EXERCISE 4.8

Solve the following system of equation.

1. $2x-y=4;2x^2-4xy-y^2=6$
2. $x+y=5;x^2+2y^2=7$
3. $3x+2y=7;3x^2=25+2y^2$
4. $x+y=5;\frac{2}{x}+\frac{3}{y}=2,x\neq 0,y\neq 0$
5. $x+y=a+b;\frac{a}{x}+\frac{b}{y}=2$
6. $3x+4y=25;\frac{3}{x}+\frac{4}{y}=2$
7. $(x-3)^2+y^2=5;2x=y+6$
8. $(x+3)^2+(y-1)^2=5;x^2+y^2+2x=9$
9. $x^2+(y+1)^2=18;(x+2)^2+y^2=21$
10. $x^2+y^2+6x=1;x^2+y^2+2(x+y)=3$

1. $2x-y=4;2x^2-4xy-y^2=6$

Solution

$2x-y=4$

$2x-4=y$

$2x^2-4x(2x-4)-(2x-4)^2=6$

$2x^2-8x^7+16x-4x^7-16+16x=6$

$$-10x^2+32x-16-6=0$$
$$-10x^2+32x-22=0$$
$$-2(5x^2-16x+11)=0$$
$$5x^2-16x+11=0$$
$$5x^2-5x-11x+11=0$$
$$5x(x-1)-11(x-1)=0$$
$$(5x-1)(x-1)=0$$

Either 5x-11=0 or x-1=0

$X=\frac{11}{5}$; x=1

Put the value of x in equation 1

$$y=2x-4$$
$$y=2\left(\frac{11}{5}\right)-4$$
$$y=\frac{22}{5}-4$$
$$y=\frac{22-20}{5}$$
$$y=\frac{2}{5}$$
$$y=2-4$$
$$y=-2$$

Hence $S.S=\left\{(1,-2),\left(\frac{11}{5},\frac{2}{5}\right)\right\}$

2. $x+y=5;x^2+2y^2=17$

Solution

x=5-y

Put the value of x in equation 2

$$(5-y)^2+2y^2=17$$
$$25+y^2-10y+2y^2=17$$
$$3y^2-10y+25-17=0$$
$$3y^2-10y+8=0$$
$$3y-6y-4y+8=0$$
$$3y(y-2)-4(y-2)=0$$
$$(y-2)(3y-4)=0$$

Either y-2=0 0r 3y-4=0

Y=2 $y=\frac{4}{3}$

Put the value of 'y'

