Exercise 3.2

Q1 If $A = [a_{ij}]_{3\times3}$ then show that

i. $I_3A = A$ **ii.** $AI_4A = A$

Solution

Let $A = [a_{ij}]_{3\times3}$

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

i. $I_3A = A$

Solution

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

L.H.S

$$I_3A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}+0+0 & a_{12}+0+0 & a_{13}+0+0 & a_{14}+0+0 \\ a_{21}+0+0 & a_{22}+0+0 & a_{23}+0+0 & a_{24}+0+0 \\ a_{31}+0+0 & a_{32}+0+0 & a_{33}+0+0 & a_{34}+0+0 \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} = A = R.H.S$$

i. $AI_4A = A$

Solution

$$AI_4 = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}+0+0 & a_{12}+0+0 & a_{13}+0+0 & a_{14}+0+0 \\ a_{21}+0+0 & a_{22}+0+0 & a_{23}+0+0 & a_{24}+0+0 \\ a_{31}+0+0 & a_{32}+0+0 & a_{33}+0+0 & a_{34}+0+0 \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} = A = R.H.S$$

Q2. Find the inverse of the following matrices

i. $\begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$ ii. $\begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$ iii. $\begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ iv. $\begin{bmatrix} 1 & 2 \\ 6 & 3 \end{bmatrix}$

i. $\begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$

Solution

$$\begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$$

Let $A = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$

$$A^{-1} = \frac{Adj\,A}{|A|}$$

$$|A| = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix} = 3+2 = 5$$

$$\text{Adj } A = \begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{Adj\,A}{|A|} = \frac{1}{5}\begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1/5 & 1/5 \\ -2/5 & 3/5 \end{bmatrix}$$

Hence inverse of matrix = $\begin{bmatrix} 1/5 & 1/5 \\ -2/5 & 3/5 \end{bmatrix}$

ii. $\begin{bmatrix} -2 & 3 \\ -4 & 5 \end{bmatrix}$

Solution

Let B = $\begin{bmatrix} -2 & 3 \\ -4 & 5 \end{bmatrix}$

$$B^{-1} = \frac{Adj\ B}{|B|}$$

|B| = $\begin{bmatrix} -2 & 3 \\ -4 & 5 \end{bmatrix}$ = -10+12 = 2

Adj B = $\begin{bmatrix} 5 & -3 \\ 4 & -2 \end{bmatrix}$

$$B^{-1} = \frac{Adj\ B}{|B|} = \frac{1}{2}\begin{bmatrix} 5 & -3 \\ 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 5/2 & -3/2 \\ 4/2 & -2/2 \end{bmatrix}$$

$$= \begin{bmatrix} 5/2 & -3/2 \\ 2 & -1 \end{bmatrix}$$

Hence inverse of matrix = $\begin{bmatrix} 5/2 & -3/2 \\ 2 & -1 \end{bmatrix}$

iii. $\begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$

Solution

Let C = $\begin{bmatrix} \mathbf{2i} & \mathbf{i} \\ \mathbf{i} & \mathbf{-i} \end{bmatrix}$

$C^{-1} = \frac{Adj\ C}{|C|}$

| C | = $\begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ = -2$i^2 - i^2$

=2(1)-(-1)

= 2+1 = 3

Adj C = $\left[\begin{bmatrix} -i & -i \\ -i & 2i \end{bmatrix}\right]$

$C^{-1} = \frac{Adj\ C}{|C|} = \frac{1}{3}\begin{bmatrix} -i & -i \\ -i & 2i \end{bmatrix}$

= $\begin{bmatrix} -i/_3 & i/_3 \\ i/_3 & 2i/_3 \end{bmatrix}$

Hence inverse of matrix = $\begin{bmatrix} -i/_3 & i/_3 \\ i/_3 & 2i/_3 \end{bmatrix}$

iv. $\begin{bmatrix} \mathbf{2} & \mathbf{1} \\ \mathbf{6} & \mathbf{3} \end{bmatrix}$

