EXERCISE 4.9

Solve the following system of equation:

1. $2x^2 = 6x + 3y^2$; $3x^2 - 5y^2 = 7$
2. $8x^2 = y^2$; $x^2 + 2y^2 = 19$
3. $2x^2 - 8 = 5y^2$; $x^2 - 13 = -2y^2$
4. $x^2 - 5xy + 6y^2 = 0$; $x^2 + y^2 = 45$
5. $12x^2 - 25xy + 12y^2 = 0$; $4x^2 + 7xy = 160$
6. $12x^2 - 11xy + 12y^2 = 0$; $2x^2 + 7xy = 60$
7. $x^2 - y^2 = 16$; $xy = 15$
8. $x^2 + xy = 9$; $x^2 - y^2 = 2$
9. $y^2 - 7 = 2xy$; $2x^2 + 3 = xy$
10. $x^2 + y^2 = 5$; $xy = 2$

1. $2x^2 = 6x + 3y^2; 3x^2 - 5y^2 = 7$

Solution

$$2x^2 = 6 + 3y^2$$
$$3(2x^2 = 6 + 3y^2)$$
$$6x^2 = 18 + 9y^2$$
$$3x^2 - 5y^2 = 7$$
$$2(3x^2 - 5y^2 = 7)$$

$6x^2 - 10y^2 = 14$
$6x^2 = 10y^2 + 14$
$18y^2 - 9y^2 = 10y^2 - 14$
$10y^2 - 9y^2 = 18 - 14$
$y^2 = 4$
$y = \pm 2$
$2x^2 = 6 + 3(2)^2$
$2x^2 = 6 + 12 = 18$
$2x^2 = 18$
$x^2 = 9$
$x = \pm 9$

Hence; $S.S = \{(\pm 3, \pm 2)\}$

or

$S.S = \{(3,2),(3,-2),(-3,2),(-3,-2)\}$

2. $8x^2 = y^2; x^2 + 2y^2 = 19$

Solution

$y^2 = 8x^2$
$x^2 + 2(8x^2) = 19$
$x^2 + 16x^2 = 19$
$17x^2 = 19$
$x^2 = \frac{19}{17}$
$x = \pm\sqrt{\frac{19}{17}}$
$y^2 = 8\left(\sqrt{\frac{19}{17}}\right)^2$

$= 8 \times \frac{19}{17} = \frac{136}{17}$

$= \pm\sqrt{\frac{136}{17}} = \pm\sqrt{\frac{38}{17}}$

Hence; $S.S = \left\{\left(\pm\frac{19}{17}, \pm 2\sqrt{\frac{38}{17}}\right)\right\}$

3. $2x^2 - 8 = 5y^2; x^2 - 13 = -2y^2$

Solution

$x^2 = 13 - 2y^2$

$2(13 - 12y^2) - 8 = 5y^2$

$26 - 4y^2 - 8 = 5y^2$

$18 = 5y^2 + 4y^2$

$9y^2 = 18$

$y^2 = \frac{18}{9} = 2$

$y = \pm\sqrt{2}$

$x^2 = 13 - 2(\pm\sqrt{2})^2$

$= 13 - (2)$

$= 13 - 4$

$= 9$

$x^2 = 9$

$x = \pm\sqrt{9} = \pm 3$

Hence; $S.S = \{\pm 3, \pm\sqrt{2}\}$

or

$S.S = \left\{\left(3, \sqrt{2}\right), \left(-3, \sqrt{2}\right), \left(3, -\sqrt{2}\right), \left(-3, -\sqrt{2}\right)\right\}$

4. $x^2-5xy+6y^2=0; x^2+y^2=45$

Solution

$$x^2-2xy-3xy+6y^2=0$$
$$x(x-2y)-3y(x-2y)=0$$
$$(x-3y)(x-2y)=0$$

Either x-3y=0 or x-2y=0

x=3y x=2y

By putting the value of x in equation

$$x^2+y^2=45$$
$$(3y)^2+y^2=45$$
$$9y^2+y^2=45$$
$$y^2=\frac{45}{10}=\frac{9}{2}$$
$$y=\pm\frac{3}{\sqrt{2}}$$
$$(2y)^2+y^2=45$$
$$4y^2+y^2=45$$
$$5y^2=45$$
$$y^2=\frac{45}{5}=9$$
$$y=\pm 3$$

therefore

$$x=3y=3\left(\pm\frac{3}{\sqrt{2}}\right)=\pm\frac{9}{\sqrt{2}}$$

$$Hence; \quad S.S=\left\{\left(\pm\frac{9}{\sqrt{2}},\frac{3}{\sqrt{2}}\right),(\pm 6,\pm 3)\right\}$$

