EXERCISE 4.5

Use the remainder when the first polynomial is divided by the second polynomial:

1. $x^2+3x+7, x+1$

2. $x^3-x^2+5x+4, x-2$

3. 3x4+4x+x-5, x+1

4. x3-2x2++3x+3, x-3

1. x^2+3x+7, x+1

Solution

$$P(x)=x^2+3x+7$$
$$diversor$$
$$x+1=0 \Rightarrow x=-1$$
$$P(-1)=(-1)^2+(-1)+7$$
$$=1-3+7=8-3=5$$
$$R=P(-1)=5$$

$$remainder=5$$

2. $x^3-x^2+5x+4, x-2$

Solution

$$P(x)=x^3-x^2+5x+4$$
$$divisor$$
$$x-2=0 \Rightarrow x=2$$
$$P(2)=(2)^3-2^2+5(2)+4$$
$$8-4+10+4=18$$

$R = P(2) = 18$

$\text{Re}\,mainder = 18$

3. $3x^4 + 4x + x - 5, x + 1$

Solution

$$P(x) = 3x^4 + 4x^3 + x - 5$$
$$divisor$$
$$x + 1 = 0 \Rightarrow x = -1$$
$$P(-1) = 3(-1)^4 + 4(-1)^3 + (-1) - 5$$
$$= 3 - 4 - 1 - 5$$
$$P(-1) = -7$$

$remainder = -7$

4. $x^3 - 2x^2 + 3x + 3, x - 3$

Solution

$$P(x) = x^3 - 2x^2 + 3x + 3$$
$$divisor$$
$$x - 3 = 0 \Rightarrow x = 3$$
$$P(3) = 3^3 - 2(3)^2 + 3(3) + 3$$
$$= 27 - 18 + 9 + 3$$
$$P(3) = 39 - 18 = 21$$

$\text{Re}\,mainder = 21$

Use the factor theorem to determine if the first polynomials the factor of the second polynomial.

5. x^2+4x-5

6. $x-2, x^3+x^2-7x+1$

7. x^3-4x^2+ax+b

8. $x-a, x^n-a^n$ **where n is a positive integer**

5. x^2+4x-5

Solution

Let P(x)= x^2+4x-5

And the factors of polynomial is

X-1=0 $\Rightarrow$ x=1

$$P(1) = \begin{aligned} &(1)^2+4(1)-5 \\ &=1+4-5 \\ &=5-5 \\ &=0 \end{aligned}$$

Hence: 'x-1' is the polynomial.

6. $x-2, x^3+x^2-7x+1$

Solution

Let P(x) = x^3+x^2-7x+1

And the factors of polynomial is

x-2=0

x=2

P(2)=2^3+2^2-7x+1

=8+4-14+1

=12-14

=-1

Hence ; 'x-2' is not the polynomial

7. $\omega+2, 2\omega^3+\omega^2-4\omega+7$

Solution

Let $P(x)=2\omega^3+\omega^2-4\omega+7$

And the factors of polynomial is

$$\omega+2=0 \Rightarrow \omega=-2$$
$$P(-2)=2(-2)^3+(-2)^2-4(-2)+7$$
$$=-16+4+8+7$$
$$=-16+19$$
$$=3\neq 0$$

Hence; 'ω+2' is not the factor of polynomial.

8. x-a,x^n-a^n where n is a positive integer

Solution

Let

$$P(x)=x^n-a^n$$
$$where$$
$$n\in z^+$$

And the factors of polynomial is

x-a=0

x=0

$$P(a)=(a)^n-a^n=0$$

Hence ; 'x-a' is the factor of polynomial.

9. x+a , x^n+a^n , where n is an odd integer.

Solution

Let $P(x) = x^n + a^n;$ *where* $n \in z^-$

x+a is the factor of x^n+a^n

x=-a

so,

$$P(-a) = (-a)^n + a^n \left[n \in z^{-1} \right]$$
$$= 0$$

Hence ; x+a is the factor of x^n+a^n.

10. when $x^4 + 2x^3 + kx^2 + 3$ is divided by x-2, the remainder is 1. Find the value of k.

