CHAPTER 3

Matrices and Determinants

Exercise 3.1

Q1. If A = $\begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 7 \\ 6 & 4 \end{bmatrix}$ then show that

i. 4A -3A = A ii. 3B-3A = 3(B-A)

Solution

i. 4A-2A = A

A = $\begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$

Then

$$4A = 4\begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 8 & 12 \\ 4 & 20 \end{bmatrix}$$

And

$$3A = 3\begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 9 \\ 3 & 15 \end{bmatrix}$$

L.H.S = R.H.S

$$= \begin{bmatrix} 8 & 12 \\ 4 & 20 \end{bmatrix} - \begin{bmatrix} 6 & 9 \\ 3 & 15 \end{bmatrix}$$

$$= \begin{bmatrix} 8-6 & 12-9 \\ 4-3 & 20-15 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$$

= A = R.H.S

Hence proved

4A -3A = 4

ii. 3B-2A = 3(B-A)

L.H.S = 3B -2A

As

$B = \begin{bmatrix} 1 & 7 \\ 6 & 4 \end{bmatrix}$ and $A = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$

Then

$3B = 3\begin{bmatrix} 1 & 7 \\ 6 & 4 \end{bmatrix}$ and $3A = 3\begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$

$3B = \begin{bmatrix} 3 & 21 \\ 18 & 12 \end{bmatrix}$ $3A = \begin{bmatrix} 6 & 9 \\ 3 & 15 \end{bmatrix}$

So

$3B - 2A = \begin{bmatrix} 3 & 21 \\ 18 & 12 \end{bmatrix} - \begin{bmatrix} 6 & 9 \\ 3 & 15 \end{bmatrix}$

$= \begin{bmatrix} 3-6 & 12-9 \\ 18-3 & 12-15 \end{bmatrix}$

$= \begin{bmatrix} -3 & 12 \\ 3 & -3 \end{bmatrix}$

$= 3\begin{bmatrix} -1 & 4 \\ 5 & -1 \end{bmatrix}$

And R.H.S = 3(B-A)

So, $B - A = \begin{bmatrix} 1 & 7 \\ 6 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$

$= \begin{bmatrix} 1-2 & 7-3 \\ 6-1 & 4-5 \end{bmatrix}$

$= \begin{bmatrix} -1 & 4 \\ 5 & -1 \end{bmatrix}$

Then $3(B-A) = 3\begin{bmatrix} -1 & 4 \\ 5 & -1 \end{bmatrix}$

= L.H.S

Hence proved

3B-3A = 3(B-A)

Q2. If = $A = \begin{bmatrix} -i & 0 \\ 1 & -i \end{bmatrix}$ then prove that $A^4 = I_2$

L.H.S = A^4

= $(A^2)^2$

=$(A.A)^2$

Solutions

So,

$$A.A = \begin{bmatrix} -i & 0 \\ 1 & -i \end{bmatrix} . \begin{bmatrix} -i & 0 \\ 1 & -i \end{bmatrix}$$

$$= \begin{bmatrix} i^2 + 0 & 0 + 0 \\ i - i & 0 + i^2 \end{bmatrix}$$

$$= \begin{bmatrix} i^2 & 0 \\ 0 & i^2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$(A^2)^2 = \left[\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\right]^2$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 + 0 & 0 + 0 \\ 0 + 0 & 0 + 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2 = R.H.S$$

Hence proved

$$A^4 = I_2$$

Q3. Find x and y if

i) $\begin{bmatrix} x + 3 & 1 \\ -3 & 3y - 4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$

ii) $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} y & 1 \\ -3 & 2x \end{bmatrix}$

Solution

i) $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$

By equating corresponding elements

We have

$x + 3 = 2$

And

$3y - 4 = 2$

$x = 2 - 3$; $3y = 2 + 4$

$x = -1$; $3y = 6$

$x = -1$; $y = \frac{6}{3} = 2$

thus

$x = -1$

and

$y = 2$

ii) $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} y & 1 \\ -3 & 2x \end{bmatrix}$

