# EXERCISE 4.7

**1. Discuss the nature of the following equation.**

**a)** $4x^2 + 6x + 1 = 0$

**b)** $x^2 - 5x + 6 = 0$

**c)** $2x^2 - 5x + 1 = 0$

**d)** $25x^2 - 30x + 9 = 0$

**a.** $4x^2 + 6x + 1 = 0$

**solution**

$4x^2 + 6x + 1 = 0$

a=4 ; b=6 ; c=1

discriminate $=b^2-4ac$

$=6^2-4(4)(1)$

$=36-16$

$=20$

Hence, roots are irrational and unequal

**b.** $x^2 - 5x + 6 = 0$

**Solution**

$x^2 - 5x + 6 = 0$

a=1 ; b =-5 ; c=6

Discriminate = $b^2$-4ac

=$(-5)^2$-4(1)(6)

=25-24

=1

Hence, roots are irrational and unequal

**c.** $2x^2 - 5x + 1 = 0$

**Solution**

$2x^2 - 5x + 1 = 0$

a=2 ; b=-5 ; c=1

Discriminate= $b^2$-4ac

=$(-5)^2$-4(2)(1)

=25-8

=1

Hence , roots are irrational and unequal .

**d.** $25x^2 - 30x + 9 = 0$

**solution**

$25x^2 - 30x + 9 = 0$

a=25 ; b=-30 ; c=9

Discriminate = $b^2$-4ac

=$(-30)^2$-4(25)(9)

=900-900

=0

Hence, roots are real and equal.

**2. Show that the roots of the following equations will be real:**

i. $x^2-2\left(m+\frac{1}{m}\right)x+3=0; m\neq 0$

ii. $(b-c)x^2+(c-a)x+(a-b)=0; a,b,c\in Q$

i. $x^2-2\left(m+\frac{1}{m}\right)x+3=0; m\neq 0$

**Solution**

a=1 ; b=-2 $\left(m+\frac{1}{m}\right)$ ; c=3

If roots of equation are real =$b^2$-4ac

Discrimination= $b^2$-4ac

$$=\left[-2\left(m+\frac{1}{m}\right)\right]^2-4(1)(3)$$

$$=4m^2+4\frac{1}{m^2}+8-12$$

$$=4m^2+4\frac{1}{m^2}-4$$

$$=4\left(m^2+\frac{1}{m^2}-1\right)$$

$$=4\left[\left(m^2+\frac{1}{m^2}-2\right)+1\right]$$

$$=4\left[\left(m^2-\frac{1}{m^2}\right)^2+1\right]>0$$

Hence , the roots are real .

b. $(b-c)x^2+(c-a)x+(a-b)=0; a,b,c \in Q$

**Solution**

a=b-c ; b=c-a ; c=a-b

If the roots of equation are real= $b^2$-4ac>0

Discrimination = $b^2$-4ac

$$\begin{aligned}
&= (c-a)^2 - 4(b-c)(a-b) \\
&= c^2 + b^2 - 2ac - 4\left[ab - b^2 - ac + bc\right] \\
&= c^2 + a^2 - 2ac - 4ab + 4b^2 + 4ac - 4bc \\
&= a^2 + 4b^2 + c^2 + 2ac - 4bc - 4bc \\
&= (a-2b+c)^2 > 0
\end{aligned}$$

Hence , roots are real.

