# EXERCISE 4.6

**If $\alpha$ , $\beta$ are the roots of 2x²-2x+4=0, find the values of**

1. $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$
2. $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$
3. $\alpha^4+\beta^4$
4. $\alpha^3+\beta^3$
5. $\frac{1}{\alpha^3}+\frac{1}{\beta^3}$
6. $\alpha^2-\beta^2$

**1.If $\alpha$ and $\beta$ are roots of equation.**

**Solution:**

$$3x^2-2x+4=0$$

$$\alpha+\beta=+\frac{2}{3},\alpha\beta=\frac{4}{3}$$

$$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{\beta^2+\alpha^2}{\alpha^2\beta^2}$$

$$=\frac{\alpha^2+\beta^2+2\alpha\beta-2\alpha\beta}{\alpha^2\beta^2}$$

$$=\frac{(\alpha+\beta)^2-2(\alpha\beta)}{(\alpha\beta)^2}$$

$$= \frac{\left(\frac{2}{3}\right)^2 - 2\left(\frac{4}{3}\right)}{\left(\frac{4}{3}\right)^2}$$

$$= \frac{\frac{4}{9} - \frac{8}{3}}{4}$$

$$= \frac{\frac{4-24}{9}}{\frac{4}{3}}$$

$$= \frac{-20 \times 3}{36}$$

$$= \frac{-20}{12}$$

$$= \frac{-5}{3}$$

**Hence;** $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = -\frac{5}{3}$

**2. let $\alpha$ and $\beta$ are roots of equation.**

$$\alpha + \beta = \frac{2}{3}$$

*and*

$$\alpha\beta = \frac{4}{3}$$

*then*

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$$

$$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$= \frac{\left(\frac{2}{3}\right)^2 - 2\left(\frac{4}{3}\right)}{\left(\frac{4}{3}\right)}$$

$$=\frac{\frac{4}{9}-\frac{8}{3}}{\frac{4}{3}}$$

$$=\frac{4-24}{9}+\frac{4}{3}$$

$$=\frac{-20}{9}\times\frac{3}{4}$$

$$=\frac{-5}{3}$$

**Hence,** $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{-5}{3}$

**3. Let $\alpha$ and $\beta$ are roots of equation.**

$$\alpha+\beta=\frac{2}{3}$$

*and*

$$\alpha\beta=\frac{4}{3}$$

**Then** $\alpha^4+\beta^4=(\alpha^2)^2+(\beta^2)^2+2\alpha^2\beta^2-2\alpha^2\beta^2$

$$=(\alpha^2)^2+(\beta^2)^2-2(\alpha\beta)^2$$

$$=\left[(\alpha+\beta)^2-2\alpha\beta\right]^2-2(\alpha\beta)^2$$

$$=\left[\left(\frac{2}{3}\right)^2-2\left(\frac{4}{3}\right)\right]^2-2\left(\frac{4}{3}\right)^2$$

$$=\left[\frac{4}{9}-\frac{8}{3}\right]^2-2\left(\frac{16}{9}\right)$$

$$=\left[\frac{4-24}{9}\right]^2-\frac{32}{9}$$

$$= \left[\frac{-20}{9}\right]^2 - \frac{32}{9}$$

$$= \frac{400}{81} - \frac{32}{9}$$

$$= \frac{400-288}{81}$$

$$= \frac{112}{81}$$

**Hence,** $\alpha^4 + \beta^4 = \frac{112}{81}$

**4. If Let $\alpha$ and $\beta$ are the roots of equation.**

**Solution**

$$\alpha + \beta = \frac{2}{3}$$

*and*

$$\alpha\beta = \frac{4}{3}$$

**Therefore**

$$(\alpha^3 + \beta^3) = (\alpha + \beta)\left[(\alpha+\beta)^2 - 3\alpha\beta\right]$$

$$= \left(\frac{2}{3}\right)\left[\left(\frac{2}{3}\right)^2 - 3\left(\frac{4}{3}\right)\right]$$

$$= \left(\frac{2}{3}\right)\left[\frac{4}{9} - \frac{12}{3}\right]$$

$$= \frac{2}{3}\left[\frac{4-36}{9}\right]$$

$$= \frac{-64}{27}$$

**Hence ,** $\alpha^3 + \beta^3 = \frac{-64}{27}$

**5. Let $\alpha$ and $\beta$ are the roots of equation.**

**Solution**

$$\alpha + \beta = \frac{2}{3}$$

*and*

$$\alpha\beta = \frac{4}{3}$$

**Therefore,**

$$\frac{1}{\alpha^3} + \frac{1}{\beta^3} = \frac{\alpha^3 + \beta^3}{\alpha^3\beta^3}$$

