Exercise 2.6

Q1. For A = {1,2,3,4}, find the following relations in A. State the domain and range of each relation. Also draw the graph of each.

i. {(x,y) I y = x} ii. {(x,y) I y+x = 5} iii. {(x,y) I x+y<5)

iv. {(x,y) I x+y>5}

Solution:

i. {(x,y) I y = x}

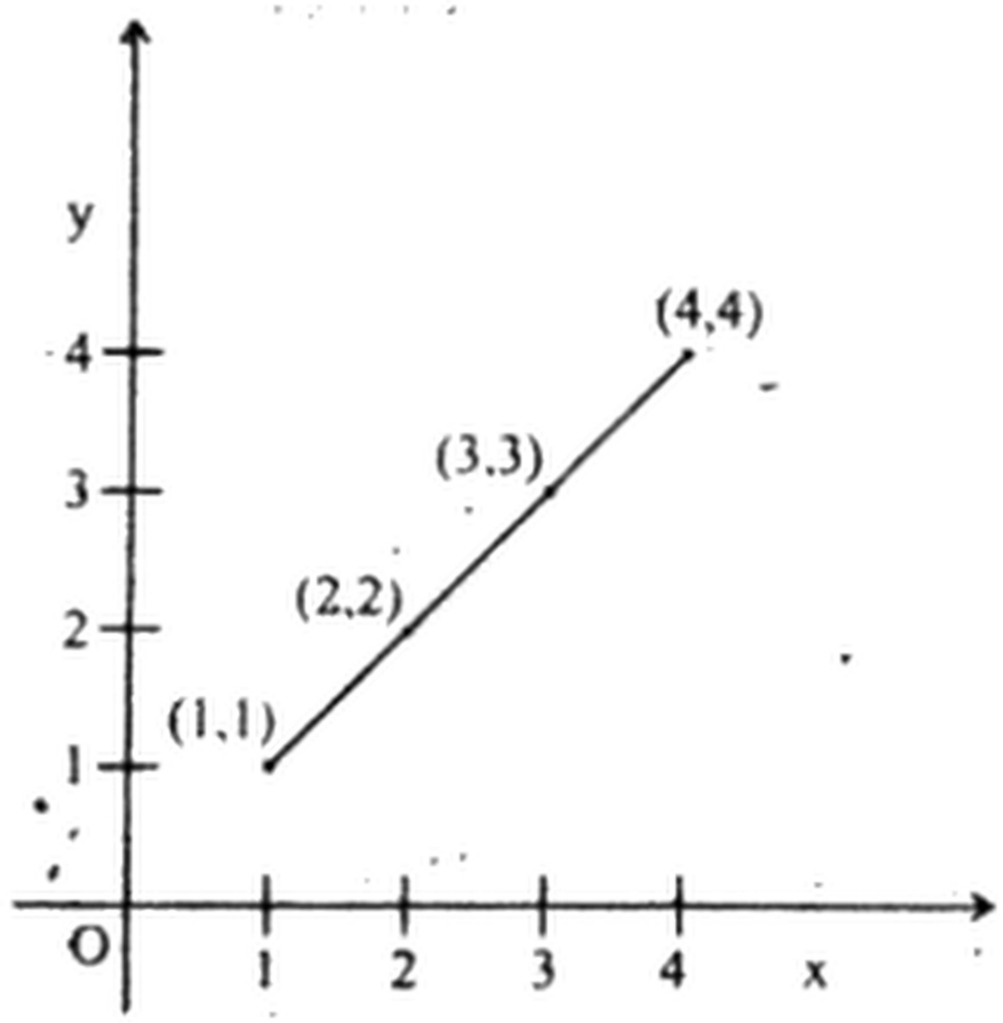
Where, A = {1,2,3,4}

So r = {(1,1), (2,2), (3,3),(4,4)}