Solution

Let P(x)= $x^4 + 2x^3 + kx^2 + 3$

And divisor is x-2 =0

x=2

$$P(2) = (2)^4 + 2(2)^3 + k(2)^2 + 3$$
$$= 16 + 16 + 4k + 3$$
$$R = P(2) = 35 + 4k$$
$$R = 1$$
$$35 + 4k = 1$$
$$4k = 1 - 35$$
$$4k = -34$$

$$k = \frac{-34}{4}$$
$$= \frac{-17}{2}$$

Hence; the value of k is $\frac{-17}{2}$

11. When the polynomial $x^3 + 2x^2 + kx + 4$ is divided by x-2, the remainder is 14. Find the value of k.

Solution

Let $P(x) = x^3 + 2x^2 + kx + 4$

And divisor is x-2 =0

X=2

$$P(2) = (2)^3 + 2(2)^2 + k(2) + 4$$
$$= 8 + 8 + 2k + 4$$
$$R = P(2)$$
$$= 20 + 2k$$
$$= 14$$
$$20 + 2k = 14$$
$$2k = 14 - 20$$
$$= -6$$
$$k = \frac{-6}{2}$$
$$= -3$$

Hence; the value of k is -3.

Use synthetic division to show that x is the solution of the polynomial and use the result to factorize the polynomial completely.

12. $x^3 - 7x + 6 = 0, x = 2$

13. $x^3 - 28x - 48 = 0, x = -4$

14. $2x^4 + 7x^3 - 4x^2 - 27x - 18, x = 2, x = -3$

12. $x^3 - 7x + 6 = 0, x = 2$

Solution

$$x^3 - 7x + 6 = 0, x = 2$$

Synthetic division

2	1	0	-7	6
	0	2	4	-6
	1	2	-3	+0

So, factors

$$x^3 - 7x + 6 = (x-2)(x^2 + 2x - 3)$$

$$= (x-2)\left[x^2 + 3x - x - 3\right]$$
$$= (x-2)\left[x(x+3) - 1(x+3)\right]$$
$$= (x-2)(x+3)(x-1)$$

Hence, factors are (x-2)(x+3)(x-1)

13. $x^3 - 28x - 48 = 0,\ x = -4$

Solution

-4	1	0	-28	-48
	0	-4	+16	48
	1	-4	-12	0

So, factors

$$x^2 - 28x - 48 = (x+4)(x^2 - 4x - 12)$$

$$= (x+4)\left[x^2 - 6x + 2x - 12\right]$$
$$= (x+4)\left[x(x-6) + 2(x-6)\right]$$
$$= (x+4)(x+2)(x-6)$$

Hence, factors are (x+4) (x+2) (x-6)

14. $-2x^4 + 7x^3 - 4x^2 - 27x - 18, x = 2$ **and x=-3**

Solution

Synthetic division

-3	2	7	-4	-27	-18
	0	-6	-3	21	18
	2	1	-7	-6	0

So, factors

$$2x^4 + 7x^3 - 4x^2 - 27x - 18 = (x+3)(2x^3 + x^2 - 7x - 6)$$

Synthetic division

2	2	1	-7	-6
	0	4	10	6
	1	-4	-12	0

So, factors

$$2x^4+7x^3-4x^2-27x-18=(x+3)(x-2)(2x^3+5x+3)$$

$$=(x+3)(x-2)\left[2x^2+2x+3x+3\right]$$
$$=(x+3)(x-2)\left[2x(x+1)+3(x+1)\right]$$
$$=(x+3)(x-2)(2x+3)(x+1)$$

Hence , factors are (x+1) (x-2) (x+3) (2x+3)

15. Use synthetic division to find the value of p and q if x+1 and x-2 are the factors of the polynomial x^3+px^2+qx+6

Solution

x^3+px^2+qx+6 **and** x=-1

Synthetic division

-1	1	p	q	6
	0	-1	-p+1	p-q-1
	1	p-1	q-p+1	p-q+5=0

If '-1' is the root of equation then p-q+5=0

And

Synthetic division

2	1	p	q	6
	0	2	2p+4	4p+2q+8
	1	p+2	2p+q+4	4p+2q+14

If '2' is the root equation.

Then $4p+2q+14=0$

$2p+q+7=0$

By adding

$3p+12=0$

And

$-4-q+5=0$

$3p=-12$

$P=-4$

$1-q=0$

$q=1$

hence, $p=-4$

and $q=1$

16.Find the values of a and b if -2 and 2 are the roots of the polynomial x^3-4x^2+ax+b

Solution

Let P(x) = x^3-4x^2+ax+b and roots are -2 & 2

Synthetic division

2	1	-4	a	B
	0	2	-4	2a-8
	1	-2	a-4	2a+b-8

If '2' is root of polynomial .

2a+b-8=0

And **synthetic division**

-2	1	-4	a	B
	0	-2	12	-2a-24
	1	-6	a++12	-2a+b-24

If -2 is the root of polynomial.

-2a+b-24=0

By adding

2a+b-8=0

-2a+b-24=0

2b-32=0

2b=32=0

B=16

Put the value of 'b' is equation

2a+16-8=0

2a+8=0

$2a = -8$

$A = -4$

Hence, $a = -4$ and $b = 16$

Is the value of polynomial.