Exercise 4.2

1. $x^4 - 6x^2 + 8 = 0$

Solution:

Let
$x^2 = y$
So
$(y)^2 - 6(y) + 8 = 0$
$y^2 - 6y + 8 = 0$
$y^2 - 4y - 2y + 8 = 0$
$y(y-4) - 2(y-4) = 0$
$(y-2)(y-4) = 0$

$y - 2 = 0$ $\quad$ $y - 4 = 0$
$y = 2$ $\quad$ $y = 4$
$Therefore;$ $\quad$ **OR**
$x^2 = 2$ $\quad$ $x^2 = 4$
$x = \pm\sqrt{2}$ $\quad$ $x = \pm 2$

Hence $\quad$ $S.S. = \left\{\sqrt{2}, -\sqrt{2}, 2, -2\right\}$

2. $x^{-2} - 10 = 3x^{-1}$

Solution:

Put
$x^{-1} = y$
So
$y^2 - 10 = 3y$
$y^2 - 3y - 10 = 0$
$y^2 - 5y + 2y - 10 = 0$

$y(y-5)+2(y-5)=0$
$(y-5)(y+2)=0$

$y+2=0$ $\qquad$ $y-5=0$
$y=-2$ $\qquad$ $y=5$

therefore **OR** *therefore*

$x^{-2}=-2$ $\qquad$ $x^{-2}=5$

$\frac{1}{x}=-2$ $\qquad$ $\frac{1}{x}=5$

$x=-\frac{1}{2}$ $\qquad$ $x=\frac{1}{5}$

Hence $S.S.=\left\{\frac{1}{2},\frac{1}{5}\right\}$

3. $x^6-9x^3+8=0$

Solution:

Let
$x^3=y$
So
$y^2-9y-8=0$
$y^2-8y-y+8=0$
$y(y-8)-1(y-8)=0$
$(y-1)(y-8)=0$

$y-1=0$ $\qquad$ $y-8=0$
$y=1$ $\qquad$ $y=8$
Therefore **OR** *Therefore*
$x^3=1$ $\qquad$ $x^3=8$
$x=1,\omega,\omega^2$ $\qquad$ $x=2,2\omega,2\omega^2$

Hence, $\text{S.S}=\{1,\omega,\omega^2,2,2\omega,2\omega^2\}$

4. $8x^6 - 19x^3 - 27 = 0$

Solution:

$$\begin{aligned}
&Let\\
&x^3 = y\\
&So\\
&8y^2 - 19y - 27 = 0\\
&y(8y-27) + 1(8y-27) = 0\\
&(y+1)(8y-27) = 0
\end{aligned}$$

$$y = -1 \qquad\qquad y = \frac{27}{8}$$

$$x^3 = -1 \qquad \text{or} \qquad x^3 = \frac{27}{8}$$

$$x = -1, -\omega, -\omega^2 \qquad\qquad x = \frac{3}{2}, \frac{3}{2}\omega, \frac{3}{2}\omega^2$$

Hence $S.S. = \left\{-1, -\omega, -\omega^2, \frac{3}{2}, \frac{3}{2}\omega, \frac{3}{2}\omega^2\right\}$

Alternative way

$$\begin{aligned}
&x^3 = -1\\
&x^3 + 1 = 0\\
&(x+1)(x^2 - x + 1) = 0
\end{aligned}$$

$$x + 1 = 0 \qquad x^2 - x + 1 = 0$$

$$x^2 - x\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 + 1 - \left(\frac{1}{2}\right)^2 = 0$$

$$x = -1 \qquad\qquad \left(x - \frac{1}{2}\right)^2 + 1 - \frac{1}{4} = 0$$

$$\left(x - \frac{1}{2}\right)^2 + \frac{3}{4} = 0$$

$$\left(x-\frac{1}{2}\right)^2=\frac{-3}{4}$$
$$x-\frac{1}{2}=\pm\sqrt{\frac{3}{2}}$$
$$x=\frac{1}{2}\pm\sqrt{\frac{3}{2}}=\frac{1\pm\sqrt{3}}{2}$$

