_\_ (ii)$$

Put x = 1 in eq. (ii)

$$(1)^2 = A(1-1)^2(1+1) + B(1-1)(1+1) + C(1+1) + D(1-1)^3$$
$$1 = A(0) + B(0) + C(2) + D(0)$$
$$1 = C2$$
$$C = \frac{1}{2}$$

Put x = -1 in eq. (ii)

$$(-1)^2 = A(-1-1)^2(-1+1) + B(-1-1)(-1+1) + C(-1+1) + D(-1-1)^3$$
$$1 = A(0) + B(0) + C(0) + D(-2)^3$$
$$1 = D(-8)$$
$$D = -\frac{1}{8}$$

$$x^2 = A(x^2-2x+1)(x+1) + B(x^2+1) + C(x+1) + D(x^3-3x^2+3x-1)$$
$$x^2 = Ax^3 - Ax^2 - Ax + A + Bx^3 + Cx + B + C + Dx^3 - 3Dx^2 + 3Dx - 1$$
$$x^2 = (A+D)x^3 + (-A+B-3D)x^2 + (-A+C+3D)x + (A+B+C-D)$$

By equating the co-efficient of x^3, x^2, x and constant

$$A + D = 0$$
$$A = 0 - D = 0 - \left(\frac{-1}{8}\right)$$
$$A = \frac{1}{8}$$
$$and$$
$$-A + B - 3D = 1$$
$$-\frac{1}{8} + B - 3\left(\frac{1}{8}\right) = 1$$
$$B = 1 + \frac{1}{8} + \frac{3}{8}$$

$$B = \frac{8+1+3}{8}$$
$$B = \frac{12}{8}$$
$$B = 3/4$$

So,

$$A = \frac{1}{8} \quad ; \quad B = \frac{3}{4} \quad ; \quad C = \frac{1}{2} \quad ; \quad D = -\frac{1}{8}$$

Therefore, by putting these values in equation (i)

$$\frac{x^2}{(x-1)^3(x+1)} = \frac{1/8}{(x-1)} + \frac{3/4}{(x-1)^2} + \frac{1/2}{(x-1)^3} + \frac{-1/8}{(x+1)}$$

Hence,

$$\frac{x^2}{(x-1)^3(x+1)} = \frac{1}{8(x-1)} + \frac{3}{4(x-1)^2} + \frac{1}{2(x-1)^3} - \frac{1}{8(x+1)}$$

Q9. $\frac{x-1}{(x-2)(x+1)^3}$

Solution

$$\frac{x-1}{(x-2)(x+1)^3} = \frac{A}{(x-2)} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2} + \frac{D}{(x+1)^3} \_\_\_\_\_ (i)$$

By multiplying $(x-2)(x+1)^3$ **both sides we get**

$$x-1 = A(x+1)^3 + B(x-2)(x+1)^2 + C(x-2)(x+1) + D(x-2) \_\_\_\_\_\_\_ (ii)$$

Put x = -1 in eq.(ii)

$$-1-1 = A(-1+1)^3 + B(-1-2)(-1+1)^2 + C(-1-2)(-1+1) + D(-1-2)$$
$$-2 = A(0) + B(0) + C(0) + D(-3)$$

$D = \frac{-2}{-3}$

$D = \frac{2}{3}$

Put x = 2 in eq. (ii)

$2-1 = A(2+1)^3 + B(2-2)(2+1)^2 + C(2-2)(2+1) + D(2-2)$

$1 = A(3)^3 + B(0) + C(0) + D(0)$

$1 = A27$

$A = \frac{1}{27}$

$x-1 = A(x^3+3x^2+3x+1) + B(x-2)(x^2+2x+1) + C(x^2-x-2) + D(x-2)$

$x-1 = (Ax^3+3Ax^2+3Ax+A) + (Bx-B2)(Bx^2+2Bx+B) + (Cx^2-Cx-2C) + (Dx-2D)$

$x-1 = (A+B)x^3 + (3A+B+C)x^2 + (3A-3B-C+D)x + (A-2B-2C+D)$

By equating the co-efficient of x^3, x^2, x and constant

$A + B = 0$

$\frac{1}{27} + B = 0$

$B = \frac{-1}{27}$

and

$3\left(\frac{1}{27}\right) - 3\left(\frac{-1}{27}\right) - C + \frac{2}{3} = 1$

$\frac{3}{27} + \frac{3}{27} - C + \frac{2}{3} = 1$

$-C = \frac{9-6-1-1}{9}$

$C = -\frac{1}{9}$

So

$A = \frac{1}{27}$; $B = \frac{-1}{27}$; $C = \frac{-1}{9}$; $D = \frac{2}{3}$

Therefore, by putting these values in equation (i)

$$\frac{x-1}{(x-2)(x+1)^3}=\frac{1/27}{(x-2)}+\frac{-1/27}{(x+1)}+\frac{-1/9}{(x+1)^2}+\frac{2/3}{(x+1)^3}$$