Solution

Let D = $\begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$

$D^{-1} = \frac{Adj\ D}{|D|}$

$adjD = \begin{bmatrix} 3 & -1 \\ -6 & 2 \end{bmatrix}$

| D | = $\begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$ = 6-6 = 0

Hence , inverse cannot be determined

Q3. Solve the following systems of linear equations

i. $\begin{cases} 2x_1 - 3x_2 = 5 \\ 5x_1 + x_2 = 4 \end{cases}$ ii. $\begin{cases} 4x_1 + 3x_2 = 5 \\ 3x_1 - x_2 = 7 \end{cases}$ iii. $\begin{cases} 3x_1 - 5y = 1 \\ -2x_2 + y = -3 \end{cases}$

i. $2x_1 - 3x_2 = 5$
$5x_1 + x_2 = 4$

Solution

$$\begin{bmatrix} 2 & -3 \\ 5 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

Let A $\begin{bmatrix} 2 & -3 \\ 5 & 1 \end{bmatrix}$; $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$; $B = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

$$Ax = B$$

$$X = A^{-1} B$$

$$A^{-1} = \frac{Adj\ A}{|A|}$$

$$\text{Adj A} = \begin{bmatrix} 2 & -3 \\ 5 & 1 \end{bmatrix} = 2+15 = 17$$

$$A^{-1} = \frac{1}{17}\begin{bmatrix} 1 & 3 \\ -5 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 1/17 & 3/17 \\ -5/17 & 2/17 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1/17 & 3/17 \\ -5/17 & 2/17 \end{bmatrix}\begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 5/17 + & 12/17 \\ -25/17 + & 8/17 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{5+12}{17} \\ \frac{-25+8}{17} \end{bmatrix} = \begin{bmatrix} 17/17 \\ -17/17 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Therefore $x = A^{-1}B$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Thus $x_1 = 1$

$x_1 = 1$

ii. $\mathbf{4x_1 + 3x_2 = 5}$
$\mathbf{3x_1 - x_2 = 7}$

Solution

Let $A = \begin{bmatrix} 4 & 3 \\ 3 & -1 \end{bmatrix}$; $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$; $B = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$

$Ax = B$

$X = A^{-1} B$

$$A^{-1} = \frac{Adj\ A}{|A|}$$

$$|A| = \begin{vmatrix} 4 & 3 \\ 3 & -1 \end{vmatrix} = -4 - 9 = -13$$

$$\text{Adj } A = \begin{bmatrix} -1 & -3 \\ -3 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{13}\begin{bmatrix} -1 & -3 \\ -3 & 4 \end{bmatrix}$$

Thus $X = A^{-1}B$

$$A^{-1} = \frac{-1}{13}\begin{bmatrix} -1 & -3 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

$$= \frac{-1}{13}\begin{bmatrix} -5 & 21 \\ -15 & 28 \end{bmatrix}$$

$$= \frac{-1}{13}\begin{bmatrix} -26 \\ 13 \end{bmatrix} = \begin{bmatrix} -26/_{-13} \\ 13/_{-13} \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

Therefore $x = A^{-1}B$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

Thus $x_1 = 2$

$x_1 = -1$

iii. $\mathbf{3x_1 - 5y = 1}$
$\mathbf{-2x_2 + y = -3}$

Solution

Let $A = \begin{bmatrix} 3 & -5 \\ -2 & 1 \end{bmatrix}$; $x = \begin{bmatrix} x \\ y \end{bmatrix}$; $B = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

Ax = B

X $= A^{-1} B$

$A^{-1} = \frac{Adj\ A}{|A|}$

| A | $= \begin{vmatrix} 3 & -5 \\ -2 & 1 \end{vmatrix}$ = 3-10 = -7

Adj A $= \begin{bmatrix} 1 & 5 \\ 2 & 3 \end{bmatrix}$

$A^{-1} = \frac{-1}{7}\begin{bmatrix} 1 & 5 \\ 2 & 3 \end{bmatrix}$

Thus X = A^{-1}B

$A^{-1} = \frac{-1}{7}\begin{bmatrix} 1 & 5 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

