Exercise 4.10

1. The product of one less a certain positive number and two less than three times the number is 14. Find the number.

Solution

Let the number = x

Given condition

$(x-1)(3x-2) = 3x^2 - 2x - 3x + 2$

$3x^2 - 5x + 2x - 14 = 0$

$3x^2 - 5x - 12 = 0$

$3x^2 - 9x + 4x - 12 = 0$

$3x(x-3) + 4(x-3) = 0$

$3x + 4 = 0$ or $x - 3 = 0$

$x = -4/3$ $x = 3$

We have a positive number i.e x = 3

Hence number = 3

2. The sum of a positive number and its square is 380. Find the number.

Solution:

Let the number = x

Given condition

$x^2 + x = 380$

$x^2 + x - 380 = 0$

$x^2 + 20x - 19x - 280 = 0$

$x(x+20) - 19(x+20) = 0$

$(x+20)(x-19) = 0$

$x-19=0$ or $x+20=0$

$x=19$ $\quad$ $x=-20$

We required only the number i.e x = 19

Hence, number = 19

3. Divide 40 into two parts such that the sum of their squares is greater than 2 times their product by 100.

Solution:

Let the number = x

Then, 1st part = x and 2nd part b= 40 – x

Given condition

$$(x)^2+(40-x)^2=2(40x-x)+100$$
$$x^2+1600+x^2-80x=80x-2x^2+100$$
$$2x^2+2x^2-80x-80x+1600-100=0$$
$$4x^2-160x+1500=0$$
$$x^2-40x+375=0$$
$$x^2-25x-15x+375=0$$
$$x(x-25)-15(x-25)=0$$
$$(x-15)(x-25)=0$$

Either

$x-15=0$ or $x-25=0$

$x=15$ $\quad$ $x=25$

Hence required parts = 15 , 25

4. The sum of positive number and its reciprocal is $\frac{26}{5}$ Find the number.

Solution:

Let the number = x

Given condition

$$x+\frac{1}{x}=\frac{26}{5}$$

$$\frac{x^2+1}{x}=\frac{26}{5}$$

$$5x^2+5=26x$$

$$5x^2-26x+5=0$$

$$5x^2-25x-x+5=0$$

$$5x(x-5)-1(x-5)=0$$

$$(5x-1)(x-5)=0$$

Either

$5x-1=0$ or $x-5=0$

$x=1/5$ $\quad$ $x=5$

$$\Rightarrow x=\frac{1}{5}$$

Hence required number = $5, 1/5$

5. A number exceed its square root by 56. Find the number

Solution:

Let the number = x

Given condition

$$x-\sqrt{x}=56$$

$$\sqrt{x}=x-56$$

$$\left(\sqrt{x}\right)^2=(x-56)^2$$

$$x=x^2-112x+3136$$

$$x^2-112x-x+3136=0$$

$$x^2-113x+3136=0$$

$$x^2-64x-49x+3136=0$$

$x(x-64)-49(x-64)=0$

$(x-64)(x-49)=0$

Either

$x-49=0$ or $x-64=0$

$x=49$ $x=64$

Hence required number = 49, 64

6. Found two consecutive numbers, whose product is 132.

(Hint; Suppose the numbers are x and x+1)

Solution:

Let the number = x

Given condition

$x(x+1)=132$

$x^2+x=132$

$x^2+x-132=0$

$x^2+12x-11x-132=0$

$x(x+12)-11(x+12)=0$

$(x-11)(x+12)=0$

Either

$x-11=0$ or $x+12=0$

$x=11$ $x=-12$

Hence required number = 11, 12 or =-11, -12

7. The difference between the cubes of two consecutive even numbers 296. Find them

(Hint; Suppose the numbers are x and x+1)

Solution:

Let the number = 2x

And other number = 2x+2

Given condition

$$(2x+2)^3-(2x)^3=296$$
$$(2x)^3+(2)^3+3(2)(2x)(2x+2)-8x^3=296$$
$$8x^3+8+24x^2+24x-8x^3=296$$
$$24x^2+24x+8-296=0$$
$$8(3x^2+3x-36)=0$$
$$3(x^2+x-12)=0$$
$$x^2+x-12=0$$
$$x^2+4x-3x-12=0$$
$$x(x+4)-3(x+4)=0$$
$$(x+4)(x-3)=0$$

Either

$x-3=0$ or $x+4=0$

$x=3$ $x=4$

So number = 2x =2(3) =6 So number =2x=2(-4) =-8

Other number = 2x+2=6+2=8 other number = 2x+2=-8+2=-6

Hence numbers = 6, 8 or -6, -8

8. A farmer bought some sheeps for Rs;9000. If he had paid Rs.100 less for each, he would have got 3 sheep more the same money. How many sheeps did he buy, when the rate in each case is uniform?