Hence

$$\frac{x-1}{(x-2)(x+1)^3}=\frac{1}{27(x-2)}-\frac{1}{27(x+1)}-\frac{1}{9(x+1)^2}+\frac{2}{3(x+1)^3}$$

Q10. $\frac{4x^3}{(x^2-1)(x+1)^2}$

Solution

$$\frac{4x^3}{(x^2-1)(x+1)^2}=\frac{4x^3}{(x-1)(x+1)^3}$$

$$\frac{4x^3}{(x-1)(x+1)^3}=\frac{A}{(x-1)}+\frac{B}{(x+1)}+\frac{C}{(x+1)^2}+\frac{D}{(x+1)^3}\_\_\_\_\_\_(i)$$

By multiplying $(x-1)(x+1)^3$ **both sides we get**

$$4x^3=A(x+1)^3+B(x-1)(x+1)^2+C(x-1)(x+1)+D(x-1)\_\_\_\_\_\_(ii)$$

Put x = 1 in eq. (ii)

$$4(1)^3=A(1+1)^3+B(1-1)(1+1)^2+C(1-1)(1+1)+D(1-1)$$
$$4=A(2)^3+B(0)+C(0)+D(0)$$
$$4=A8$$
$$A=\frac{4}{8}$$
$$A=\frac{1}{2}$$

And put x= -1 in eq. (ii)

$$4(-1)^3 = A(-1+1)^3 + B(-1-1)(-1+1)^2 + C(-1-1)(-1+1) + D(-1-1)$$
$$-4 = A(0) + B(0) + C(0) + D(-2)$$
$$-4 = D(-2)$$
$$D = 2$$

$$4x^3 = A\left(x^3+3x^2+3x+1\right) + B(x-1)\left(x^2+2x+1\right) + C\left(x^2-1\right) + D(x-1)$$
$$4x^3 = A\left(x^3+3x^2+3x+1\right) + B\left(x^3+2x^2+x-x^2-2x-1\right) + C\left(x^2-1\right) + D(x-1)$$
$$4x^3 = (A+B)x^3 + (3A+B+C)x^2 + (3A-B+D)x + (A-B-C-D)$$

By equating the co-efficient of x^3, x^2, x and constant

$$A + B = 4$$
$$B = 4 - A$$
$$B = 4 - \frac{1}{2}$$
$$B = \frac{8-1}{2} = \frac{7}{2}$$
and
$$A - B - C - D = 0$$
$$\frac{1}{2} - \frac{7}{2} - C - 2 = 0$$
$$C = \frac{1-7-4}{2}$$
$$C = -5$$

So

$$A = 1/2 \quad ; \quad B = 7/2 \quad ; \quad C = -5 \quad ; \quad D = 2$$

Therefore, by putting the values in equation (i)

$$\frac{4x^3}{(x-1)(x+1)^3} = \frac{1/2}{(x-1)} + \frac{7/2}{(x+1)} + \frac{-5}{(x+1)^2} + \frac{2}{(x+1)^3}$$

Hence
$$\frac{4x^3}{(x-1)(x+1)^3} = \frac{1}{2(x-1)} + \frac{7}{2(x+1)} - \frac{5}{(x+1)^2} + \frac{2}{(x+1)^3}$$

Q11. $\frac{2x+1}{(x+3)(x-1)(x+2)^2}$

Solution

Let $\frac{2x+1}{(x+3)(x-1)(x+2)^2}=\frac{A}{(x+3)}+\frac{B}{(x-1)}+\frac{C}{(x+2)}+\frac{D}{(x+2)^2}$ ______ (i)

By multiplying $(x+3)(x-1)(x+2)^2$ **both sides we get**

$2x+1=A(x-1)(x+2)^2+B(x+3)(x+2)^2+C(x+3)(x-1)(x+2)+D(x+3)(x-1)$ ______ (ii)

Put x=1 in eq. (ii)

$$2(1)+1=A(1-1)(1+2)^2+B(1+3)(1+2)^2+C(1+3)(1-1)(1+2)+D(1+3)(1-1)$$
$$3=A(0)+B(4)(3)^2+C(0)+D(0)$$
$$3=B36$$
$$B=\frac{3}{36}$$
$$B=\frac{1}{12}$$

And put x=-2 in eq. (ii)

$$2(-2)+1=A(-2-1)(-2+2)^2+B(-2+3)(-2+2)^2+C(-2+3)(-2-1)(-2+2)+D(-2+3)(-2-1)$$
$$-4+1=A(0)+B(0)+C(0)+D(1)(-3)$$
$$-3=D(-3)$$
$$D=1$$