$= \frac{-1}{7}\begin{bmatrix} 1 & - & 15 \\ 2 & - & 9 \end{bmatrix}$

$= \frac{-1}{7}\begin{bmatrix} -14 \\ -7 \end{bmatrix}$

= $\begin{bmatrix} -14/-7 \\ -7/-7 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

Therefore x = $A^{-1}B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Thus $x = 2$

$y = 1$

Q4. If A = $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 5 \\ -1 & 0 & 4 \end{bmatrix}$, B = $\begin{bmatrix} 2 & 1 & -1 \\ 1 & 3 & 4 \\ -1 & 2 & 1 \end{bmatrix}$ and C = $\begin{bmatrix} 1 & 3 & -2 \\ -1 & 2 & 0 \\ 3 & 4 & -1 \end{bmatrix}$ then find

i. A-B ii. B -A iii. (A-B)-C iv. A-(B-C)

i. A-B

Solution

$$A-B = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 5 \\ -1 & 0 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 1 & -1 \\ 1 & 3 & 4 \\ -1 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2 & -1-1 & 2+1 \\ 3-1 & 2-3 & 5-4 \\ -1+1 & 0-2 & 4-1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -2 & 3 \\ 2 & -1 & 1 \\ 0 & -2 & 3 \end{bmatrix}$$

Hence $A - B = \begin{bmatrix} -1 & -2 & 3 \\ 2 & -1 & 1 \\ 0 & -2 & 3 \end{bmatrix}$

ii. B-A

Solution

$$B - A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 3 & 4 \\ -1 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 5 \\ -1 & 0 & 4 \end{bmatrix}$$

$$=\begin{bmatrix} 2-1 & 1+1 & -1-2 \\ 1-3 & 3-2 & 4-5 \\ -1+1 & 2-0 & 1-4 \end{bmatrix}$$

$$=\begin{bmatrix} 1 & 2 & -3 \\ -2 & 1 & -1 \\ 0 & 2 & -3 \end{bmatrix}$$

Hence $B-A = \begin{bmatrix} 1 & 2 & -3 \\ -2 & 1 & -1 \\ 0 & 2 & -3 \end{bmatrix}$

iii. (A-B)-C

Solution

We know that $A-B = \begin{bmatrix} -1 & -2 & 3 \\ 2 & -1 & 1 \\ 0 & -2 & 3 \end{bmatrix}$

So, $(A-B)-C = \begin{bmatrix} -1 & -2 & 3 \\ 2 & -1 & 1 \\ 0 & -2 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 3 & -2 \\ -1 & 2 & 0 \\ 3 & 4 & -1 \end{bmatrix}$

$$=\begin{bmatrix} -1-1 & -2-3 & 3+2 \\ 2+1 & -1-2 & 1-0 \\ 0-3 & -2-4 & 3+1 \end{bmatrix}$$

$$=\begin{bmatrix} -2 & -5 & 5 \\ 3 & -3 & 1 \\ -3 & -6 & 4 \end{bmatrix}$$

iv. A-(B-C)

Solution

$B-C = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 3 & 4 \\ -1 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 3 & -2 \\ -1 & 2 & 0 \\ 3 & 4 & -1 \end{bmatrix}$

$$=\begin{bmatrix} 2-1 & 1-3 & -1+2 \\ 1+1 & 3-2 & 4-0 \\ -1-3 & 2-4 & 1+1 \end{bmatrix}$$

$$=\begin{bmatrix}1 & -2 & 1\\2 & 1 & 4\\-4 & -2 & 2\end{bmatrix}$$

Then $A-(B-C) = \begin{bmatrix}1 & -1 & 2\\3 & 2 & 5\\-1 & 0 & 4\end{bmatrix} - \begin{bmatrix}1 & -2 & 1\\2 & 1 & 4\\-4 & -2 & 2\end{bmatrix}$

$$= \begin{bmatrix}1-1 & -1+2 & 2-1\\3-2 & 2-1 & 5-4\\-1+4 & 0+2 & 4-2\end{bmatrix}$$

$$=\begin{bmatrix}0 & 1 & 1\\1 & 1 & 1\\3 & 2 & 2\end{bmatrix}$$

Hence $A-(B-C) = \begin{bmatrix}0 & 1 & 1\\1 & 1 & 1\\3 & 2 & 2\end{bmatrix}$

Q5. If $A = \begin{bmatrix}i & 2i\\1 & -i\end{bmatrix}$ $B = \begin{bmatrix}2i & -1\\-i & i\end{bmatrix}$ and $C = \begin{bmatrix}2i & -1\\-i & i\end{bmatrix}$ then show that

i) (AB)C = A(BC) ii) (A+B)C = AC + BC

Solution

i) (AB)C = A(BC)