Solution:

Let the number = 2x

And other number = 2x+2

Given condition

Let number of sheeps farmer buy = x

Price of one sheep=Rs $\frac{900}{x}$

If the price is Rs.100 less than the original price then new price.

New price = Rs $\left[\frac{900}{x}-100\right]$

$$\left(\frac{900}{x}-100\right)(x+3)=900$$

$$\left(\frac{(9000-100x)(x+3)}{x}\right)=9000$$

$$(9000-100x)(x+3)=9000x$$

$$100(90-x)(x+3)=9000x$$

$$(90-x)(x+3)=90x$$

$$90x+270-x^2-3x=90x$$

$$270-x^2-3x=0$$

$$x^2+3x-270=0$$

$$x^2+18x-15x-270=0$$

$$x(x+18)-15(x+18)=0$$

$$(x+18)(x-15)=0$$

Either

$x+18=0$, $x=-18$ or $x-15=0$, $x=15$

Sheep cannot be negative; therefore x=-18 not possible.

Hence farmer buy sheep = 15

9. A man sold his stock of eggs for Rs=240. If he had 2 dozen more, he would get the same money by selling the whole for Rs=0.50 per dozen cheaper. How many dozen eggs did he sell?

Solution:

Let total stock of eggs=x dozen

Man sold his stock of x dozen eggs=Rs 240

So , price of '1' dozen eggs = Rs;240/x

Rate of per dozen eggs=240/x=0.50

According to given condition

(number of eggs in dozen) (rate per dozen) = Rs 240

$$(x+2)\left(\frac{240}{x}-0.50\right)=240$$
$$(x+2)\left(\frac{240-0.50}{x}\right)=240$$
$$(x+2)(240-0.50)=240x$$
$$240x-0.50x^2+480-x=0$$
$$-0.50x^2-x+480=0$$
$$x^2+2x-960=0$$
$$x^2+32x-30x-960=0$$
$$x(x+32)-30(x+32)=0$$
$$(x+32)(x-30)=0$$

Either

$x-30=0$ or $x+32=0$

$x=30$ $x=-32$

Eggs cannot be negative in number i.e x= -32 not possible

Hence, the man sold 30 dozen eggs.

10. A cyclist traveled 48km at a uniform speed. Had he traveled 2km/h slower, he would have taken 2 hrs. more to perform the journey. How long did he take to cover 48 km?

Solution:

Let the speed of cycle=x hrs

Time taken = $\frac{dis\tan ce}{speed}=\frac{48}{x}hrs$

If he decreases the speed

Then $\text{time} = \frac{distance}{speed} = \frac{48}{x} hrs$

Given condition

(time with decrease speed)-(time with original speed) = 2 hrs

$t_2 - t_1 = 2$

$$\frac{48}{x-2} - \frac{48}{2} = 2$$

$$\frac{48x - 48(x-2)}{x(x-2)} = 2$$

$$\frac{48x - 48x + 96}{x(x-2)} = 2$$

$$96 = 2x(x-2)$$

$$96 = 2x^2 - 4x$$

$$2x^2 - 4x - 96 = 0$$

$$x^2 - 2x - 48 = 0$$

$$x^2 - 8x + 6x - 48 = 0$$

$$x(x-8) + 6(x-8) = 0$$

$$(x-8)(x+6) = 0$$

Either

$x + 6 = 0$, $x = -6$ or $x - 8 = 0$, $x = 8$

speed of cyclist = 8km/h

'-6' speed is not possible

i.e time taken by cyclist = 48/8 = 6 hrs

hence, the time taken by cyclist = 6 hours

11. **The area of a rectangular field is 297 square meters, had it been 3 meters longer and one meter shorter, the area would been 3 square meter more. Find its length and breath.**

Solution:

Let the length = x meter

And x . y= 297 m^2

$length = (x+3) meter$

$breath = (y-1) meter$

$(x+3)(y-1) = 297+3$

$xy - x + 3y - 3 = 300$

We know that xy=297 and x=297/y

$297 - \frac{297}{y} + 3y - 3 = 300$

$294 - \frac{297}{y} + 3y = 300$

$\frac{-297+3y^2}{y} = 300 - 294$

$\frac{-297+3y^2}{y} = 6$

$-297 + 3y^2 = 6y$

$6y - 297 + 3y^2 = 0$

$3(y^2 - 2y - 99) = 0$

$(y^2 - 2y - 99) = 0$

$y^2 - 11y - 9y - 99 = 0$

$y(y-11) + 9(y-11) = 0$

$(y-11)(y+9) = 0$

Either

$y - 11 = 0$ or $y + 9 = 0$

$y = 11$ $y = -9$

So breath = 11 meter

And length = 27 meter

Negative value of breath y= -9 is not possible.