And

$$x^3=\frac{27}{8}$$
$$x^3-\frac{27}{8}=0$$
$$\left(x-\frac{3}{2}\right)\left(x^2\pm\frac{3}{2}x+\frac{9}{4}\right)=0$$

$$x-\frac{3}{2}=0$$
$$x=\frac{3}{2}$$

;

$$x^2+\frac{3}{2}x+\frac{9}{4}=0$$
$$x=\frac{\frac{-3}{2}\pm\sqrt{\frac{9}{4}-9}}{2}$$
$$=\frac{\frac{-3}{2}\pm\sqrt{\frac{-27}{4}}}{2}$$
$$=\frac{-3\pm3\sqrt{3i}}{4}$$

Hence $S.S.=\left\{-1,\frac{1+\sqrt{3}}{2},\frac{1-\sqrt{3}}{2},\frac{3}{2},\frac{-3+\sqrt{3i}}{2},\frac{-3-3\sqrt{3i}}{2}\right\}$

5. $x^{2/3}+8=6x^{\frac{1}{5}}$

Solution:

$$Let$$
$$x^{\frac{1}{5}}=y$$
$$So,$$
$$y^2+8=6y$$

$y^2-4y-2y+8=0$

$y(y-4)-2(y-4)=0$

$(y-4)(y-2)=0$

$y-2=0$		$y-4=0$
$y=2$		$y=4$
$x^{\frac{1}{5}}=2$	or	$x^{\frac{1}{5}}=4$
$x=(2)^5$		$x=(4)^5$
$x=32$		$x=1024$

Hence

$S.S=\{32,1024\}$

6. $(x+1)(x+2)(x+3)(x+4)=24$

Solution:

$(x+1)(x+2)(x+3)(x+4)=24$

$\left[(x+4)(x+1)\right]\left[(x+3)(x+2)\right]=24$

$(x^2+5x+4)(x^2+5x+6)=24$

Let

$x^2+5x=y$

$(y+4)(y+6)=24$

$y^2+10y+24=24$

$y^2+10y=0$

$y(y+10)=0$

$y=0$	or	$y+10=0$
		$y=-10$

Therefore

$x^2+5x=0$

$x(x+5)=0$

$$x = 0; x + 5 = 0$$
$$x = 0; x = -5$$

$$x^2 + 5x = -10$$
$$x^2 + 5x + 10 = 0$$
$$= \frac{-5 \pm \sqrt{25 - 40}}{2}$$
$$x = \frac{-5 \pm \sqrt{-15}}{2}$$

Hence

$$S.S = \left\{0, -5, \frac{-5 \pm \sqrt{-15}}{2}\right\}$$

7. $(x-1)(x+5)(x+8)(x+2) - 880 = 0$

Solution:

$$(x-1)(x+5)(x+8)(x+2) - 880 = 0$$
$$[(x-1)(x+8)][(x+5)(x+2)] - 880 = 0$$
$$(x^2 + 7x - 8)(x^2 + 7x + 10) - 880 = 0$$

Let

$$x^2 - 7x = y$$

$$(y-8)(y+10) - 880 = 0$$
$$y^2 - 8y + 10y - 8 - -880 = 0$$
$$y^2 + 2y - 960 = 0$$
$$y(y+32) - 30(y+32) = 0$$

$y - 30 = 0$ **or** $y + 32 = 0$

$y = 30$ $\quad$ $y = -32$

Therefore

$x^2 + 7x = 30$; $x^2 + 7x = -32$

$$x^2+7x=3=$$
$$x^2+7x-30=0$$
$$x^2+10x-3x-30=0$$

$$x(x+10)-3(x+10)=0$$
$$(x-3)(x+10)=0$$

$x-3=0$ **or** $x+10=0$

$x=3$ $\quad$ $x=-10$

$$and$$
$$x^2+7x=-32$$
$$x^2+7x+32=0$$
$$x=\frac{-7\pm\sqrt{49-128}}{2}$$
$$x=\frac{-7\pm\sqrt{-79}}{2}$$