L.H.S = A(BC)

$AB = \begin{bmatrix}i & 2i\\1 & -i\end{bmatrix}\begin{bmatrix}2i & -1\\-i & i\end{bmatrix}$

$$=\begin{bmatrix}-i^2+4i^2 & -i^2+2i^2\\i^2-2i^2 & 1-i^2\end{bmatrix}$$

$$= \begin{bmatrix}1-4 & 2-2\\-1+2 & 1+1\end{bmatrix}$$

$$=\begin{bmatrix}-3 & i-2\\i+2 & 2\end{bmatrix}$$

$(AB)C = \begin{bmatrix}-3 & i-2\\i+2 & 2\end{bmatrix}\begin{bmatrix}2i & -1\\-i & i\end{bmatrix}$

$$=\begin{bmatrix}-6-i-i(i-2) & 3+i(i-2)\\2i(-i+2)-2i & -1(-i+2)+2i\end{bmatrix}$$

$$= \begin{bmatrix} -6i - i^2 + 2i & 3 - 1 - 2i \\ -2i^2 + 4i - 2i & i - 2 + 2i \end{bmatrix}$$

$$= \begin{bmatrix} -4i + 1 & -2i + 2 \\ 2 + 2i & 3i - 2 \end{bmatrix}$$

R.H.S = A(BC)

$$BC = \begin{bmatrix} 2i & -1 \\ -\text{i} & i \end{bmatrix} \begin{bmatrix} i & 1 \\ 2\text{i} & i \end{bmatrix}$$

$$= \begin{bmatrix} 2i - i^2 & i + i \\ 4i^2 - i^2 & -2i + i^2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 - i & 2i \\ -4 + 1 & -2i - 1 \end{bmatrix}$$

$$BC = \begin{bmatrix} 2 - i & 2i \\ -3 & -2i - 1 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} i & 2i \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 2 - i & 2i \\ -3 & -2i - 1 \end{bmatrix}$$

$$= \begin{bmatrix} i(i-2) - 6i & i(2i) + 2i(-2i - 1) \\ 1(2 - i) + 3i & 1(2i) - i(-2i - 1) \end{bmatrix}$$

$$= \begin{bmatrix} 2i - i^2 + 6i & 2i^2 - 4i^2 - 2i \\ 2 - i + 3i & 2i + 2i^2 + i \end{bmatrix}$$

$$= \begin{bmatrix} -4i + 1 & -2i + 2 \\ 2 + 2i & 3i - i \end{bmatrix}$$

= R.H.S

Hence proved

(AB)C = A(BC)

ii) (A+B)C = AC +BC

L.H.S = (A+B)C

$$A+B = \begin{bmatrix} i & 2i \\ 1 & -i \end{bmatrix} + \begin{bmatrix} 2i & -1 \\ -\text{i} & i \end{bmatrix}$$

$$= \begin{bmatrix} i - i & 2i + 1 \\ 1 + 2i & -i + i \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2i + 1 \\ 2i + 1 & 0 \end{bmatrix}$$

$$(A+B)C = \begin{bmatrix} 0 & 2i + 1 \\ 2i + 1 & 0 \end{bmatrix} \begin{bmatrix} 2i & -1 \\ -\text{i} & i \end{bmatrix}$$

$$= \begin{bmatrix} 0 + (2i + 1)(-i) & 0 + i(2i + 1) \\ 2i(2i + 1) + 0 & -1(2i + 1) + 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 - 2i - i^2 & 2i^2 + 1 \\ 4i^2 + 2i & -2i - 1 \end{bmatrix}$$