**3. Show that the roots of the following equations will be rational:**

**a)** $(p+q)x^2 - px - q = 0$

**b)** $Px^2 - (p-q)x - q = 0$

**a)** $(p+q)x^2 - px - q = 0$

**Solution**

a=p+q ; b=-q ; c=-q

Discrimination = $b^2$-4ac

$$= (-p)^2 - 4(p+q)(-q)$$
$$= p^2 + 4pq + 4q^2$$
$$= (p+2q)^2$$

Hence, the roots are rational

**b)** $Px^2 - (p-q)x - q = 0$

**Solution**

a=p ; b= -(p-q) ; c=-q

Discrimination = $b^2$-4ac

$$= [-(p-q)]^2 - 4(p)(-q)$$
$$= p^2 + q^2 - 2pq + 4pq$$
$$= (p+q)^2$$

Hence, the roots are rational

**4. For what values of m will the roots of the following equations be equal?**

**a)** $(m+1)x^2 + 2(m+3)x + m + 8 = 0$

**b)** $x^2 - 2(1+3m)x + 7(3+3m) = 0$

**c)** $(1+m)x^2 - 2(1+3m)x + (1+8m) = 0$

**a.** $(m+1)x^2 + 2(m+3)x + m + 8 = 0$

**Solution:**

a=m+1 ; b=2 (m+3) ; c=m+8

discrimination= $b^2$-4ac

$$= \left[2\left(m+2\right)\right]^2 - 4(m+1)(m+8)$$
$$= 4(m^2+6m+9) - 4(m^2+9m+8)$$
$$= -12m - 4$$

If the roots are equal then discrimination =0

-12m-4=0

12m+4=0

$$m = \frac{-4}{12} = \frac{-1}{3}$$

Hence, the value of m is $\frac{-1}{3}$

**b.x²-2(1+3m)x+7(3+3m)=0**

**Solution**

a=1 ; b=-2 (1+3m) ; c=7(3+2m)

discrimination= $b^2$-4ac

$$= \left[-2(1+3m)\right]^2 - 4(1)(7(3+2m))$$
$$= 4(1+9m^2+6m) - 28(3+2m)$$
$$= 4+36m^2+24m-84-56m$$
$$= -4(9m^2-8m-20)$$

If the roots are equal then

Discrimination = 0

$9m^2$-8m-20=0

$$m = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(9)(-20)}}{2(9)}$$

$$= \frac{8 \pm \sqrt{(-8)^2 - 4(9)(-20)}}{2(9)}$$

$$= \frac{8 \pm \sqrt{784}}{18}$$

$$= \frac{8 \pm 28}{18}$$

$$m = \frac{8+28}{18} \qquad \text{or} \qquad m = \frac{8-28}{18}$$

$$m = \frac{36}{18} \qquad \qquad m = \frac{-20}{18}$$

$$m = 2 \qquad \qquad m = \frac{-10}{9}$$

Hence, the value of 'm' is 2, $\frac{-10}{9}$

**c.(1+m)x²-2(1+3m)x+(1+8m)=0**

**Solution**

a=(1+m) ; b=-2(1+3m) ; c=1+8m

Discrimination = $b^2$-4ac

$$= \left[-2(1+3m)\right]^2 - 4(1+m)(1+8m)$$
$$= 4(1+9m^2+6m) - 4(1+m)(1+8m)$$
$$= 4+36m^2+24m-4-36m-32m^2$$
$$= 4m^2-12m$$
$$= 4m(m-3)$$

If the roots are equal then

Discrimination=0

4m(m-3)=0

M=0 or m=3

Hence, the value of m is 0,3

5. **Show that the roots $x^2+(mx+c)^2=a^2$ will be equal , if $c^2=a^2(1+m^2)$**

**Solution**

$$x^2+(mx+c)^2=a^2$$
$$x^2+(m^2x^2+c^2+2mxc)=a^2$$
$$x^2+m^2x^2+2mxc+c^2-a^2=0$$
$$(1+m^2)x^2+2mcx+c^2-a^2=0$$
$$a=(1+m^2);b=2mc;c=c^2-a^2$$
$$D=b^2-4ac$$
$$=(2mc)^2-4(1+m^2)(c^2-a^2)$$
$$=4m^2c^2-4(c^2-a^2+m^2c^2+4a^2m^2$$
$$=4a^2+4a^2m^2-4c^2$$

If the roots are equal then

Discrimination=0

$$4a^2+4a^2m^2-4c^2=0$$
$$a^2(1+m^2)-c^2=0$$
$$a^2(1+m^2)=c^2$$

Hence , it is required to proved.