Hence $\quad S.S=\left\{3,-10-\dfrac{-7\pm\sqrt{-79}}{2}\right\}$

8. $(x-5)(x-7)(x+6)(x+4)-504=0$

Solution:

$$(x-5)(x-7)(x+6)(x+4)-504=0$$
$$\left[(x-5)(x+4)\right]\left[(x-7)(x+6)\right]-504=0$$
$$\left(x^2-x-20\right)\left(x^2-x-42\right)-504=0$$

$$let \qquad x^2-x=y$$
$$(y-20)(y-42)-504=0$$
$$y^2-63y+840-504=0$$

$$y^2-62y+336=0$$
$$y^2-56y-6y+336=0$$
$$y(y-56)-6(y-56)=0$$
$$(y-6)(y-56)=0$$

$y-6=0$ **or** $y-56=0$
$y=6$ $\quad$ $y=56$

Therefore

$x^2-x=6$ **or** $x^2-x=56$

$$x^2-x-6=0$$
$$x^2-3x+2x-6=0$$
$$(x+2)(x-3)=0$$

$x+2=0$ **or** $x-3=0$
$x=-2$ $\quad$ $x=3$

$$x^2-x-56=0$$
$$x^2-8x+7x-56=0$$
$$x(x-8)+7(x-8)=0$$
$$(x-8)(x+7)=0$$

$x+7=0$ **or** $x-8=0$
$x=-7$ $\quad$ $x=8$

Hence $S.S=\{-2,3,-7,8\}$

9. $(x-1)(x-2)(x-8)(x+5)+360=0$

Solution:

$$(x-1)(x-2)(x-8)(x+5)+360=0$$
$$[(x-1)(x-2)][(x-8)(x+5)]+360=0$$
$$(x^2-3x+2)(x^2-3x-40)+360=0$$

Let $x^2 - 3x = y$

$(y+2)(y-40)+360=0$

$y^2-38y-80+360=0$

$y^2-287-10y+280=0$

$y(y-28)-10(y-28)=0$

$(y-10)(y-28)=0$

$y-28=0$ **or** $y-10=0$

$y=28$ $y=10$

Therefore

$x^2-3x=28$ $x^2-3x=10$

$x^2-3x-28=0$ $x^2-3x-10=0$

$x^2-3x-10=0$

$x^2-5x+2x-10=0$

$x(x-5)+2(x-5)=0$

$(x-5)(x+2)=0$

$y-28=0$

$y=28$

or

$y-10=0$

$y=10$

Therefore

$x^2-3x=28$

$x^2-3x-28=0$

$x^2-3x=10$

$2-3x-10=0$

$x^2-3x-10=0$

$x^2-5x+2x-28=0$

$x(x-5)+2(x-5)=0$

$(x+2)(x-5)=0$

$x+2=0$ **or** $x-5=0$

$x=-2$ $x=5$

And

$x^2-3x-28=0$

$x-7x+4x-28=0$

$x(x-7)+4(x-7)=0$

$(x-7)(x+4)=0$

$x+4=0$ **or** $x-7=0$

$x=-4$ $x=7$

Hence $S.S=\{-4,7,-2,5\}$

10. $(x+1)(2x+3)(2x+5)(x+3)=945$

Solution:

$(x+1)(2x+3)(2x+5)(x+3)=945$

$[(x+1)(x+3)][(2x+3)(2x+5)]=945$

$(x^2+4x+3)(4x^2+16x+15)=945$

Let

$x^2+4x=y$

$4x^2+16x=4y$

$(y+3)(4y+15)=945$

$4y^2+15y+12y+45=945$

$4y^2+27y+45-945=0$

$4y^2+27y-900=0$

$4y^2+27y-48y-900=0$

$y(4y+75)-12(4y+75)=0$

$(y-12)(4y+75)=0$

$y-12=0$ *Or* $4y+75=0$

$y=12$ *Or* $y=-75/4$

Therefore

$x^2+4x=12$ *Or* $x^2+4x={}^{-75}\!/_{4}$

$x^2+4x-12=0$

$x^2+6x-2x-12=0$

$x(x+6)-2(x+6)=0$

$(x-2)(x+6)=0$

$x-2=0$ *Or* $x+6=0$

$x=2$ *Or* $x=-6$

And

$$x^2+4x=\frac{-75}{4}$$

$4x^2+16x=-75$

$4x^2+16x+75=0$

$$x=\frac{-16\pm\sqrt{(16)^2-4(4)(75)}}{2(4)}$$

$$=\frac{-16\pm\sqrt{256-1200}}{8}$$

$$=\frac{-16\pm\sqrt{-944}}{8}$$

$$=\frac{-16\pm4\sqrt{-59}}{8}$$

Hence $S.S=\left\{2,-6,\dfrac{-4\pm\sqrt{-59}}{2}\right\}$

11. $(2x-7)(x^2-9)(2x+5)-91=0$

Solution:

$(2x-7)(x^2-9)(2x+5)-91=0$

$(2x-7)(x-3)(x+3)(2x+5)-91=0$

$[(2x-7)(x+3)][(x-3)(2x+5)]-91=0$

$$(2x^2 - x - 21)(2x^2 - x - 15) - 91 = 0$$

Let

$$2x^2 - x = y$$

$$(y - 21)(y - 15) - 91 = 0$$

$$y^2 - 21y - 15 + 315 - 91 = 0$$

$$y^2 - 36y + 224 = 0$$

$$y^2 - 28y - 8y + 224 = 0$$

$$(y - 8)(y - 28) = 0$$

$y - 8 = 0$ **or** $y - 28 = 0$

$y = 8$ $\quad$ $y = 28$

Therefore

$$2x^2 - x = 8$$

$$2x^2 - x = 28$$

$$2x^2 - x - 8 = 0$$

$$x = \frac{1 \pm \sqrt{1 + 4(2)(8)}}{2(2)}$$

$$= \frac{1 \pm \sqrt{1 + 64}}{4}$$

$$= \frac{1 \pm \sqrt{65}}{4}$$

and

$$2x^2 - x = 28$$

$$2x^2 - x - 28 = 0$$

$$2x^2 - 8x + 7x - 28 = 0$$

$$2x(x - 4) + 7(x - 4) = 0$$

$$(x - 4)(2x + 7) = 0$$

$2x + 7 = 0$ **or** $x - 4 = 0$

$x = -7/2$ $\quad$ $x = 4$

Hence $\quad S.S = \left\{ \frac{-7}{2}, 4, \frac{1 \pm \sqrt{65}}{2} \right\}$

12. $\left(x^2+6x+8\right)\left(x^2+14x+48\right)=105$

Solution:

$\left(x^2+6x+8\right)\left(x^2+14x+48\right)=105$
$\left(x^2+4x+2x+8\right)\left(x^2+6x+8x+48\right)=105$
$\left[x\left(x+4\right)+2\left(x+4\right)\right]\left[x\left(x+6\right)+8\left(x+6\right)\right]=105$
$\left(x+2\right)\left(x+4\right)\left(x+6\right)\left(x+8\right)=105$
$\left(x^2+10x+16\right)\left(x^2+10x+24\right)=105$

Let
$x^2+10x=y$
$\left(y+16\right)\left(y+24\right)=105$
$y^2 24y+16y+384-105=0$
$y^2+40y+279=0$
$y^2+31y+9y+279=0$
$y\left(y+31\right)+9\left(y+31\right)=0$
$\left(y+31\right)\left(y+9\right)=0$

$y+9=0$ **or** $y+31=0$
$y=-9$ $y=-31$

Therefore

$x^2+10x=-9$
$x^2+9x+x+9=0$
$x\left(x+9\right)+1\left(x+9\right)=0$
$\left(x+9\right)\left(x+1\right)=0$

$x+1=0$ **or** $y+9=0$
$x=-1$ $y=-9$

And

$$x^2+10x+31=0$$
$$x=\frac{-10\pm\sqrt{100-4(1)(31)}}{2(1)}$$
$$=\frac{-10\pm\sqrt{100-124}}{2}$$
$$=\frac{-10\pm\sqrt{-24}}{2}$$
$$=\frac{-10\pm2\sqrt{6}}{2}$$
$$=-5\pm\sqrt{6}$$