$$= \frac{(\alpha^3 + \beta^3)}{(\alpha\beta)^3}$$

$$= \frac{\left(\frac{-64}{27}\right)}{\left(\frac{4}{3}\right)^3}$$

$$= \frac{\left(\frac{-64}{27}\right)}{\left(\frac{64}{27}\right)}$$

$$= -1$$

**Hence ,** $\frac{1}{\alpha^3} + \frac{1}{\beta^3} = -1$

**6. Let $\alpha$ and $\beta$ are roots of equation.**

**Solution**

$$\alpha + \beta = \frac{2}{3}$$

*and*

$$\alpha\beta = \frac{4}{3}$$

**Therefore** $\alpha^2 - \beta^2 = (\alpha + \beta)(\alpha - \beta)$

**And** $\alpha - \beta = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$

$$= \sqrt{\left(\frac{2}{3}\right)^2 - 4\left(\frac{4}{3}\right)}$$

$$= \sqrt{\frac{4}{9} - \frac{16}{3}}$$

$$= \sqrt{\frac{4 - 48}{9}}$$

$$= \sqrt{\frac{-44}{9}}$$

$$= \frac{2}{3}\sqrt{-11}$$

$$\alpha^2 - \beta^2 = (\alpha + \beta)(\alpha - \beta)$$

$$= \left(\frac{2}{3}\right)\left(\frac{2}{3}\sqrt{-11}\right)$$

$$= \frac{4}{9}\sqrt{-11}$$

**Hence ;** $\alpha^2 - \beta^2 = \frac{4}{9}\sqrt{-11}$

**2. If $\alpha, \beta$ are the roots of $x^2$-px –p-c=0, prove that** $(1+\alpha)(1+\beta) = 1 - c$

**Solution**

$\alpha + \beta = \frac{p}{1}$

$\alpha\beta = -\frac{(p+c)}{1}$

$L.H.S = (1+\alpha)(1+\beta)$

$= 1 + \beta + \alpha + \alpha\beta$
$= 1 + (\alpha + \beta) + (\alpha\beta)$
$= 1 + (p) + [-(p+c)]$
$= 1 + p - p - c$
$= 1 - c$
$= R.H.S$

**Hence ,** $(1+\alpha)(1+\beta) = 1 - c$

**3. Find the condition that one roots of $x^2+px+q=0$ is**

- **i. double the other**
- **ii. square of the other**
- **iii. additive inverse of the other**
- **iv. multiplicative inverse of the other.**

**1.Let $\alpha, \beta$ are the roots of equation.**

**Solution**

$\alpha + \beta = -P$
&
$\alpha\beta = q$

**i. double the other.**

$\alpha ; 2\alpha$

$\alpha + 2\alpha = -P$

$\Rightarrow a = -\frac{-p}{3}$

$and(\alpha)(2\alpha) = 2\alpha^2 = q$

$2\left(\frac{-p}{3}\right)^2 = q$

$\frac{2}{9}p^2 = q$

$\Rightarrow 2p^2 = 9q$

$Hence; \ 2P^2 = 9q$

**ii. Square of the other**

$\alpha ; \alpha^2$

$and$

$\alpha.\alpha^2 = \alpha^3 = q$

$\Rightarrow a = (q)^{\frac{1}{3}}$

$q^{\frac{1}{3}} + q^{\frac{2}{3}} = -P$

$\left[q^{\frac{1}{3}} + q^{\frac{2}{3}}\right]^3 = (-P)^3$

$q + q^2 + 3q(-P) = -P^3$

$q^2 + q - 3Pq + P^3 = 0$

$Hence; \ q^2 + q - 3Pq + P^3 = 0$

**iii. Additive inverse of other**

$$\alpha + (-\alpha) = -P$$
$$and$$
$$(\alpha)(-\alpha) = q$$
$$\alpha - \alpha = -P$$
$$0 = -P$$
$$\Rightarrow P = 0$$

$$Hence; \quad P = 0$$

**iv. Multiplicative inverse of other**

$$\alpha, \frac{1}{\alpha}$$
$$\alpha.\frac{1}{\alpha} = -P$$
$$and$$
$$\alpha\frac{1}{\alpha} = q$$
$$1 = q$$

$$Hence; \quad q = 1$$

**If $\alpha$ and $\beta$ are roots of equation.**

$$3x^2 - 2x + 4 = 0$$
$$\alpha + \beta = \frac{2}{3}$$
$$\alpha\beta = \frac{4}{3}$$