Hence breath = 11 meter

And length n= 27 meter

12. The length of a rectangular price of paper exceeds its breath by 5 cm. If a strip 0.5 cm wide be cut all around the piece of paper, the area of the remaining part would be 500 square cms. Find its original dimensions.

Solution:

Let breath =x

New breath = x-0.5-0.5=(x-1) cm

Length = (x+5) cm

New length = x+5-0.5-0.5=(x+4) cm

By given condition

$$(x-1)(x+4)=500$$
$$x^2+4x-x-4=500$$
$$x^2+3x-4=500$$
$$x^2+3x-4-500=0$$
$$x^2+3x-504=0$$
$$x^2+24x-21x-504=0$$
$$x(x+24)-21(x+24)=0$$
$$(x+24)(x-21)=0$$

Either

$x-1=0$ or $x+24=0$

$x=1$ $x=-24$

Negative value of 'x' is not possible so

Therefore

Breath = 21cm

And

Length = 26cm

13. A number consists of two digits whose product is 18. If the digits are interchanged, the new number becomes 27 less than the original number. Find the number.

Solution:

Let unit digit =x

Ten digits = y

Therefore number=10y+x

If they are interchanged = 10x+y

x=18/y

$$(10y+x)-(10x+y)=27$$
$$10y+x-10x-y=27$$
$$9y-9x=27$$
$$9(y-x)=27$$
$$y-x=3$$

By putting the value of x

$$y-\frac{18}{y}=3$$
$$\frac{y^2-18}{y}=3$$
$$y^2-18=3y$$
$$y^2-3y-18=0$$
$$y^2-6y+3y-18=0$$
$$y(y-6)+3(y-6)=0$$
$$(y-6)(y+3)=0$$

Either

$y-6=0$, $y=6$ or $y+3=0$, $y=-3$

Therefore, unit digit = 18/6 =3

So, unit digit (x)=3

And tens digit (y)=6

Hence number = x+10y

$=3+60=63$

Number=63

14. A number consist of two digits whose product is 14. If the digits are interchanged, the new number become 45 less than the original number. Find the number

Solution:

Let unit digit = x

And ten digits = y

Therefore number = x+10y

1st given condition

$$(y+10x)-(x+10y)=45$$
$$y+10x-x-10y=45$$
$$9x-9y=45$$
$$9(x-y)=45$$
$$x-y=5$$
$$we\_know\_that\_x=\frac{14}{y}$$
$$\frac{14}{y}-y=5$$
$$\frac{14-y^2}{y}-y=5$$
$$14-y^2=5y$$
$$14-y^2-5y=0$$
$$y^2+7x-2x-14=0$$
$$y(y+7)-2(y+7)=0$$
$$(y+7)(y-2)=0$$

Either

$y-2=0$, $y=2$ or $y+7=0$, $y=-7$

The negative value of 'y' is not possible

So y=2

And $x = \frac{14}{y} = \frac{14}{2} = 7$

Therefore number = x+10y

=7+2(10)

=7+20=27

Hence number = 27

15. The area of a right triangle is 210 square meters. If its hypotenuse is 37 meters long. Find the length of the base and the altitude.

Solution:

Let the length of base triangle = x meter

And altitude of triangle = y meter

By 1st condition

$$\frac{1}{2}.x.y = 210m^2$$

$$\frac{xy}{2} = 210$$

$$xy = 420m^2$$

And 2nd condition

$$x^2 + y^2 = (37)^2$$

$$x^2 + y^2 = 1369$$

$$(x)^2 + (y)^2 + 2xy = 1369 + 3xy$$

$$(x+y)^2 = 1369 + 2xy$$

$$(x+y)^2 = 1369 + 2(42)$$

$$(x+y)^2 = 1369 + 840 = 2209$$

$$(x+y)^2 = 2209$$

$$x + y = \sqrt{2209}$$

$$x + y = 47$$

Negative value is emitted

And

$(x)^2+(y)^2-2xy=1369-2xy$

$(x-y)^2=1369-2(420)$

$=1369-840=529$

$(x-y)=\sqrt{529}=23$

So

$x+y=47$

$x-y=23$

$2x=70$

And

$x+y=47$

$x-y=23$

$2y=24$

y = 12

hence base = 35 meter

and altitude=12 meter

16. The area of a rectangle is 1680 square meters. If its diagonal is 37 meters long. Find the length and the breadth of the rectangle.