5. $12x^2 - 16xy - 9xy + 12y^2 = 0; 4x^2 + 7y^2 = 148$

Solution

$12x^2 - 16xy - 9xy + 12y^2 = 0$

$4x(3x - 4y) - 3y(3x - 4y) = 0$

$(4x - 3y)(3x - 4y) = 0$

Either

$4x - 3y = 0 \quad \Rightarrow \quad 4x = 3y \quad \Rightarrow \quad x = 3/4\, y$

$3x - 4y = 0 \quad \Rightarrow \quad 3x = 4y \quad \Rightarrow \quad x = 3/4\, y$

Put the values of 'x' in equation ii

$4\left(\frac{3}{4}y\right) + 7y = 148$

$4 \times \frac{9}{16}y^2 + 7y^2 = 148$

$\frac{9}{4}y^2 + 7y^2 = 148$

$9y^2 + 28y^2 = 4(148) = 592$

$37y^2 = 592$

$y^2 = \frac{592}{37} = 16$

$y = \pm\sqrt{16} = \pm 4$

Therefore

$x = \frac{3}{4}y = \frac{3}{4}(\pm 4) = \pm 3$

And

$4\left(\frac{4}{3}y\right)^2 + 7y^2 = 148$

$4\frac{16}{9}y^2 + 7y^2 = 148$

$64y^2 + 63y^2 = 148(9)$

$127y^2 = 1332$

$$y^2 = \frac{1332}{127}$$

$$y = \pm\sqrt{\frac{1332}{127}} = \pm 6\sqrt{\frac{37}{127}}$$

Therefore

$$x = \frac{4}{3}\left(\pm 6\sqrt{\frac{37}{127}}\right)$$

$$= \pm 8\sqrt{\frac{37}{127}}$$

$$Hence; \qquad S.S. = \left\{\left(\pm 8\sqrt{\frac{37}{127}}, \pm 6\sqrt{\frac{37}{127}}\right), (\pm 3, \pm 4)\right\}$$

6. $12x^2 - 11xy + 12y^2 = 0$; $2x^2 + 7xy = 60$

Solution

$$12x^2 - 11xy + 12y^2 = 0$$

$$12x^2 - 8xy - 3xy + 12y^2 = 0$$

$$4x(3x - 2y) - y(3x - 2y) = 0$$

$$(4x - y)(3x - 2y) = 0$$

Either

$$4x - y = 0 \quad => \quad 4x = y \quad => \quad x = y/4$$

$$3x - 2 = 0 \quad => \quad 3x = 2y \quad => \quad x = 2/3\, y$$

By putting the value of 'x' in equation

$$2\left(\frac{y}{4}\right)^2 + 7\left(\frac{y}{4}\right)y = 60$$

$$\frac{2y^2}{16} + \frac{7}{4}y^2 = 60$$

$$\frac{y^2}{8} + \frac{7y^2}{y} = 60$$

$$\frac{y^2+14y^2}{8}=60$$

$$15y^2=(60)(8)$$

$$y^2=\frac{60(8)}{15}=32$$

$$y=\pm\sqrt{32}=\pm4\sqrt{2}$$

$$x=\frac{y}{4}=\frac{4\sqrt{2}}{4}=\pm\sqrt{2}$$

And

$$2\left(\frac{3}{2}y\right)^2+7\left(\frac{3}{2}y\right)y=60$$

$$\frac{8}{9}y^2+\frac{14}{13}y^2=60$$

$$\frac{8y^2+42y^2}{9}=60$$

$$50y^2=60$$

$$y^2=\frac{60}{50}=\frac{6}{5}$$

$$y=\pm\sqrt{\frac{6}{5}}$$

And

$$x=\frac{2}{3}y=\frac{2}{3}\left(\pm\sqrt{\frac{6}{5}}\right)$$

$$=\pm\sqrt{\frac{(4)(6)}{(9(5))}}=\pm\sqrt{\frac{8}{15}}=\pm2\sqrt{\frac{2}{15}}$$

$$Hence; \qquad S.S.=\left\{\left(\pm2\sqrt{\frac{2}{15}\pm\sqrt{\frac{6}{5}}}\right),\left(\pm\sqrt{2},\pm4\sqrt{2}\right)\right\}$$

7. $x^2-y^2=16 \quad ; \quad xy=15$

Solution

$xy = 15$ => x=15/y

By putting the value of 'x'

$$\left(\frac{15}{y}\right)^2 - y^2 = 16$$
$$\frac{225}{y^2} - y^2 = 16$$
$$225 - y^4 = 16y^2$$
$$225 - y^4 - 16y^2 = 0$$
$$y^4 + 25y^2 - 9y^2 - 225 = 0$$
$$y^2\left(y^2 - 25\right) - 9\left(y^2 - 25\right) = 0$$
$$\left(y^2 - 25\right)\left(y^2 - 9\right) = 0$$