**4. If the roots of the equation $x^2$-px+q=0 differ by unity, prove that**

**$P^2$=4q+1.**

**Solution**

Let $\alpha$ and $\beta$ be the roots of equation.

$$\alpha+\beta=P$$
*and*
$$\alpha\beta=q$$
*roots*
$$\alpha \,\&\, \alpha-1$$
$$\alpha+\alpha-1=P$$
$$2\alpha-1=P$$

$$2\alpha=P+1$$
$$\Rightarrow a=\frac{P+1}{2}$$
*and*
$$\alpha(\alpha-1)=\alpha^2-\alpha$$

$$=\left(\frac{P+1}{2}\right)^2-\left(\frac{P+1}{2}\right)$$
$$=\left(\frac{P+1}{2}\right)\left[\frac{P+1-2}{2}\right]$$
$$=\left(\frac{P+1}{2}\right)\left(\frac{P-1}{2}\right)$$
$$=\frac{P^2-1}{4}$$
$$\alpha(\alpha-1)=q$$
$$\frac{P^2-1}{4}=q$$
$$\Rightarrow P^2-1=4q$$

**5. Find the condition that $\frac{a}{x-a}+\frac{b}{x-b}=$ may have roots equal in magnitude but opposite in signs.**

**Solution**

$$\frac{a}{x-a}+\frac{b}{x-b}=5$$

$$a(x-b)+b(x-a)=5(x-a)(x-b)$$
$$ax-ab+bx-ab=5(x^2-ax-bx+ab)$$
$$ax-ab+bx-ab=5x^2-5ax-5bx+5ab$$
$$5x^2-5ab-ax-5bx-bx+5ab-2ab=0$$
$$5x^2-6ax-6bx+3ab=0$$
$$5x^2-(6a+6b)x+3ab=0$$

Sum of roots = $\left[-\frac{(6a+6b)}{5}\right]=\frac{6a+6b}{5}$

Product of roots **=** $\frac{3ab}{5}$

**Let** $\alpha$ **&** $\beta$ are the of equation

$$\beta=-\alpha$$
$$\alpha+\beta=\alpha+(-\alpha)=\frac{6a+6b}{5}$$
$$0=\frac{6a+6b}{5}$$
$$a+b=0$$

Hence; condition of equation is

a+b=0

**6. If the roots of Px²+qx+q=0 are $\alpha$ and $\beta$ then prove that**

$$\sqrt{\frac{\alpha}{\beta}}+\sqrt{\frac{\beta}{\alpha}}+\sqrt{\frac{q}{p}}=0$$

**Solution**

Let $\alpha$ and $\beta$ be the roots of equation

$$\alpha+\beta=\frac{-q}{p};\alpha\beta=\frac{q}{p}$$

$$\frac{\alpha+\beta}{\sqrt{\alpha\beta}}=\frac{\frac{-q}{p}}{\sqrt{\frac{q}{p}}}$$

$$\frac{\alpha}{\sqrt{\alpha\beta}}+\frac{\beta}{\sqrt{\alpha\beta}}=-\frac{\frac{-q}{p}}{\sqrt{\frac{q}{p}}}$$

$$\sqrt{\frac{\alpha}{\beta}}+\sqrt{\frac{\beta}{\alpha}}=-\sqrt{\frac{q}{p}}$$

$$\sqrt{\frac{\alpha}{\beta}}+\sqrt{\frac{\beta}{\alpha}}+\sqrt{\frac{q}{p}}=0$$

**Hence proved** $\sqrt{\frac{\alpha}{\beta}}+\sqrt{\frac{\beta}{\alpha}}+\sqrt{\frac{q}{p}}=0$

**7.If $\alpha$ , $\beta$ are the roots of the equation ax$^2$+bx +c=0 , from the equation whose roots are**

- **i.** $\alpha^2,\beta^2$
- **ii.** $\frac{1}{\alpha},\frac{1}{\beta}$
- **iii.** $\frac{1}{\alpha^2},\frac{1}{\beta^2}$
- **iv.** $\alpha^3,\beta^3$
- **v.** $\frac{1}{\alpha^3},\frac{1}{\beta^3}$
- **vi.** $\alpha+\frac{1}{\alpha},\beta+\frac{1}{\beta}$