By equating corresponding elements

We have

$x + 3 = y$

And

$3y - 4 = 2x$

$x+3 = y$; $3y - 4 = 2x$

$x-y = 3$; $3y-2x = 4$

$x-y = 3$; $-2x+3y = 4$

multiply eq. "1" by "2"

$2x - 2y = 3$

Add it in eq"2"

$2x-2y = 3$

$-2 + 3y = 4$

$y = 7$

put the value of "y" in eq. '1'

$x-7 = 3$

$x = 3+7 = 10$

$x = 10$

thus

$x = 10$

$y = 7$

Q4. **IF** $A = \begin{bmatrix} -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix}$ **and** $B = \begin{bmatrix} 0 & 3 & 2 \\ 1 & -1 & 2 \end{bmatrix}$

i) **4A -3B** **ii)** **A +3(B-A)**

Solution

i) $A = \begin{bmatrix} -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix}$

Then

$$4A = 4\begin{bmatrix} -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 8 & 12 \\ 4 & 0 & 8 \end{bmatrix}$$

And

$$3B = 3\begin{bmatrix} 0 & 3 & 2 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 9 & 6 \\ 3 & -3 & 6 \end{bmatrix}$$

Then

$$4A - 3B = \begin{bmatrix} -4 & 8 & 12 \\ 4 & 0 & 8 \end{bmatrix} - \begin{bmatrix} 0 & 9 & 6 \\ 3 & -3 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} -4 - 0 & 8 - 9 & 12 - 6 \\ 4 - 3 & 0 + 3 & 8 - 6 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & -1 & 6 \\ 1 & 3 & 2 \end{bmatrix}$$

Hence $4A - 3B = \begin{bmatrix} -4 & -1 & 6 \\ 1 & 3 & 2 \end{bmatrix}$

ii) A+3(B-A)

$$B-A = \begin{bmatrix} 0 & 3 & 2 \\ 1 & -1 & 2 \end{bmatrix} - \begin{bmatrix} -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 + 1 & 3 - 2 & 2 - 3 \\ 1 - 1 & -1 - 0 & 2 - 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$3(B-A) = 3\begin{bmatrix} 1 & 1 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 3 & -3 \\ 0 & -3 & 0 \end{bmatrix}$$

$$A+3(B-A) = \begin{bmatrix} -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 3 & -3 \\ 0 & -3 & 0 \end{bmatrix}$$

$$=\begin{bmatrix} -1+13 & 2+3 & 3-3 \\ +0 & 0-3 & 2+0 \end{bmatrix}$$

$$=\begin{bmatrix} 2 & 5 & 0 \\ 1 & -3 & 2 \end{bmatrix}$$

Hence

$$A+3(B-A) = \begin{bmatrix} 2 & 5 & 0 \\ 1 & -3 & 2 \end{bmatrix}$$

Q5. Find x and y

$$\begin{bmatrix} 2 & 0 & x \\ 1 & y & 3 \end{bmatrix} + 2\begin{bmatrix} 1 & x & y \\ 0 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 6 & 1 \end{bmatrix}$$

Solution

$$\begin{bmatrix} 2 & 0 & x \\ 1 & y & 3 \end{bmatrix} + 2\begin{bmatrix} 1 & x & y \\ 0 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 6 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & x \\ 1 & y & 3 \end{bmatrix} + \begin{bmatrix} 2 & 2x & 2y \\ 0 & 4 & -2 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 6 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2+2 & 0+2x & x+2y \\ 1+0 & y+4 & 3-3 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 6 & 1 \end{bmatrix}$$