Hence $S.S=\left\{-1,-9,-5\pm\sqrt{6}\right\}$

13. $\left(x^2+6x-27\right)\left(x^2-2x-35\right)=386$

Solution:

$$\left(x^2+6x-27\right)\left(x^2-2x-35\right)=386$$
$$\left(x^2+9x-3x-27\right)\left(x^2-7x+5x-35\right)=385$$
$$\left[x(x+9)-3(x+9)\right]\left[x(x-7)+5(x-7)\right]=385$$
$$(x-3)(x+9)(x+5)(x-7)=385$$
$$\left(x^2+5x-3x-15\right)\left(x^2+9x-7x-63\right)=385$$
$$\left(x^2+2x-15\right)\left(x^2+2x-63\right)=385$$

Let
$$x^2+2x=y$$
$$(y-15)(y-63)=385$$
$$y^2-63y-15y+945=385$$
$$y^2-78y+945-385=0$$
$$y^2-70y-8y+560=0$$
$$y(y-70)-8(y-70)=0$$
$$(y-8)(y-70)=0$$

$$y-8=0 \qquad \text{or} \qquad y-70=0$$
$$y=8 \qquad\qquad y=70$$

Therefore

$$x^2+2x=8$$
$$x^2+2x=70$$
$$x^2+2x-8=0$$
$$x^2+4x-2x-8=0$$
$$x(x+4)-2(x+4)=0$$
$$(x+4)(x-2)=0$$

$$x-2=0 \qquad \text{or} \qquad x+4=0$$
$$x=2 \qquad\qquad x=-4$$

And

$$x^2+2x=70$$
$$x^2+2x-70=0$$
$$x=\frac{-2\pm\sqrt{4-4(1)(-70)}}{2(1)}$$
$$=\frac{-2\pm2\sqrt{1+70}}{2}$$
$$=-1\pm\sqrt{71}$$

Hence $S.S.=\{2,-4,-1\pm\sqrt{71}\}$

14. $4.2^{2x+1}-9.2^x+1=0$

Solution:

$$4.2^{2x}.2-9.2^x+1=0$$
$$8.2^{2x}-9.2^x+1=0$$
$$Let \qquad 2^x=y$$
$$8y^2-9y+1=0$$

$$8y^2-8y-y+1=0$$
$$8y(y-1)-1(y-1)=0$$
$$(8y-1)(y-1)=0$$

$8y-1=0$ or $y-1=0$

$y=\frac{1}{8}$ $\quad$ $y=1$

Therefore

$$2^x=(8)^{-1}$$
$$2^x=(2^3)^{-1}$$
$$2^x=2^{-3}$$
$$x=-3$$
$$2^x=1$$
$$2^x=2^0$$
$$x=0$$

Hence $\quad S.S=\{0,-3\}$

15. $2^x+2^{-x+6}-20=0$

Solution:

$$2^x+2^{-x}.2^6-20=0$$
$$2^x+64.2^{-x}-20=0$$
Let
$$2^x=y$$
$$y+64y^{-1}-20=0$$
$$y+64.\frac{1}{y}-20=0$$
$$y^2+64-20y=0$$
$$y^2-20y+64=0$$
$$y^2-16y-4y+64=0$$

$y(y-16)-4(y-16)=0$
$(y-4)(y-16)=0$

$y-4=0$ **or** $y-16=0$
$y=4$ $y=16$

Therefor

$2^x=4$
$2^x=2^2$
$x=2$
$2^x=2^4$
$x=4$

Hence $S.S=\{2,4\}$

16. $4^x-3.2^{x+3}+128=0$

Solution:

$(2^2)^x-3.2^x.2^3+128=0$
$2^{2x}-24.2^x+128=0$

Let
$2^x=y$
$y^2-24y+128=0$
$y^2-16y-8y+128=0$
$y(y-16)-8(y-16)=0$
$(y-8)(y-16)=0$

$y-8=0$ **or** $y-16=0$
$y=8$ $y=16$

Therefore;

$2^x = 8$ **and** $2^x = 16$

$2^x = 2^3$
$x = 3$
$2^x = 2^4$
$x = 4$

Hence $S.S = \{3, 4\}$

17. $3^{2x-1} - 12.3^x + 81 = 0$

Solution:

$3^{2x}.3^{-1} - 12.3^x + 81 = 0$
$Let \quad 3^x = y$
$3^{-1}.y^2 - 12y + 81 = 0$
$\frac{1}{3}y^2 - 12y + 81 = 0$
$y^2 - 36y + 81 = 0$
$y^2 27y - 9y + 243 = 0$
$y(y-27) - 9(y-27) = 0$
$(y-9)(y-27) = 0$