**vii.** $(\alpha-\beta)^2, (\alpha+\beta)^2$

**viii.** $-\frac{1}{\alpha^3}, -\frac{1}{\beta^3}$

**Let $\alpha$ and $\beta$ be the roots of equation**

$$\alpha+\beta=\frac{-b}{a} \; \& \; \alpha\beta=\frac{c}{a}$$

**1.** $\alpha^2+\beta^2$

**Solution**

$$S=\alpha^2+\beta^2$$
$$=(\alpha+\beta)^2-2\alpha\beta$$
$$=\left(-\frac{b}{a}\right)^2-2\left(\frac{c}{a}\right)$$
$$=\frac{b^2}{a^2}-\frac{2c}{a}$$
$$=\frac{b^2-2ac}{a^2}$$
$$p=\alpha^2+\beta^2=(\alpha\beta)^2=\left(\frac{c}{a}\right)^2=\frac{c^2}{a^2}$$

**Therefore, equation**

$$x^2+\left[-\frac{\left(b^2-2ac\right)}{a^2}\right]x+\frac{c^2}{a^2}=0$$

$$Hence, \quad a^2x^2-(b^2-2ac)x+c^2=0$$

**2.** $\frac{1}{\alpha}, \frac{1}{\beta}$

**Solution**

$$s = \frac{1}{\alpha} + \frac{1}{\beta}$$

$$= \frac{\alpha + \beta}{\alpha\beta}$$

$$= \frac{\frac{-b}{a}}{\frac{c}{a}}$$

$$= \frac{-b}{c}$$

$$P = \frac{1}{\alpha} . \frac{1}{\beta}$$

$$= \frac{\alpha + \beta}{\alpha\beta}$$

$$= \frac{1}{\frac{c}{a}}$$

$$= \frac{a}{c}$$

**Therefore equation**

$$x^2 + \left(-\frac{b}{c}\right)x + \frac{a}{c} = 0$$

$$Hence; \quad cx^2 - bx + a = 0$$

**3.** $\frac{1}{\alpha^2}, \frac{1}{\beta^2}$

**Solution**

$$s = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$$

$$= \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2}$$

$$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$$

$$= \frac{\left(\frac{-b}{a}\right)^2 - 2\left(\frac{c}{a}\right)}{\left(\frac{c}{a}\right)^2}$$

$$= \frac{\frac{b^2}{a^2} - 2\frac{c}{a}}{\frac{c^2}{a^2}}$$

$$= \frac{b^2 - 2ac}{a^2\left(\frac{c^2}{a^2}\right)}$$

$$= \frac{b^2 - 2ac}{c^2}$$

$$P = \frac{1}{\alpha^2} \cdot \frac{1}{\beta^2}$$

$$= \frac{1}{\left(\frac{c}{a}\right)^2}$$

$$= \frac{a^2}{a^2}$$

**Therefore equation**

$$x^2 - \left(\frac{b^2 2ac}{c^2}\right)x + \frac{a^2}{c^2} = 0$$

$$Hence; \quad c^2x^2 - (b^2 - 2ac)x + a^2 = 0$$

**4.** $\alpha^3, \beta^3$

**Solution**

$S = \alpha^3 + \beta^3$

$= (\alpha + \beta)\left[(\alpha + \beta)^2 - 3\alpha\beta\right]$

$= \left(\frac{-b}{a}\right)\left[\left(\frac{-b}{a}\right)^2 - 3\left(\frac{c}{a}\right)\right]$