Equating the corresponding element

2x = -2 ; x+2y =2 and y+4 = 6

$x = \frac{-2}{2}$ y = 6-4

x= -1 ; y = 2

hence x= -1

and y = 2

Q6. If A = $|a_{ij}|_{3\times3}$, Show that

i) $\lambda(\mu A) = (\lambda\mu)A$ **iii)** $(\lambda+\mu)A = \lambda A + \mu A$

ii) $\lambda A - A = (\lambda - 1)A$

Solution

i) **A =** $[a_{ij}]_{3\times3}$

L.H.S

$$\mu A = \mu[a_{ij}]_{3\times3}$$

$$=[\mu a_{ij}]_{3\times3}$$

$$\lambda(\mu A) = \lambda[\mu a_{ij}]_{3\times3}$$

$$= [\lambda\mu a_{ij}]_{3\times3}$$

$$= ((\lambda\mu a_{ij})_{3\times3}$$

$$= (\lambda\mu)[a_{ij}]_{3\times3}$$

R.H.S

Hence proved

$$\boldsymbol{\lambda(\mu A) = (\lambda\mu)A}$$

ii) $(\lambda + \mu)A = \lambda A + \mu A$

L.H.S

$$(\lambda + \mu)A = (\lambda + \mu)[a_{ij}]_{3\times3}$$

$$=\lambda[a_{ij}]_{3\times3} + \mu\ [a_{ij}]_{3\times3}$$

$$= \lambda A + \mu A$$

= R.H.S

Hence proved

$$(\lambda + \mu)A = \lambda A + \mu A$$

iii) $\boldsymbol{\lambda A - A = (\lambda - 1)A}$

L.H.S

$$\lambda A - A = \lambda[a_{ij}]_{3\times3} - [a_{ij}]_{3\times3}$$

$$= (\lambda - 1)[a_{ij}]_{3\times3}$$

$$= (\lambda - 1)A$$

Q7. If A = $|a_{ij}|_{2\times3}$ and B = $|b_{ij}|_{2\times3}$

Solution

L.H.S $= \lambda(A + B)$

$$\lambda(A + B) = \lambda\left[\,[[a_{ij}]_{3\times3} + [b_{ij}]_{3\times3}\right]$$

$$= \lambda[a_{ij}]_{3\times3}[b_{ij}]_{3\times3}$$

$$= \lambda A + \lambda B$$

= R.H.S

Hence proved

$$\lambda(A + B) = \lambda A + \lambda B$$

Q8. If A = $\begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix}$ and $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ find the value of a and b.

Solution

Then

$$A^2 = A.A = \begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix}\begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix}$$

$$= \begin{bmatrix} 1 + 2a & 2 + 2b \\ a + ab & 2a + b^2 \end{bmatrix}$$

And

$$A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

By equating

$$\begin{bmatrix} 1+2a & 2+2b \\ a+ab & 2a+b^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Corresponding elements are same

So, 1+2a = 0 ; 2+2b = 0

2a = -1 2b = -2

$a = \frac{-1}{2}$ $b = \frac{-2}{2} = -1$

Hence

$a = \frac{-1}{2}$

b = -1

Q9. If A = $\begin{bmatrix} 1 & -1 \\ a & b \end{bmatrix}$ and $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ find the value of a and b.

Solution

Then

$$A^2 = A - A = \begin{bmatrix} 1 & -1 \\ a & b \end{bmatrix}\begin{bmatrix} 1 & -1 \\ a & b \end{bmatrix}$$

$$= \begin{bmatrix} 1-a & -a-b \\ a-ab & -a+b^2 \end{bmatrix}$$

And

$$A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By equating

$$\begin{bmatrix} 1-a & -a-b \\ a+ab & -a+b^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Corresponding elements are same

So, 1-a = 1 ; -a-b = 0

a = 1-1 -1-b = -2

a =0 b = -1

Hence

a = 0

b = -1

Q10. If A = $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & -1 \end{bmatrix}$ then show that $(A+B)^t = A^t + B^t$