$$(A+B)C = \begin{bmatrix} 2 - i & -2 + i \\ -4 + 2i & -2i - 1 \end{bmatrix}$$

R.H.S = AC +BC

$$AC = \begin{bmatrix} i & 2i \\ 1 & -i \end{bmatrix} \begin{bmatrix} 2i & -1 \\ -\mathrm{i} & i \end{bmatrix}$$

$$= \begin{bmatrix} 2i^2 - 2i^2 & -i + 2i^2 \\ 2i + i^2 & -1 - i^2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -i - 2 \\ 2i - 1 & 0 \end{bmatrix}$$

$$AC = \begin{bmatrix} 0 & -i - 2 \\ 2i - 1 & 0 \end{bmatrix}$$

$$BC = \begin{bmatrix} 2i & -1 \\ -\mathrm{i} & i \end{bmatrix} \begin{bmatrix} i & 1 \\ 2\mathrm{i} & i \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{2i - i^2} & \mathbf{i + i} \\ \mathbf{4i^2 - i^2} & \mathbf{-2i + i^2} \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{2 - i} & \mathbf{2i} \\ \mathbf{-4 - 1} & \mathbf{-2i - 1} \end{bmatrix}$$

$$BC = \begin{bmatrix} 2 - i & 2i \\ -5 & -2i - 1 \end{bmatrix}$$

$$AC+BC = \begin{bmatrix} 0 & -i - 2 \\ 2i - 1 & 0 \end{bmatrix} + \begin{bmatrix} 2 - i & 2i \\ -5 & -2i - 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 - i & -i + 2 \\ -4i + 2 & -2i - 1 \end{bmatrix}$$

= R.H.S

Hence proved

(A+B)C = AC +C

Q6. If A and B are square matrices of the same order , then explain why in general

i) $(A+B)^2 \neq A^2 + 2AB + B^2$

ii) $(A-B)^2 \neq A^2 - 2AB + B^2$

iii) **(A+B)(A-B)** $\neq A^2 - B^2$

Solution

L.H.S $= (A+B)^2$

=(A+B)(A+B)

=A(A+B)+B(A+B)

=A.A +A.B+B.A+B.B

$= A^2 + A.B + B.A + B^2$

As, we know that matrix doesn't has commutative property

Therefore A.B $\neq B.A$

Hence it is clear that $(A+B)^2 \neq A^2 + 2AB + B^2$

i) $(A-B)^2 \neq A^2 - 2AB + B^2$

L.H.S $= (A-B)^2$

=(A-B)(A-B)

=A(A-B)-B(A-B)

=A.A -A.B-B.A-B.B

$= A^2 - A.B - B.A - B^2$

As, we know that matrix doesn't has commutative property

Therefore A.B $\neq B.A$

Hence it is clear that $(A-B)^2 \neq A^2 - 2AB + B^2$

ii) **(A+B)(A-B)** $\neq A^2 - B^2$

L.H.S = (A+B)(A-B)

=A(A-B)+B(A-B)

=A.A -A.B +B.A-B.B

$= A^2 - A.B + B.A - B^2$

As, we know that matrix doesn't has commutative property

Therefore A.B $\neq B.A$

Hence it is clear that $(A+B)(A-B) \neq A^2 - B^2$

Q7. If A = $\begin{bmatrix} 2 & -1 & 3 & 0 \\ 1 & 0 & 4 & -2 \\ -3 & 5 & 2 & -1 \end{bmatrix}$ **then find AA^t and A^tA**