And put x = -3 in eq.(ii)

$$2(-3)+1=A(-3-1)(-3+2)^2+B(-3+3)(-3+2)^2+C(-3+3)(-3-1)(-3+2)+D(-3+3)(-3-1)$$
$$-6+1=A(-4)(-1)^2+B(0)+C(0)+D(0)$$
$$-5=A(-4)$$
$$A=5/4$$

$$2x+1=A(x-1)(x^2+4x+4)+B(x+3)(x^2+4x+4)+C(x^2+5x+6)(x-1)+D(x^2+2x-3)$$
$$2x+1=A(x^3+3x^2-4)+B(x^3+7x^2+16x+12)+C(x^3+4x^2+x+6)+D(x^2+2x-3)$$
$$2x+1=(A+B+C)x^3+(3A+7B+4C+D)x^2+(16B+C+2D)x+(-4A+12B)+6(6C-3D)$$

By equating the co-efficient of x^3, x^2, x and constant

$$A+B+C=0$$
$$\frac{5}{4}+\frac{1}{2}+C=0$$
$$\frac{15+1}{12}+C=0$$
$$C=\frac{-16}{12}$$
$$C=\frac{-4}{3}$$

So,

$A=\frac{5}{4}$; $B=\frac{1}{12}$; $C=\frac{-4}{3}$ **and** $D=1$

Therefore, by putting the values in equation (i)

$$\frac{2x+1}{(x+3)(x-1)(x+2)^2}=\frac{5/4}{(x+3)}+\frac{1/12}{(x-1)}+\frac{-4/3}{(x+2)}+\frac{1}{(x+2)^2}$$

Hence

$$\frac{2x+1}{(x+3)(x-1)(x+2)^2}=\frac{5}{4(x+3)}+\frac{1}{12(x-1)}-\frac{4}{3(x+2)}+\frac{1}{(x+2)^2}$$

Q12. $\frac{2x^4}{(x-3)(x+2)^2}$

Solution

$$\frac{2x^4}{(x-3)(x^2+4x+4)}=\frac{2x^4}{x^3+4x^2-4x-3x^2-12x-12}$$

$$= \frac{3x^4}{x^3 + x^2 - 8x - 12}$$

Since the fraction is improper because the order of numerator is 4 and denominator's order is 3. Convert the improper fraction to proper fraction.

$$x^3 + x^2 - 8x - 12 \overline{)\, 2x^4}$$
$$\underline{\pm 2x^4 \pm 2x^3 \mp 16x^2 \mp 24x}$$
$$+2x^3 + 16x^2 + 24x$$
$$\underline{\mp 2x^3 \mp 2x^2 \pm 16x + 24}$$
$$18x^3 + 8x + 24x$$

$$\frac{2x^4}{(x-3)(x+2)^2} = (2x-2) + \frac{18x^2 - 8x - 24}{(x-3)(x+2)^2}$$

$$\frac{18x^2 - 8x - 24}{(x-3)(x+2)^2} = \frac{A}{(x-3)} + \frac{B}{(x+2)} + \frac{C}{(x+2)^2} \text{ ________ } (i)$$

By multiplying $(x-3)(x+2)^2$ **both sides we get**

$$18x^2 + 8x - 24 = A(x+2)^2 + B(x-3)(x+2) + C(x-3) \text{ _______ } (ii)$$

Put x = 3 in eq. (ii)

$$18(3)^2 + 8(3) - 24 = A(3+2)^2 + B(3-3)(3+2) + C(3-3)$$
$$162 + 24 - 24 = A(5)^2 = B(0) + C(0)$$
$$162 = A25$$
$$A = \frac{162}{25}$$

Put x = -2 in eq. (ii)

$$18(-2)^2 + 8(-2) - 24 = A(-2+2)^2 + B(-2-3)(-2+2) + C(-2-3)$$
$$72 - 16 - 24 = A(0) + B(0) + C(-5)$$
$$32 = -5C$$
$$C = -\frac{32}{5}$$

$18x^2+8x-24=A\left(x^2+4x+4\right)+B\left(x^2-x-6\right)+C\left(x-3\right)$

$18x^2+8x-24=\left(Ax^2+4Ax+4A\right)+\left(Bx^2-Bx-6B\right)+\left(Cx-3C\right)$

$18x^2+8x-24=\left(A+B\right)x^2+\left(4A-B+C\right)x+\left(4-6B-3C\right)$

By equating the co-efficient of x^3, x^2, x **and constant**

$A+B=18$

$\frac{162}{25}+B=B$

$B=\frac{400-162}{25}$

$B=\frac{288}{25}$

So,

$A=\frac{162}{25}$; $B=\frac{288}{25}$; **and** $C=-\frac{32}{5}$

Therefore, putting these values in eq, (i)

$$\frac{18x^2-8x-24}{(x-3)(x+2)^2}=\frac{162/25}{(x-3)}+\frac{288/25}{(x+2)}+\frac{-32/5}{(x+2)^2}$$

Hence

$$\frac{18x^2-8x-24}{(x-3)(x+2)^2}=\frac{162}{25(x-3)}+\frac{288}{25(x+2)}-\frac{32}{5(x+2)^2}$$