Solution

If A = $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & -1 \end{bmatrix}$

L.H.S = $(A+B)^t$

A+B = $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & -1 \end{bmatrix}$

$= \begin{bmatrix} 1+2 & -1+3 & 2+0 \\ 0+1 & 3+2 & 3-1 \end{bmatrix}$

$= \begin{bmatrix} 3 & 2 & 2 \\ 1 & 5 & 2 \end{bmatrix}$

$(A+B)^t = \begin{bmatrix} 3 & 2 & 2 \\ 1 & 5 & 2 \end{bmatrix}^t$

$= \begin{bmatrix} 3 & 1 \\ 2 & 5 \\ 2 & 2 \end{bmatrix}$

R.H.S = $A^t + B^t$

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 3 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 3 \end{bmatrix}^t$$

$$B^t = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & -1 \end{bmatrix}^t$$

$$= \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 0 & -1 \end{bmatrix}$$

$$A^t + B^t = \begin{bmatrix} 3 & 1 \\ 2 & 5 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+3 & 0+1 \\ -1+3 & 3+2 \\ 2+0 & 3-1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 1 \\ 2 & 5 \\ 2 & 2 \end{bmatrix}$$

$$(A+B)^t = A^t + B^t$$

Q11. Find A^3 if $A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$

Solution

$$A.A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$$

$$A^2 = A.A = \begin{bmatrix} 1+5-6 & 1+2-3 & 3+6-9 \\ 5+10-12 & 5+4-6 & 15+12-18 \\ -2-5+6 & -2-2+3 & -6-6+9 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 0 & 0 \\ 3 & +3 & 9 \\ -1 & -1 & -3 \end{bmatrix}$$

$$A^3 = A.A^2 = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 3 & 3 & 9 \\ -1 & -1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+3-3 & 0+3-3 & 0+9-9 \\ 0+6-6 & 0+6-6 & 0+18-18 \\ 0-3+3 & 0-3+3 & 0-9+9 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Q12. Find the matrix X if:

i) $X \begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ -12 & 3 \end{bmatrix}$ ii. $\begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix} X = \begin{bmatrix} 2 & 1 \\ 5 & 10 \end{bmatrix}$

Solution

i) $X \begin{bmatrix} 2 & 1 \\ 5 & 10 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix}$

$$X = \frac{\begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix}}{\begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix}}$$

$$X = \begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix} \begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix}^{-1}$$

LET $A = \begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix}$

$$A^{-1} = \frac{Adj\ A}{|A|}$$

$$|A| = \begin{vmatrix} 5 & 2 \\ -2 & 1 \end{vmatrix} = 5-(-4) = 5+4 = 9$$

$$\text{Adj } A = \begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{Adj\ A}{|A|} = \frac{1}{9}\begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}$$

$$X = \begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix} \frac{1}{9}\begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}$$

$$=\frac{1}{9}\begin{bmatrix}-1+10 & -2+25\\ 12+6 & -24+15\end{bmatrix}$$

$$=\frac{1}{9}\begin{bmatrix}9 & 27\\ 18 & -9\end{bmatrix}$$

$$=\begin{bmatrix}9/9 & 27/9\\ 18/9 & -9/9\end{bmatrix}$$

$$=\begin{bmatrix}1 & 3\\ 2 & -1\end{bmatrix}$$

Thus value of matrix 'x' = $\begin{bmatrix}1 & 3\\ 2 & -1\end{bmatrix}$

ii) $\begin{bmatrix}\mathbf{5} & \mathbf{2}\\ \mathbf{-2} & \mathbf{1}\end{bmatrix} X = \begin{bmatrix}\mathbf{2} & \mathbf{1}\\ \mathbf{5} & \mathbf{10}\end{bmatrix}$