6. **Show that the roots of $(mx+c)^2=4ax$ will be equal, if $c=\frac{a}{m};m\neq 0$**

**Solution**

$$(mx+c)^2=4ax$$
$$m^2x^2+2mcx+c^2=4ax$$
$$m^2x^2+(2mc-4a)x+c^2=0$$

$a = m^2; b = 2mc - 4a; c = c^2$
$D = b^2 - 4ac$
$= (2mc - 4a)^2 - 4(m^2)(c)^2$
$= 4m^2c^2 + 16a^2 - 16mca - 4m^2c^2$
$= 16a^2 - 16mca$
$= 16a(a - mc)$

If the roots are equal than

Discrimination =0

16a(a-mc)=0

a-mc=0

$c = \frac{a}{m}$

Hence , it is required to proved

**7. Prove that $\frac{x^2}{a^2} + \frac{(mx+c)^2}{b^2} = 1$ will have equal roots , if $c^2 = a^2m^2 + b^2; a \neq 0, b \neq 0$**

**Solution**

$$\frac{x^2}{a^2} + \frac{(mx+c)^2}{b^2} = 1$$
$$\frac{x^2}{a^2} + \frac{m^2x^2 + c^2 + 2mcx}{b^2} = 1$$
$$\frac{x^2}{a^2} + \frac{m^2x^2}{b^2}c + \left(\frac{c^2}{b^2} - 1\right) = 0$$
$$\left(\frac{1}{a^2} + \frac{m^2}{b^2}\right)x^2 + \frac{2mc}{b^2}c + \left(\frac{c^2}{b^2} - 1\right) = 0$$
$$a = \frac{1}{a^2} + \frac{m^2}{b^2}; b = \frac{2mc}{b^2}; c = \frac{c^2}{b^2} - 1$$

$D = b^2 - 4ac$

$= \left(\frac{2mc}{b^2}\right) - 4\left(\frac{1}{a^2} + \frac{m^2}{b^2}\right)\left(\frac{c^2}{b^2} - 1\right)$

If the roots are equal then

Discrimination =0

$\frac{4m^2c^2}{b^4} - 4\left(\frac{b^2 + a^2m^2}{a^2b^2}\right)\left(\frac{c^2 - b^2}{b^2}\right) = 0$

$\frac{4}{a^2b^4}\left[a^2m^2c^2 - \frac{(b^2 + a^2m^2)(c^2 - b^2)}{a^2}\right] = 0$

$a^2m^2c^2 - \left[b^2c^2 - b^4 + a^2m^2c^2 - a^2b^2m^2\right] = 0$

$-b^2c^2 + b^4 + a^2b^2m^2 = 0$

$b^2(-c^2 + b^2 + a^2m^2) = 0$

$-c^2 + b^2 + a^2m^2 = 0$

$b^2 + a^2m^2 = c^2$

Hence, it is required to proved .

8. **Show that the roots of the equation** $(a^2 - bc)x^2 + 2(b^2 - ca)x + (c^2 - ab) = 0$ **will be equal, if either** $a^3 + b^3 + c^3 = 3abc$ **or b=0.**

**Solution**

$(a^2 - bc)x^2 + 2(b^2 - ca)x + (c^2 - ab) = 0$

$a = a^2 - bc; b = 2(b^2 - ca); c = c^2 - ab$

$D = b^2 - 4ac$

$= \left[2\left(b^2 - ca\right)\right]^2 - 4(a^2 - bc)(c^2 - ab)$

$= 4\left(b^4 + c^2a^2 - 2b^2ca\right) - 4(a^2c^2 - a^3b - bc^3 + ab^2c)$

$= 4\left[b^4 + c^2a^2 - 2b^2ca - a^2c^2 + a^3b + bc^3 - ab^2c\right]$

If the roots are equal then

D=0

$$4\left[b^4 - 3b^2ca + a^3b + bc^3\right] = 0$$

$$b\left(b^3 - 3bca + a^3 + c^3\right) = 0$$

$$b^3 + a^3 + c^3 - 3bca = 0$$

$$a^3 + b^3 + c^3 - 3abc = 0$$

Hence, it is required to proved.