and

Dom (r) = {1,2,3,4}

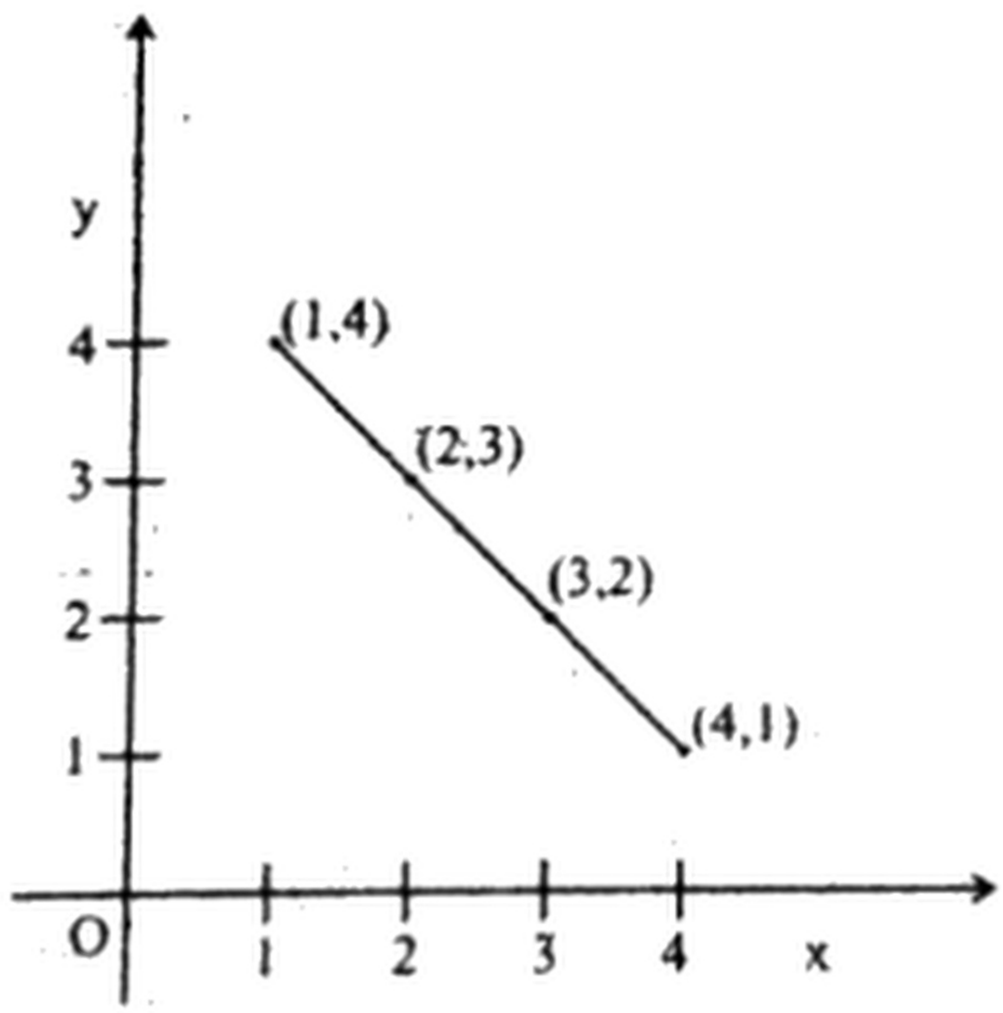
And range (r) = {1,2,3,4}



ii. **{(x,y) I y+x = 5}; Where A = {1,2,3,4}**

r = {(1,4), (2,3), (3,2), (2,1)}

So, Dom (r) = {1,2,3,4} = {1,2,3,4}

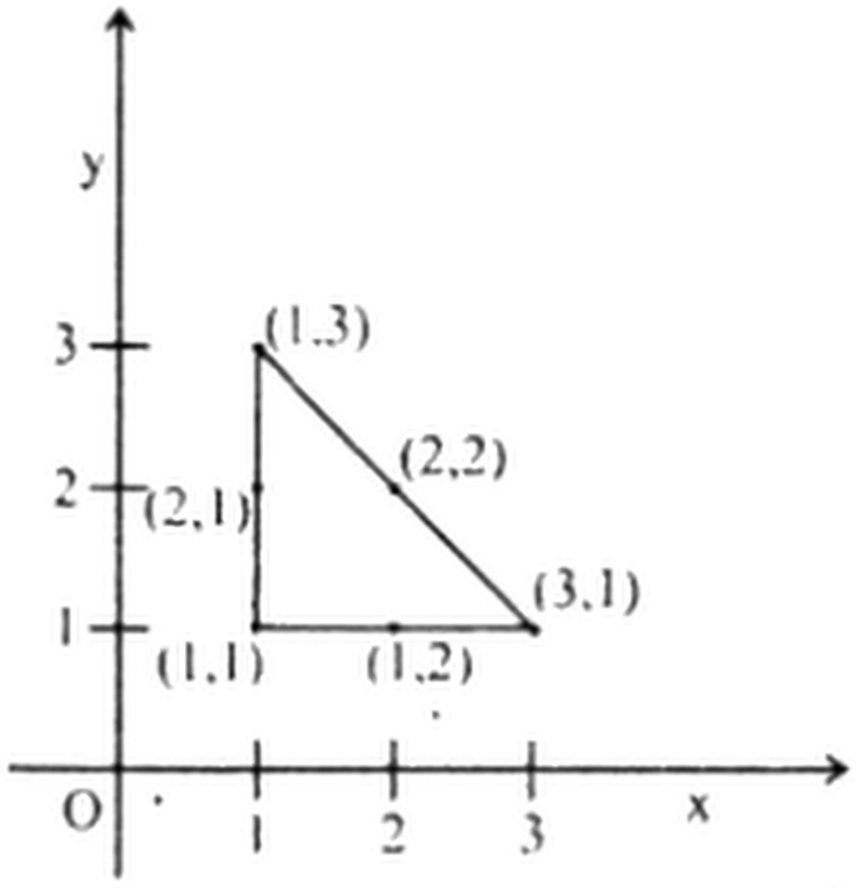


iii. {(x,y) I x+y<5), where A = {1,2,3,4}

r ={(1.1),(1,2),(1,3),(2,1), (2,2). (3.1)}