Solution:

Let the length of the rectangle = x meter

And breath = y meter

By 1st condition

$xy=1680$

$x^2+y^2=(58)^2$

$x^2+y^2=3364$

$x^2+y^2-2xy=3364+2(1680)$

$(x-y)^2=3364-3360=4$

$x-y=\sqrt{4}=2$

$$x + y = 82$$
$$x - y = 2$$
$$2x = 84$$

So , $x = \frac{84}{2} = 42$

And $x + y = 82$
$x - y = 2$

$2y = 80$

⇨ y = 40

hence length of rectangle = 42 meter

and breath of rectangle = 40 meter

17. To do a piece of work, A takes 10 days more than B. Together they finish the work in 12 days. How long would B take to finish it alone? 84

(Hint: If someone takes x days to finish a work. The one day's work will be $\frac{84}{2}$)

Solution:

Let 'B' finishes the work alone = x days

Then 'B' finish work in 1 day= 1/x

And let 'A' finishes the work alone = (x + 10) days

Then 'A' finish work in 1 day = $\frac{1}{x+10}$

By given condition

$$\frac{1}{x} - \frac{1}{x+10} = \frac{1}{12}$$
$$\frac{x+10+x}{x(x+10)} = \frac{1}{12}$$

$$12(2x+10) = x(x+10)$$
$$24x+120 = x^2+10x$$
$$24x+120-x^2-10x = 0$$
$$x^2-20x+6x-120 = 0$$
$$x(x-20)+6(x-20) = 0$$
$$(x-20)(x+6) = 0$$

Either

$x-20=0$ or $x+6=0$

$x=20$ $x=-6$

-ve value of x is not possible

So, x = 20

Hence 'B' alone finished the work = 20 days

18. To complete a job, A and B take 4 days working together. An alone takes twice as long as B alone to finish the same job. How long would each one alone takes to do the job?

Solution:

Let 'B' finishes work = x days

So, 'B' finish work in 1 day = $\frac{1}{x} days$

And 'A' finishes work = 2x days

So, 'A' finish work in 1 day = $\frac{1}{2x} days$

By given condition

$$\frac{1}{x}+\frac{1}{2x} = 4$$

$$\frac{2x+x}{2x^2}=\frac{1}{4}$$

$$2x+x=\frac{1}{4}(2x)^2$$

$$4(3x)=2x^2$$

$$2x^2-12x=0$$

$$x(2x-12)=0$$

So, 'B' finished the work = 6 days

And 'A' finished the same work = 12 days

Hence B's completed work in 6 days while A's completed the same work in 12 days.

19. An open box is to be made form a square piece of tin by cutting a piece 2 dm square from each cover and then folding the sides of the remaining piece. If the capacity of the box is to be 128 c.dm, find the length of the side of the piece.

Solution:

Let width of box = x dm

So its length of box = x dm

And its height = 2 dm

Therefore

The volume of box = $(x)(x)(2)$

$$2x^2=128$$

$$x^2=\frac{128}{2}=64$$

$$x=\sqrt{64}$$

$$=\pm 8dm$$

Negative value of x is not possible.

Therefore, x = 8 dm

Hence, length and breadth of the side of the piece = 12 cm

20. A man invests Rs. 100,000 in two companies. His total profit is Rs. 5060. If he receives Rs. 1980 from one company and at the rate 1% more from the other, find the amount of each investment.

Solution:

Let original amount(P) = Rs x

And interest(I) =Rs. 1980

Rate (R)= y%

Time (T) = 1 year

We know the formula $I = \frac{PRT}{100}$

$$1980 = \frac{x.y.1}{100}$$
$$xy = 1980 \times 198000$$

⇨ **From 2nd company**

$$P = Rs(100,000 - x)$$
$$\text{int}\,erest(I) = Rs(5060 - 1980)$$
$$= Rs.3080$$
$$Rate(R) = (y+1)\%$$
$$Time(T) = 1.year$$

We know that

$$I = \frac{PRT}{100}$$
$$3080 = \frac{(100,000 - x)(y+1)(1)}{100}$$
$$3080(100) = (100,000 - x)(y+1)(1)$$
$$3080(100) = 100,000y + 100,000 - 198000 - \frac{19800}{y}$$

$$308 = 100y + 100 - 198 - \frac{198}{y}$$

$$308 + 98 = 100y - \frac{198}{y}$$

$$406 = 100y - \frac{198}{y}$$

$$406y = 100y^2 - 198$$

$$=> 100y^2 - 406y - 198 = 0$$

$$or \_\_\_\_\_ 50y^2 - 203y - 99 = 0$$

$$y = \frac{203 \pm \sqrt{(203)^2 - 4(50)(99)}}{2(50)}$$

$$= \frac{203 \pm \sqrt{41209 + 19800}}{100}$$

$$= \frac{203 \pm \sqrt{61009}}{100}$$

Either

$$y = \frac{203 + 247}{100}$$

$$y = \frac{450}{100}$$

$$y = 4.5$$

or

$$y = \frac{203 - 247}{100}$$

Not _ Possible

Therefore $x = \frac{198000}{4.5} = 44000$