X=5-2

X=3

And

$$x=5-\frac{4}{3}$$
$$=\frac{15-4}{3}$$
$$=\frac{11}{3}$$

Hence: $S.S=\left\{(3,2),\left(\frac{11}{3},\frac{4}{3}\right)\right\}$

3. $3x+2y=7;3x^2=25+2y^2.$

Solution

3x=7-2y

$$x=\frac{7-2y}{3}$$

Put the value of 'x' in equation 2

$$3\left(\frac{7-2y}{3}\right)^2 = 25+2y^2$$

$$\frac{3}{9}(49+4y^2-28y) = 25+2y^2$$

$$(4y^2-28y+49) = 3(25+2y^2)$$

$$4y^2-28y+49 = 75+6y^2$$

$$6y^2-4y^2+28y+75-49 = 0$$

$$2y^2+28y+26 = 0$$

$$2(y^2+14y+13) = 0$$

$$y^2+14y+13 = 0$$

$$y^2+13y+y+13 = 0$$

$$y(y+13)+1(y+13) = 0$$

$$(y+1)(y+13) = 0$$

Either y+1=0 or y+13=0

Y=-1 y=-13

Put the value of y in equation

$$x = \frac{7-2y}{3}$$

$$= \frac{7-2(-13)}{3}$$

$$= \frac{7+26}{3}$$

$$= \frac{33}{3}$$

$$= 11$$

Hence; $S.S = \{(3,-1),(11,-13)\}$

4. $x+5=5; \frac{2}{x}+\frac{3}{y}=2; x\neq 0; y\neq 0$

Solution

x=5-y

Put the value of x in equation 2

$$\frac{2}{5-y}+\frac{3}{y}=2$$

$$\frac{2y+3(5-y)}{y(5-y)}=2$$

$$2y+15-3y=2y(5-y)$$

$$15-y=10y-2y^2$$

$$2y^2-10y-y+15=0$$

$$2y^2-11y+15=0$$

$$2y^2-6y-4y+15=0$$

$$2y(y-3)-5(y-3)=0$$

$$(2y-5)(y-3)=0$$

Either 2y-5=0 or y-3=0

$$y=\frac{5}{2} \qquad y=3$$

Put the value of y in equation,

$$x=5-y$$

$$=5-3$$

$$=2$$

and

$$x=5-\frac{5}{2}$$

$$=\frac{10-5}{2}$$

$$=\frac{5}{2}$$

Hence; $S.S=\left\{(2,3),\left(\frac{5}{2},\frac{5}{2}\right)\right\}$

5. $x+y=a+b; \frac{a}{x}+\frac{b}{x}=2$

Solution

x=a+b-y

Putting the value of x in equation

$$\frac{a}{(a+b-y)} = \frac{b}{y} = 2$$

$$\frac{ay+b(a+b-y)}{y(a+b-y)} = 2$$

$$ay+ab+b^2-by = 2y(a+b-y)$$

$$ay+ab+b^2-by = 2ay+2by+2y^2$$

$$2y^2+ay-2ay-by-2by+ab \neq b^2 = 0$$

$$2y^2-ay-3by+ab \neq b^2 = 0$$

$$y = \frac{-\left[-(a+3b)\right] \pm \sqrt{\left[-(a+3b)\right]^2 - 4(2)(ab+b^2)}}{4}$$

$$= \frac{(a+3b) \pm \sqrt{a^2+9b^2+6ab-8ab-8b^2}}{4}$$

$$= \frac{(a+3b) \pm \sqrt{a^2+b^2-2ab}}{4}$$

$$= \frac{(a+3b) \pm \sqrt{(a-b)^2}}{4}$$

$$= \frac{(a+3b) \pm (a-b)}{4}$$

either

$$y = \frac{(a+3b)+(a-b)}{4}$$

$$= \frac{a+3b+a-b}{4}$$

$$= \frac{2a+2b}{4}$$

$$= \frac{2(a+b)}{4}$$

$$= \frac{a+b}{2}$$

$$= \frac{a+3b-a+b}{4}$$

$$= \frac{4b}{4}$$

$$= b$$

By using the value of y in equation

x=a+b-y

= a+b-b

=a

And

$$x = a+b-\left(\frac{a+b}{2}\right)$$

$$x = \frac{2a+2b-a-b}{2} = \frac{a+b}{2}$$

Hence; $S.S = \left\{(a,b),\left(\frac{a+b}{2},\frac{a+b}{2}\right)\right\}$

6. $3x+4y=25; \frac{3}{x}+\frac{4}{y}=2$

Solution

$$3x+4y=25$$

$$3x=25-4y$$

$$x=\frac{25-4y}{3}$$

Put the value of x in equation.