$= \frac{-b}{a}\left[\frac{b^2}{a^2} - \frac{3c}{a}\right]$

$= \frac{-b}{a}\left[\frac{b^2 - 3ac}{a^3}\right]$

$= \frac{-b(b^2 - 3ac)}{a^3}$

$P = \alpha^3\beta^3$

$= (\alpha\beta)^3$

$= \left(\frac{c}{a}\right)^3$

$= \frac{c^3}{a^3}$

**Therefore**

$x^2 + \frac{b(b^2 - 3ac)x}{a^3} + \frac{c^3}{a^3} = 0$

*Hence;* $a^3x^3 + b(b^2 - 3ac)x + c^3 0$

**5.** $\frac{1}{\alpha^3}, \frac{1}{\beta^3}$

**Solution**

$S = \frac{1}{\alpha^3} + \frac{1}{\beta^3}$

$= \frac{\alpha^3 + \beta^3}{(\alpha\beta)^3}$

$$= \frac{\frac{-b(b^2-3ac)}{a^3}}{\frac{c^3}{a^3}}$$

$$= \frac{-b(b^2-3ac)}{c^3}$$

$$P = \frac{1}{\alpha^3} \cdot \frac{1}{\beta^3}$$

$$= \frac{1}{(\alpha\beta)^3}$$

$$= \frac{c^3}{a^3}$$

$$= \frac{c^3}{a^3}$$

**Therefore**

$$x^2 - \left[\frac{-b(b^2-3ac)}{c^3}\right]x + \frac{c^3}{a^3} = 0$$

$$Hence; \quad e^3x^2 + b(b^2-3ac)x + a^3 = 0$$

**6.** $\alpha + \frac{1}{\alpha}; \beta + \frac{1}{\beta}$

**Solution**

$$S = \alpha + \frac{1}{\alpha} + \beta + \frac{1}{\beta}$$

$$= \alpha + \beta + \frac{\alpha+\beta}{\alpha\beta}$$

$$= \left(\frac{-b}{a}\right) + \frac{\left(\frac{-b}{a}\right)}{\left(\frac{c}{a}\right)}$$

$$= \left(\frac{-b}{a}\right) - \frac{b}{c}$$

$$= -b\left[\frac{c+a}{ac}\right]$$

$$P = \left(\alpha + \frac{1}{\alpha}\right)\left(\beta + \frac{1}{\beta}\right)$$

$$= \alpha\beta + \frac{\alpha}{\beta} + \frac{\beta}{\alpha} + \frac{1}{\alpha\beta}$$

$$= \alpha\beta + \frac{1}{\alpha\beta} + \frac{\alpha^2 + \beta^2}{\alpha\beta}$$

$$= \left(\frac{c}{a}\right) + \frac{1}{\left(\frac{c}{a}\right)} + \frac{\left[(\alpha+\beta)^2 - 2\alpha\beta\right]}{\alpha\beta}$$

$$= \frac{c}{a} + \frac{a}{c} + \left[\frac{\left(\frac{-b}{a}\right)^2 - 2\frac{c}{a}}{\frac{c}{a}}\right]$$

$$= \frac{\left[c^2 + a^2\right]}{ac} + \left[\frac{b^2 - 2ac}{a^2\frac{c}{a}}\right]$$

$$= \frac{\left[c^2 + a^2\right]}{ac} + \left[\frac{b^2 - 2ac}{2ac}\right]$$

$$= \frac{c^2 + a^2 + c^2 - 2ac}{ac}$$

$$= \frac{\left[a^2 + b^2 + c^2 - 2ac\right]}{ac}$$

**Therefore**

$$x + b\frac{(c+a)}{ac}x - \frac{\left[a^2 + b^2 + c^2 - 2ac\right]}{ac} = 0$$

$$\textit{Hence;}\quad acx + b(c+a)x + (a^2 + b^2 + c^2 - 2ac) = 0$$

**7.** $(\alpha - \beta)^2, (\alpha + \beta)^2$

**Solution**

$$S=(\alpha-\beta)^2+(\alpha+\beta)^2$$
$$=2\left[\alpha^2+\beta^2-2\alpha\beta\right]$$
$$=2\left[\left(\frac{-b}{a}\right)^2-2\frac{c}{a}\right]$$
$$=2\left[\frac{b^2}{a^2}-2\frac{c}{a}\right]$$
$$=\frac{c}{a}\left[b^2-2ac\right]$$

$$P=(\alpha-\beta)^2\times(\alpha+\beta)^2$$
$$=\left[\alpha^2+\beta^2-2\alpha\beta\right]\left[\frac{-b^2}{a^2}\right]$$
$$=\left[(\alpha-\beta)^2-4\alpha\beta\right]\left[\frac{-b}{a^2}\right]^2$$
$$=\left[\frac{b^2-4ac}{a^4}\right]\left(\frac{b^2}{a^3}\right)$$
$$P=\frac{b^2(b^2-4ac)}{a^4}$$