$y - 9 = 0$ **or** $y - 27 = 0$
$y = 0$ $y = 27$

Therefore

$3^x = 9$ **and** $3^x = 27$

$3^x = 3^2$
$3^x = 3^3$
$x = 3$

Hence, $S.S = \{2, 3\}$

18. $\left(x+\frac{1}{x}\right)^2-3\left(x\frac{1}{x}\right)-4=0$

Solution:

$$x+\frac{1}{x}=y$$
$$y^2-3y-4=0$$
$$y^2-4y+y-4=0$$
$$y(y-4)+1(y-4)=0$$
$$(y-4)(y+1)=0$$

$y+1=0$ **and** $y-4=0$

$y=-1$ $y=4$

Therefore,

$x+\frac{1}{x}=-1$ **and** $x_-\frac{1}{x}=4$

$x^2+1=-x$ $x^2+1=4x$

$$x^2+x+1=0$$
$$x=\frac{-1\pm\sqrt{1-4(1)(1)}}{2(1)}$$
$$=\frac{-1\pm\sqrt{-3}}{2}$$
$$=\frac{-1\pm\sqrt{-3}}{2},,,or,,,\omega,\omega^2$$

And $x^2-4x+1=0$

$$x=\frac{-4\pm\sqrt{16-4}}{2}$$
$$=\frac{-4\pm\sqrt{12}}{2}$$
$$=\frac{4\pm\sqrt{12}}{2}$$

$$= \frac{4 \pm 2\sqrt{3}}{2}$$
$$= 2 \pm \sqrt{3}$$

Hence,

$$S.S. = \left\{2 \pm \sqrt{3}, \frac{-1 \pm \sqrt{-3}}{2}\right\}$$
$$Or = \left\{2 \pm \sqrt{3}, \omega, \omega^2\right\}$$

19. $x^2 + x - 4 + \frac{1}{x} + \frac{1}{x^2} = 0$

Solution:

$$x^2 + \frac{1}{x^2} + 2 + x + \frac{1}{x} - 4 - 2 = 0$$
$$\left(x + \frac{1}{x}\right)^2 + \left(x + \frac{1}{x}\right) - 6 = 0$$

Let

$$x + \frac{1}{x} = y$$

$$y^2 + y - 6 = 0$$
$$y^2 + 3y - 2y - 6 = 0$$
$$y(y+3) - 2(y+3) = 0$$
$$(y+3)(y-2) = 0$$

$y - 2 = 0$ **and** $y + 3 = 0$

$y = 2$ $\quad$ $y = -3$

Therefore,

$x + \frac{1}{x} = 2$ **and** $x + \frac{1}{x} = -3$

$x^2 + 1 = 2x$ $\quad$ $x^2 + 1 = -3x$

$$x^2 - 2x + 1 = 0$$
$$x^2 - x - x + = 0$$
$$x(x-1) - 1(x-1) = 0$$
$$(x-1)(xx-1) = 0$$

$$x - 1 = 0 \quad \textbf{and} \quad x - 1 = 0$$
$$x = 1 \qquad\qquad x = 1$$

$$x^2 + 3x + 1 = 0$$
$$= \frac{-3 \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$$
$$= \frac{-3 \pm \sqrt{9-4}}{2}$$
$$x = \frac{-3\sqrt{5}}{2}$$

Hence, $S.S = \left\{1, \frac{-3 \pm \sqrt{5}}{2}\right\}$

20. $\left(x - \frac{1}{x}\right)^2 + 3\left(x + \frac{1}{x}\right) = 0$

Solution:

$$x^2 + \frac{1}{x^2} - 2 + 3\left(x + \frac{1}{x}\right) - 4 = 0$$
$$\left(x + \frac{1}{x}\right)^2 + 3\left(x + \frac{1}{x}\right) - 4 = 0$$
let
$$x + \frac{1}{x} = y$$
$$y^2 + 3y - 4 = 0$$
$$y^2 + 4y - y - 4 = 0$$