$$\frac{3}{\frac{25-4y}{3}}+\frac{4}{y}=2$$

$$\frac{9}{25-4y}+\frac{4}{y}=2$$

$$\frac{9y+4(25-4y)}{y(25-4y)}=2$$

$$9y+100-16y=2y(25-4y)$$

$$100-7y=50y-8y^2$$

$$8y^2-7y-50y+100=0$$

$$8y^2-57y+100=0$$

$$8y^2-32y-25y+100=0$$

$$8y(y-4)-25(y-4)=0$$

$$(8y-25)(y-4)=0$$

Either 8y-25=0 or y-4=0

$$y=\frac{25}{8} \qquad y=4$$

By using the value of y in equation

$$x=\frac{25-4y}{3}$$

$$x=\frac{25-4(4)}{3}$$

$$=\frac{25-16}{3}$$

$$=\frac{9}{3}$$

$$=3$$

and

$$x=\frac{25-4\left(\frac{25}{8}\right)}{3}$$

$$= \frac{50-25}{3(2)} = \frac{25}{6}$$

Hence; $S.S = \left\{(3,4), \left(\frac{25}{6}, \frac{25}{8}\right)\right\}$

7. $(x-3)^2 + y^2 = 5; 2x = y+6$

Solution

y=2x-6

Put the value of y in equation i

$(x-3)^2 + (2x-6)^2 = 5$

$x^2 + 9 - 6x + 4x^2 + 36 - 24x = 5$

$5x^2 - 30x + 45 - 5 = 0$

$5x^2 - 30x + 40 = 0$

$5(x^2 - 6x + 8) = 0$

$x^2 - 6x + 8 = 0$

$x^2 - 2x - 4x + 8 = 0$

$x(x-2) - 4(x-2) = 0$

$(x-4)(x-2) = 0$

Hence; $S.S = \{(2,-2), (4,2)\}$

8. $(x+3)^2 + (y-1)^2 = 5; x^2 + y^2 + 2x = 9$

Solution

$$(x+3)^2+(y-1)^2=5$$
$$x^2+6x+9+y^2+1-2y=5$$
$$x^2+y^2+6x-2y+5=0$$
$$x^2+y^2=9-2x$$
$$9-2x=6x-2y+5$$

$$4x-2y+14=0$$
$$2x-y+7=0$$
$$y=2x+7$$

Put the value of y in equation 2

$$x^2+(2x+7)^2+2x=9$$
$$x^2+4x^2+49+28x+2x=9$$
$$5x^2+30x+40=0$$
$$5(x^2+6x+8)$$
$$x^2+6x+8=0$$
$$x^2+2x+4x+8=0$$
$$(x+4)(x+2)=0$$

Either x+4=0 or x+2=0

x=-4 x=-2

Put the value of x in equation

$$y=2x+7$$
$$y=2(-4)+7=-8+7=-1$$
$$y=2(-2)+7=-7+7=3$$

Hence; $S.S=\{(-2,3),(-4,-1)\}$

9. $x^2+(y+1)^2=18;(x+2)^2+y^2=21$

Solution

$$x^2+(y+1)^2=18$$
$$x^2+y^2+2y+1=18$$
$$x^2+y^2=18-1-2y=17-2y$$
$$x^2+y^2=17-2y$$

and

$$(x+2)^2+y^2=21$$

$$x^2+4x+4+y^2=21$$
$$x^2+y^2=21-4-4x=17-4x$$
$$x^2+y^2=17-4x$$
$$-2y=-4x$$
$$y=2x$$

Put the value of y in equation 1

$$x^2+[2x+1]^2=18$$
$$x^2+4x^2+4x+1=18$$
$$5x^2+4x+1-18=0$$
$$5x^2+4x-17=0$$
$$x=\frac{-4\pm\sqrt{16-4(5)(-17)}}{2(5)}$$
$$=\frac{-4\pm\sqrt{16+340}}{10}$$
$$=\frac{-4\pm\sqrt{356}}{10}$$
$$=\frac{-2\pm\sqrt{89}}{5}$$

Put the value of x in equation y=2

$$y=2\left[\frac{-2\pm\sqrt{89}}{5}\right]=\frac{-4\pm2\sqrt{89}}{5}$$

Hence; $$S.S=\left\{\left(\frac{-2\pm\sqrt{89}}{5},\frac{-4\pm2\sqrt{89}}{5}\right)\right\}$$

10. $x^2+y^2+6x=1; x^2+y^2+2(x+y)=3$

Solution

$$x^2+y^2+6x=1$$
$$x^2+y^2=1-6x$$
$$x^2+y^2+2(x+y)=3$$
$$x^2+y^2=3-2(x+y)$$
$$3-2(x+y)=1-6x$$
$$3-2x-2y-1+6x=0$$
$$4x-2y+2=0$$
$$2x-y+1=0$$
$$y=2x+1$$

Put the value of y in equation i

$$x^2+(2x+1)^2+6x=1$$
$$x^2+4x^2+1+4x+6x=1$$
$$5x^2+10x=0$$
$$5x(x+2)=0$$

X=0 or x=-2

Put the value of x in equation

$$y=2x+1$$
$$=2(0)+1=1$$
$$y=2(-2)+1$$
$$=-4+1=-3$$

Hence; $S.S=\{(0,1),(-2,-3)\}$