Solution

$$A = \begin{bmatrix} 2 & -1 & 3 & 0 \\ 1 & 0 & 4 & -2 \\ -3 & 5 & 2 & -1 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 2 & -1 & 3 & 0 \\ 1 & 0 & 4 & -2 \\ -3 & 5 & 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 & 3 & 0 \\ 1 & 0 & 4 & -2 \\ -3 & 5 & 2 & -1 \end{bmatrix}$$

$$A.\ A^t = \begin{bmatrix} 2 & -1 & 3 & 0 \\ 1 & 0 & 4 & -2 \\ -3 & 5 & 2 & -1 \end{bmatrix} \begin{bmatrix} 2 & 1 & -3 \\ -1 & 0 & 5 \\ 3 & 4 & 2 \\ 0 & -2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1+9+0 & 2+0+12+0 & -6-6+6+0 \\ 2+0+12+0 & 1+0+16+4 & -3+0+8+2 \\ -6-5+6+0 & -3+0+8+2 & 9+25+4+1 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 14 & -5 \\ 14 & 21 & 7 \\ -5 & 7 & 39 \end{bmatrix}$$

Hence $AA^t = \begin{bmatrix} 14 & 14 & -5 \\ 14 & 21 & 7 \\ -5 & 7 & 39 \end{bmatrix}$

$$A^tA = \begin{bmatrix} 2 & 1 & -3 \\ -1 & 0 & 5 \\ 3 & 4 & 2 \\ 0 & -2 & -1 \end{bmatrix} \begin{bmatrix} 2 & -1 & 3 & 0 \\ 1 & 0 & 4 & -2 \\ -3 & 5 & 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1+9 & -2+0-15 & 6+4-6 & 0-2+3 \\ -2+0-15 & 1+0+25 & -3+0+10 & 0+0-5 \\ 6+4-6 & -3+0+10 & 9+16+4 & 0-8-2 \\ 0-2+3 & 0+0-5 & 0-8-2 & 0+4+1 \end{bmatrix}$$

Hence $A^tA = \begin{bmatrix} 14 & -17 & 4 & 1 \\ -17 & 26 & +7 & -5 \\ 4 & 7 & 29 & -10 \\ 1 & -5 & -10 & 5 \end{bmatrix}$

Q8. Solve the following matrix equations for X

i) 3X -2A = B If $A = \begin{bmatrix} 2 & 3 & -2 \\ -1 & 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -3 & 1 \\ 5 & 4 & -1 \end{bmatrix}$

ii) 2X-3A = B If $A = \begin{bmatrix} 1 & -1 & 2 \\ -2 & 4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$

Solution

i) 3X -2A = B

$3X-2\begin{bmatrix} 2 & 3 & -2 \\ -1 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 2 & -3 & 1 \\ 5 & 4 & -1 \end{bmatrix}$

$3X-\begin{bmatrix} 4 & 6 & -4 \\ -2 & 2 & 10 \end{bmatrix} = \begin{bmatrix} 2 & -3 & 1 \\ 5 & 4 & -1 \end{bmatrix}$

$$3X = \begin{bmatrix} 4 & 6 & -4 \\ -2 & 2 & 10 \end{bmatrix} + \begin{bmatrix} 2 & -3 & 1 \\ 5 & 4 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2+4 & -3+6 & 1-4 \\ 5-2 & 4+2 & -1+10 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 3 & -3 \\ 3 & 6 & 9 \end{bmatrix}$$

$$X = \frac{1}{3}\begin{bmatrix} 6 & 3 & -3 \\ 3 & 6 & 9 \end{bmatrix}$$

$$X = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 2 & 3 \end{bmatrix}$$

Hence the value of $X = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 2 & 3 \end{bmatrix}$

ii) 2X-2A = B

$$2X - 3\begin{bmatrix} 1 & -1 & 2 \\ -2 & 4 & 5 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$$

$$2X - \begin{bmatrix} 3 & -3 & 6 \\ -6 & 12 & 15 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$$

$$2X = \begin{bmatrix} 3 & -3 & 6 \\ -6 & 12 & 15 \end{bmatrix} + \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3+3 & -1-3 & 0+6 \\ 4-6 & 2+12 & 1+15 \end{bmatrix}$$

$$2X = \begin{bmatrix} 4 & -4 & 6 \\ -2 & 14 & 16 \end{bmatrix}$$

$$X = \frac{1}{2}\begin{bmatrix} 4 & -4 & 6 \\ -2 & 14 & 16 \end{bmatrix}$$

$$X = \begin{bmatrix} 3 & -2 & 3 \\ -1 & 7 & 8 \end{bmatrix}$$

Hence, $X = \begin{bmatrix} 3 & -2 & 3 \\ -1 & 7 & 8 \end{bmatrix}$

Q9. Solve the following matrix equations for A:

i) $\begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix}$ **A-** $\begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$ **=** $\begin{bmatrix} -1 & -4 \\ 3 & 6 \end{bmatrix}$ **ii) A** $\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$ **-** $\begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix}$ **=** $\begin{bmatrix} 2 & 0 \\ -1 & 5 \end{bmatrix}$