Either

$$y^2 - 25 = 0 \quad \Rightarrow \quad y^2 = 25 \quad \Rightarrow \quad y = \pm 5$$
$$y^2 - 9 = 0 \quad \Rightarrow \quad y^2 = 9 \quad \Rightarrow \quad y = \pm 3$$

Therefore

$$x = \frac{15}{\pm 3} = \pm 5$$
$$x = \frac{15}{\pm 3} = \pm 3$$

$$Hence; \quad S.S. = \left\{\left(\pm 3, \pm 5\right), \left(\pm 5, \pm 3\right)\right\}$$

8. $x^2 + xy = 9 \quad ; \quad x^2 - y^2 = 2$

Solution

$$x^2 + xy = 9$$
$$x\left(x + y\right) = 9$$
$$x^2 - y^2 = 2$$
$$\left(x - y\right)\left(x + y\right) = 2$$

By dividing the equation.

$$\frac{(x-y)(x+y)}{x(x+y)}=\frac{2}{9}$$

$$\frac{x-y}{x}=\frac{2}{9}$$

$$9(x-y)=2x$$

$$9x-9y=2x$$

$$9x-2x=9y$$

$$7x=9y$$

$$x=\frac{9y}{7}$$

By putting the value of 'x'

$$\left(\frac{9}{7}y\right)^2+\left(\frac{9}{7}y\right)y=9$$

$$\frac{81}{49}y^2+\frac{9y^2}{7}=9$$

$$\frac{81y^2+63y^2}{49}=9$$

$$144y^2=(9)(49)$$

$$y^2=\frac{9(49)}{144}$$

$$y=\pm\sqrt{\frac{9(49)}{144}}\left(\pm\sqrt{\frac{49}{16}}\right)=\pm\frac{7}{4}$$

$$x=\pm\frac{9}{4}$$

Hence,

$$S.S.=\left\{\left(\pm\frac{9}{4},\pm\frac{7}{4}\right)\right\}$$

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$$S.S.=\left\{\left(\frac{9}{4},\frac{7}{4}\right),\left(\frac{9}{4},\frac{-7}{4}\right),\left(\frac{-9}{4},\frac{7}{4}\right),\left(\frac{-9}{4},\frac{-7}{4}\right)\right\}$$

9. $y^2 - 7 = 2xy \quad ; \quad 2x^2 + 3 = xy$

Solution

$y^2 - 7 = 2xy$

Multiplying by 3

$3y^2 - 6xy - 21 = 0$
$3x^2 - xy + 3 = 0$

Multiplying by 7

$14x^2 - 7xy + 21 = 0$

Adding

$14x^2 - 7xy + 21 = 0$

$+3y^2 - 6xy - 21 = 0$

$14x^2 + 3y^2 - 13xy = 0$
$14x^2 - 13xy + 3y^2 = 0$
$14x^2 - 6xy - 7xy + 3y^2 = 0$
$2x(7x - 3y) - y(7x - 3y) = 0$
$(7x - 3y)(2x - y) = 0$

Either

$2x - y = 0 \quad \Rightarrow \quad y = 2x$
$7x - 3y = 0 \quad \Rightarrow \quad 3y = 7x \quad \Rightarrow \quad y = \frac{7}{3}x.$

By putting the values of y in equation.

$(2x)^2 - 7 = 2(2x)x$
$4x^2 - 7 = 4x^2$
$4x^2 - 4x^2 = 7$

No solution

Let $y = \frac{7}{3}x$

$\left(\frac{7}{3}x\right)^2 - 7 = 2\left(\frac{7}{3}x\right)x$

$\frac{49}{9}x^2 - \frac{14}{3}x^2 = 7$

$\frac{49x^2 - 42x^2}{9} = 7$

$7x^2 = 7(9)$

$x^2 = \frac{7(9)}{7} = 9$

So,

$y = \frac{7}{3}(\pm 3) = \pm 7$

Hence; $S.S. = \{\pm 3, \pm 7\}$

or

$S.S. = \{(3,7),(-3,-7)\}$

10. $x^2 + y^2 = 5$; $xy = 2$

By putting the value of 'x'

$\left(\frac{y}{2}\right)^2 + y^2 = 5$

$\frac{y^2}{4} + y^2 = 5$

$\frac{5y^2}{4} = 5$

$y^2 = \frac{5}{5}(4) = 4$

$y = \pm\sqrt{4} = \pm 2$

$x = \pm\dfrac{2}{2} = \pm 1$

Hence; $S.S. = \{(1,2),(-1,-2)\}$

Or

$S.S = \{(\pm 1, \pm 2)\}$