$y(y+4)-1(y+4)=0$
$(y-1)(y+4)=0$

$y-1=0$ and $y+4=0$
$y=1$ $y=-4$

Therefore,

$x+\frac{1}{x}=1$ and $x+\frac{1}{x}=-4$
$x^2+1=x$ $x^2+1=-4x$
$x^2+x+1=0$ $x^2+1=-4x$

$x^2-x+1=0$

$x=\frac{+1\pm\sqrt{1-4}}{2}$
$x=\frac{1\pm\sqrt{-3}}{2}$
and
$x^2+4x+1=0$
$x=\frac{-4\pm\sqrt{16-4}}{2}$
$=\frac{-4\pm 2\sqrt{-3}}{2}$
$=2\pm\sqrt{3}$

Hence, $S.S>=\left\{\frac{1\pm\sqrt{-3}}{2};-2\pm\sqrt{3}\right\}$

21. $2x^2-x^3+x^2-x+2=0$

Solution:

Dividing $2x^2-x^3+x^2-x+2=0$ by x^2

$$\frac{2x^4}{x^2}-\frac{x^3}{x^2}+\frac{x^2}{x^2}-\frac{x}{x^2}+\frac{2}{x^2}=0$$

$$2x^2-x+1-\frac{1}{x}+2\frac{1}{x^2}=0$$

$$2\left(x^2+\frac{1}{x^2}\right)-\left(x+\frac{1}{x}\right)+1=0$$

let

$$x+\frac{1}{x}=y$$

then

$$x^2+\frac{1}{x^2}+2=y^2$$

$$2\left(y^2-2\right)-y+1=0$$

$$2y^2-4-y+1=0$$

$$2y^2-y--3=0$$

$$2y^2-3y+2y-3=0$$

$$y(2y-3)+1(2y-3)=0$$

$$(y+1)(2y-3)=0$$

$y-1=0$ or $2y-3=0$

$y=1$ $\qquad$ $y=\frac{3}{2}$

Therefore,

$x+\frac{1}{x}=1$ **or** $x+\frac{1}{x}=\frac{3}{2}$

$$x^2+1=x$$

$$x^2-x+1=0$$

$$x=\frac{-1\pm\sqrt{1-4}}{2}$$

$$x=\frac{-1\pm\sqrt{-3}}{2}$$

$$x+\frac{1}{x}=\frac{3}{2}$$

$$2x^2+2=3x$$

$2x^2-3x+2=0$

$2x^2-4x+x+2=0$

$2x(x-2)+1(x+2)=0$

$(2x+1)(x-2)=0$

$2x+1=0 \Rightarrow x=-\dfrac{1}{2} \Rightarrow x-2=0 \Rightarrow x=2$

Hence, $S.S.=\left\{2,-\dfrac{1}{2},\dfrac{-1\pm\sqrt{-3}}{2}\right\}$

22. $2x^4+3x^3-4x^2-3x+2=0$

Solution:

Dividing $2x^4+3x^3-4x^2-3x+2=0$ by x^2

$\dfrac{2x^4}{x^2}+\dfrac{3x^3}{x^2}-\dfrac{4x^2}{x^2}-\dfrac{3x}{x^2}+\dfrac{2}{x^2}=0$

$2x^2+3x-4-3\dfrac{1}{x}+2\dfrac{1}{x^2}=0$

$2x^2+2\dfrac{1}{x^2}+3x-3\dfrac{1}{x}2\dfrac{1}{x^2}=0$ \

$2x^2+2\dfrac{1}{x^2}+3x-3\dfrac{1}{x}-4=0$

$2\left(x^2+\dfrac{1}{x^2}\right)+3\left(x-\dfrac{1}{x}\right)-4=0$

Let

$x-\dfrac{1}{x}=y$

$x^2-\dfrac{1}{x^2}-2=y^2$

$x^2+\dfrac{1}{x^2}=y^2+2$

$2(y^2+2)+3(y)-4=0$

$$2y+4+3y-4=0$$
$$2y^2+3y=0$$
$$y(2y+3)=0$$

$y=0$ **or** $y=\dfrac{-3}{2}$

Therefore,

$x-\dfrac{1}{x}=0$ **or** $x-\dfrac{1}{x}=\dfrac{-3}{2}$

$x^2-1=0$ $\quad$ $2x^2-2=-3x$

$$x^2=1$$
$$x=\pm 1$$
$$2x^2-2+3x=0$$
$$2x^2+3x-2=0$$
$$2x^2+4x-x-2=0$$
$$2x(x+2)-2(x+2)=0$$
$$(x+2)(2x-1)=0$$