LET A = $\begin{bmatrix}5 & 2\\ -2 & 1\end{bmatrix}$

Then equation becomes

Ax= $\begin{bmatrix}2 & 1\\ 5 & 10\end{bmatrix}$

Multiply equation by '' A^{-1}''

$$A^{-1}[Ax] = A^{-1}\begin{bmatrix}2 & 1\\ 5 & 10\end{bmatrix}$$

$$[A^{-1}A]x = A^{-1}\begin{bmatrix}2 & 1\\ 5 & 10\end{bmatrix}$$

$$A = \begin{bmatrix}5 & 2\\ -2 & 1\end{bmatrix}$$

$$A^{-1} = \frac{Adj\ A}{|A|}$$

$$|A| = \begin{vmatrix}5 & 2\\ -2 & 1\end{vmatrix} = 5-(-4) = 5+4 = 9$$

$$Adj\ A = \begin{bmatrix}1 & -2\\ 2 & 5\end{bmatrix}$$

$$A^{-1} = \frac{Adj\ A}{|A|} = \frac{1}{9}\begin{bmatrix}1 & -2\\ 2 & 5\end{bmatrix}$$

$$=\frac{1}{9}\begin{bmatrix}1 & -2\\2 & 5\end{bmatrix}\begin{bmatrix}2 & 1\\5 & 10\end{bmatrix}$$

$$=\frac{1}{9}\begin{bmatrix}2-10 & 1-20\\4+25 & 2+50\end{bmatrix}$$

$$=\frac{1}{9}\begin{bmatrix}-8 & -19\\29 & 52\end{bmatrix}$$

$$=\begin{bmatrix}-8/_9 & -19/_9\\29/_9 & 52/_9\end{bmatrix}$$

Thus value of matrix 'x' = $\begin{bmatrix}-8/_9 & -19/_9\\29/_9 & 52/_9\end{bmatrix}$

Q13. Find the matrix A if

i) $\begin{bmatrix}\mathbf{5} & \mathbf{-1}\\\mathbf{0} & \mathbf{0}\\\mathbf{3} & \mathbf{1}\end{bmatrix}\boldsymbol{A}=\begin{bmatrix}\mathbf{3} & \mathbf{-7}\\\mathbf{0} & \mathbf{0}\\\mathbf{7} & \mathbf{2}\end{bmatrix}$ **ii)** $\begin{bmatrix}\mathbf{2} & \mathbf{-1}\\\mathbf{-1} & \mathbf{2}\end{bmatrix}\boldsymbol{A}=\begin{bmatrix}\mathbf{0} & \mathbf{-3} & \mathbf{8}\\\mathbf{3} & \mathbf{-3} & \mathbf{7}\end{bmatrix}$

Solution

$$\begin{bmatrix}\mathbf{5} & \mathbf{-1}\\\mathbf{0} & \mathbf{0}\\\mathbf{3} & \mathbf{1}\end{bmatrix}\boldsymbol{A}=\begin{bmatrix}\mathbf{3} & \mathbf{-7}\\\mathbf{0} & \mathbf{0}\\\mathbf{7} & \mathbf{2}\end{bmatrix}$$

Let B = $\begin{bmatrix}5 & -1\\0 & 0\\3 & 1\end{bmatrix}$

The equation becomes

$$B.A=\begin{bmatrix}3 & -7\\0 & 0\\7 & 2\end{bmatrix}$$

Multiply both sides by "B^{-1}"

$$B^{-1}\, B.A = B^{-1}\begin{bmatrix}3 & -7\\0 & 0\\7 & 2\end{bmatrix}$$

$$A = B^{-1}\begin{bmatrix}3 & -7\\0 & 0\\7 & 2\end{bmatrix}$$

$$B^{-1} = \frac{Adj\, B}{|B|}$$

$$|B| = \begin{bmatrix}3 & -7\\0 & 0\\7 & 2\end{bmatrix} = 5\text{-}(-1)(3)=5+3 = 8$$

$$\text{Adj B} = \begin{bmatrix}1 & 0 & 1\\-3 & 0 & 5\end{bmatrix}$$

$$B^{-1} = \frac{Adj\, B}{|B|} = \frac{1}{8}\begin{bmatrix}1 & 0 & 1\\-3 & 0 & 5\end{bmatrix}$$

$$=\frac{1}{8}\begin{bmatrix}1 & 0 & 1\\-3 & 0 & 5\end{bmatrix}\begin{bmatrix}3 & -7\\0 & 0\\7 & 2\end{bmatrix}$$