**Therefore**

$$x^2-\frac{c}{a^2}(b^2-2ac)x+\frac{b^2(b^2-4ac)}{a^4}=0$$

*Hence;* $a^4x^2-a^2c(b^2-2ac)x+b^2(b^2-4ac)=0$

**8.** $-\frac{1}{\alpha^3}-\frac{1}{\beta^3}$

**Solution**

$$S=-\frac{1}{\alpha^3}-\frac{1}{\beta^3}$$

$$= -\left[\frac{\alpha^3+\beta^3}{(\alpha\beta)^3}\right]$$

$$= -\left[\frac{\frac{-b(b^2-3ac)}{a^3}}{\frac{c^3}{a^3}}\right]$$

$$= \frac{b(b^2-3ac)}{c^3}$$

*and*

$$P = \left(-\frac{1}{\alpha^3}\right)\left(-\frac{1}{\beta^3}\right)$$

$$= \frac{1}{\alpha^3\beta^3}$$

$$= \frac{1}{\frac{c^3}{a^3}}$$

$$= \frac{a^3}{c^3}$$

**Therefore**

$$x - \frac{b(b^2-3ac)}{c^3}x + \frac{a^3}{c^3} = 0$$

*Hence*; $c^3x - b(b^2-3ac)x + a^3 = 0$

**8. If $\alpha$ , $\beta$ are the roots of 5x²-x-2=0 , form the equation whose roots are** $\frac{3}{\alpha}$ *and* $\frac{3}{\beta}$.

**Solution**

**Let $\alpha$ , $\beta$ are the roots of equation**

$$\alpha + \beta = \frac{-1}{5}$$

and

$$\alpha\beta = \frac{-2}{5}$$

**If roots are** $\frac{3}{\alpha}$ *and* $\frac{3}{\beta}$

**Then**

$$S = \frac{3}{\alpha} + \frac{3}{\beta}$$
$$= 3\left[\frac{\alpha+\beta}{\alpha\beta}\right]$$

$$= 3\left[\frac{\frac{-1}{5}}{\frac{-2}{5}}\right]$$
$$= \frac{3}{2}$$

*and*

$$P = \left(\frac{3}{\alpha}\right)\left(\frac{3}{\beta}\right)$$
$$= \frac{9}{\alpha\beta}$$
$$= \frac{9}{\left(\frac{-2}{5}\right)}$$
$$= \frac{-45}{2}$$

**Therefore**

$$x^2 - \frac{3}{2}x - \frac{45}{2} = 0$$

*Hence;* $2x^2 - 3x - 45 = 0$

**9. If $\alpha$ and $\beta$ are the roots of $x^2$-3x+5=0 form the equation whose roots are $\frac{1-\alpha}{1+\alpha}$ *and* $\frac{1-\beta}{1+\beta}$.**

**Solution**

**Let $\alpha$ and $\beta$ are the roots of equation.**

$$\alpha+\beta=3$$
*and*
$$\alpha\beta=5$$

**If roots of equation are** $\frac{1-\alpha}{1+\alpha}+\frac{1-\beta}{1+\beta}$

**Then**

$$S=\frac{1-\alpha}{1+\alpha}+\frac{1-\beta}{1-\beta}$$
$$=\frac{(1+\beta)(1-\alpha)+(1+\alpha)(1-\beta)}{(1+\alpha)(1+\beta)}$$
$$=\frac{1-\alpha+\beta-\alpha\beta+1+\alpha-\beta-\alpha\beta}{1+\alpha+\beta+\alpha\beta}$$
$$=\frac{2-2\alpha\beta}{1+\alpha+\beta+\alpha\beta}$$
$$=\frac{2-2(5)}{1+3+5}$$
$$=\frac{2-10}{1+3+5}$$
$$=\frac{-8}{9}$$
$$=-\frac{8}{9}$$
*and*
$$P=\left(\frac{1-\alpha}{1+\alpha}\right)\left(\frac{1-\beta}{1+\beta}\right)$$
$$=\frac{1-\alpha-\beta+\alpha\beta}{1+\alpha+\beta+\alpha\beta}$$

$$= \frac{1-(\alpha+\beta)+(\alpha\beta)}{1+(\alpha+\beta)+(\alpha\beta)}$$

$$= \frac{1-3+5}{1+3+5}$$

$$= \frac{3}{9}$$

$$= \frac{1}{3}$$

**Therefore**

$$x^2 - \left(\frac{-8}{9}\right)x + \frac{1}{3} = 0$$

*Hence*; $9x^2 + 8x + 3 = 0$