Solution

i) $\begin{bmatrix}4 & 3\\2 & 2\end{bmatrix}$ **A-** $\begin{bmatrix}2 & 3\\-1 & -2\end{bmatrix}$ **=** $\begin{bmatrix}-1 & -4\\3 & 6\end{bmatrix}$

$$\begin{bmatrix}4 & 3\\2 & 2\end{bmatrix} A = \begin{bmatrix}-1 & -4\\3 & 6\end{bmatrix} + \begin{bmatrix}2 & 3\\-1 & -2\end{bmatrix}$$

$$= \begin{bmatrix}1+2 & -4+2\\3-1 & 6-2\end{bmatrix}$$

$$= \begin{bmatrix}1 & -1\\2 & 4\end{bmatrix}$$

$$\begin{bmatrix}4 & 3\\2 & 2\end{bmatrix} A = \begin{bmatrix}1 & -1\\2 & 4\end{bmatrix}$$

Let $B = \begin{bmatrix}4 & 3\\2 & 2\end{bmatrix}$

$$BA = \begin{bmatrix}1 & -1\\2 & 4\end{bmatrix}$$

$$A = B^{-1}\begin{bmatrix}1 & -1\\2 & 4\end{bmatrix}$$

$$B^{-1} = \frac{Adj\ B}{|B|}$$

$$|B| = \begin{vmatrix}4 & 3\\2 & 2\end{vmatrix} = 8-6 = 2$$

$$Adj\ B = \begin{bmatrix}2 & -3\\-2 & 4\end{bmatrix}$$

$$B^{-1} = \frac{Adj\ B}{|B|} = \frac{1}{2}\begin{bmatrix}2 & -3\\-2 & 4\end{bmatrix}$$

$$A = \frac{1}{2}\begin{bmatrix}1 & -1\\2 & 4\end{bmatrix}\begin{bmatrix}2 & -3\\-2 & 4\end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix}2-6 & -2-12\\-2+8 & 2+16\end{bmatrix}$$

$$= \begin{bmatrix}-4/2 & -14/2\\6/2 & 18/2\end{bmatrix}$$

$$A = \begin{bmatrix} -2 & -7 \\ 3 & 9 \end{bmatrix}$$

Thus value of matrix 'A' = $\begin{bmatrix} -2 & -7 \\ 3 & 9 \end{bmatrix}$

ii) $\mathbf{A\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ -1 & 5 \end{bmatrix}}$

$$A\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ -1 & 5 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 0+2 \\ -1+3 & 5+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}$$

$$A\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}$$

Let $B = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$

$$A.B = \begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}$$

The value of A is $A = \begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix} B^{-1}$

$$B^{-1} = \frac{Adj\ B}{|B|}$$

$$|B| = \begin{vmatrix} 3 & 1 \\ 4 & 2 \end{vmatrix} = 6-4 = 2$$

$$Adj\ B = \begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix}$$

$$B^{-1}B = \frac{Adj\ B}{|B|} = \frac{1}{2}\begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix}$$

$$A = \frac{1}{2}\begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 2-8 & -1+6 \\ 4-28 & 2+18 \end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix}-6 & 5\\-20 & 16\end{bmatrix}$$

$$=\begin{bmatrix}-6/2 & 5/2\\-20/2 & 16/2\end{bmatrix}$$

$$A = \begin{bmatrix}-3 & \frac{5}{2}\\-10 & 8\end{bmatrix}$$

Thus value of matrix 'A' = $\begin{bmatrix}-3 & \frac{5}{2}\\-10 & 8\end{bmatrix}$