$$=\frac{1}{9}\begin{bmatrix}3+0+7 & -7+0+12\\-3+0+5 & 21+0+10\end{bmatrix}$$

$$=\frac{1}{8}\begin{bmatrix}10 & -5\\26 & 31\end{bmatrix}$$

$$=\begin{bmatrix}10/_8 & -5/_8\\26/_8 & 31/_8\end{bmatrix}$$

Thus value of matrix 'x' = $\begin{bmatrix}10/_8 & -5/_8\\26/_8 & 31/_8\end{bmatrix}$

ii) $\begin{bmatrix}\mathbf{2} & \mathbf{-1}\\\mathbf{-1} & \mathbf{2}\end{bmatrix}\boldsymbol{A} = \begin{bmatrix}\mathbf{0} & \mathbf{-3} & \mathbf{8}\\\mathbf{3} & \mathbf{-3} & \mathbf{7}\end{bmatrix}$

Let $B = \begin{bmatrix}2 & -1\\-1 & 2\end{bmatrix}$

The equation becomes

$$B.A = \begin{bmatrix} 0 & -3 & 8 \\ 3 & 3 & -7 \end{bmatrix}$$

Multiply both sides by "B^{-1}"

$$B^{-1}\, B.A = B^{-1} \begin{bmatrix} 0 & -3 & 8 \\ 3 & 3 & -7 \end{bmatrix}$$

$$A = B^{-1} \begin{bmatrix} 0 & -3 & 8 \\ 3 & 3 & -7 \end{bmatrix}$$

$$B^{-1} = \frac{Adj\ B}{|B|}$$

$$|B| = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = 4\text{-}1 = 3$$

$$\text{Adj B} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$B^{-1} = \frac{Adj\ B}{|B|} = \frac{1}{3}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$A = \frac{1}{3}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 0 & -3 & 8 \\ 3 & 3 & -7 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 0+3 & -6+3 & 16-7 \\ 0+6 & -3+6 & 8-14 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 3 & -3 & 9 \\ 6 & 3 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} 3/3 & -3/3 & 9/3 \\ 6/3 & 3/3 & -6/3 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 1 & -2 \end{bmatrix}$$

Thus value of matrix 'A' = $\begin{bmatrix} 1 & -1 & 3 \\ 2 & 1 & -2 \end{bmatrix}$

Q14 $\begin{bmatrix} r\cos\phi & 0 & -\sin\phi \\ 0 & r & 0 \\ r\sin\phi & 0 & r\cos\phi \end{bmatrix}\begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & r & 0 \\ -r\sin\phi & 0 & r\cos\phi \end{bmatrix} = r\,I^3$

Solution

$$L.H.S = \begin{bmatrix} r\cos\phi & 0 & -\sin\phi \\ 0 & r & 0 \\ r\sin\phi & 0 & r\cos\phi \end{bmatrix}\begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & r & 0 \\ -r\sin\phi & 0 & r\cos\phi \end{bmatrix}$$

$$= \begin{bmatrix} r\,cos^2\phi + 0 + r\,sin^2\phi & 0+0+0 & r\sin\phi\cos\phi + 0 - r\sin\phi \\ 0+0+0 & 0+r+0 & 0+0+0 \\ r\sin\phi\cos\phi + 0 - r\sin\phi\cos\phi & 0+0+0 & r\,sin^2\phi + r\,cos^2\phi \end{bmatrix}$$

$$= \begin{bmatrix} r\,cos^2\phi + r\,sin^2\phi & 0 & r\sin\phi\cos\phi - r\sin\phi \\ 0 & r & 0 \\ r\sin\phi\cos\phi - r\sin\phi\cos\phi & 0 & r\,sin^2\phi + r\,cos^2\phi \end{bmatrix}$$

$$= \begin{bmatrix} r(cos^2\phi + sin^2\phi) & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & r(sin^2\phi + cos^2\phi) \end{bmatrix}$$

$$= \begin{bmatrix} r & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & r \end{bmatrix}$$

$$= r\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= rI^3 = R.H.S$$

Hence proved