$x+2=0$ **or** $2x=1$

$x=-2$ $\quad$ $x=\dfrac{1}{2}$

Hence $S.S.=\left\{-1,1,-2,\dfrac{1}{2}\right\}$

23. $6x^2-35x^3+62x^2-35x+6=0$

Solution:

Divide equation by x^2

$$\frac{-6x^4}{x^2}-\frac{35x^3}{x^2}+\frac{62x^2}{x^2}-\frac{35x}{x^2}+\frac{6}{x^2}=0$$
$$6x^2-35x+62-35\frac{1}{x}+\frac{6}{x^2}=0$$

$$6\left(x^2+\frac{1}{x^2}\right)-35\left(x+\frac{1}{x}\right)+62=0$$

Let

$$x+\frac{1}{x}=y$$
$$x^2+\frac{1}{x^2}+2=y^2$$
$$x^2+\frac{1}{x^2}=y^2-2$$

$$6y^2-12-35y+62=0$$
$$6y^2-35y+62=0$$

So $$6y^2-15y-20y+50=0$$
$$3y(2y-5)-10(2y-5)=0$$
$$(3y-10)(2y-5)=0$$

$\Rightarrow$ 3y-10=0 or 2y-5=0

$y=\frac{10}{3}$ Or $y=\frac{5}{2}$

Therefore,

$x+\frac{1}{x}=\frac{10}{3}$; $x+\frac{1}{x}=\frac{5}{2}$

$3x^2+3=10x$ $2x^2+2=5x$

So,

$3x^2-10x+3=0$

$3x^2-9x-x+3=0$

$3x(x-3)-1(x-3)=0$

$3x-1=0$; $x-3=0$

$3x=1$

$\Rightarrow \qquad x+\frac{1}{3} \qquad\qquad$ x=3

And

$$2x^2-5x+2=0$$
$$2x^2-4x-x+2=0$$
$$2x(x-2)-1(x-2)=0$$
$$(2x-1)(x-2)=0$$

2x-1=0 or x-2=0

$x=\frac{1}{2}$ x=2

Hence, S.S = $\left\{3,2,\frac{1}{2},\frac{1}{3}\right\}$

24. $x^4-6x^2+10-\frac{6}{x^2}+\frac{1}{x^4}=0$

Solution:

$$x^4+\frac{1}{x^4}-6\left(x^2+\frac{1}{x^2}\right)+10=0$$

Let

$$x^2+\frac{1}{x^2}=y$$
$$x^4+\frac{1}{x^4}+2=y^2$$
$$x^4+\frac{1}{x^4}=y^2-2$$
$$y^2-2-6y+10=0$$
$$y^2-6y+8=0$$
$$y^2-4y-2y+8=0$$
$$y(y-4)-2(y-4)=0$$
$$(y-2)(y-4)=0$$

y-2=0 0r y-4=0

y=2 y=4

therefore

$x+\frac{1}{x}=2$ or $x+\frac{1}{x}=4$

$x^2+1=2x$ $x^2+1=4x$

$$x^2-2x+1=0$$

$$x^2-4x+1=0$$
$$x=\frac{2\pm\sqrt{4-4(1)(1)}}{2}$$
$$x=\frac{2\pm\sqrt{4-4}}{2}$$
$$x=\frac{2\pm\sqrt{0}}{2}$$
$$x=\frac{2}{2}$$
$$x=1$$

And

$$x^2-4x+1=0$$
$$x=\frac{\pm4\pm\sqrt{16-4(1)(1)}}{2}$$
$$x=\frac{4\pm\sqrt{16-4}}{2}$$
$$x=\frac{4\pm\sqrt{12}}{2}$$
$$x=\frac{4\pm2\sqrt{3}}{2}$$
$$x=2\pm\sqrt{3}$$

Hence, S.S = $\left\{1,2\pm\sqrt{3}\right\}$