So. Dom(r)= {1.2,3}

& range(r)= {1,2,3}

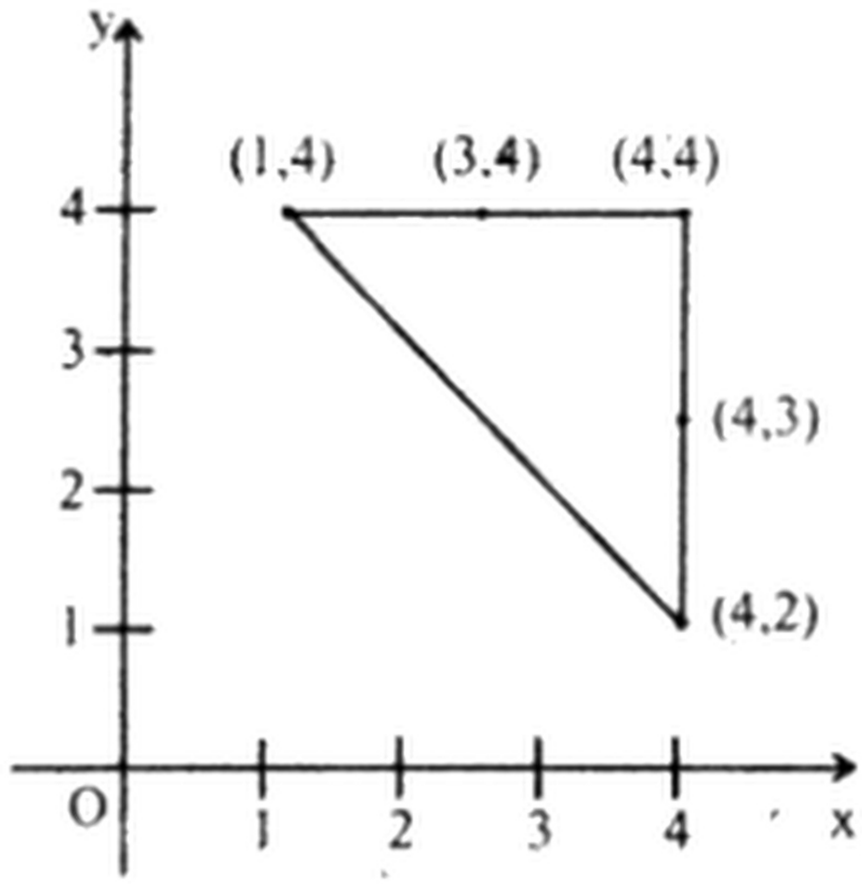


iv. {(x,y) | x+y>5}

r ={(2.4), (3,3), (3.4), (4,2), (4,3), (4,4)}

So Dom (r)= {2,3,4}

& Range (r)= {4,3,2} or {2,3,4}



Q2. Repeat Q-1 when A = R the set of real number. Which of the real lines are functions?

Solution:

i. A ϵ R

r ={(x,y) | y = x}

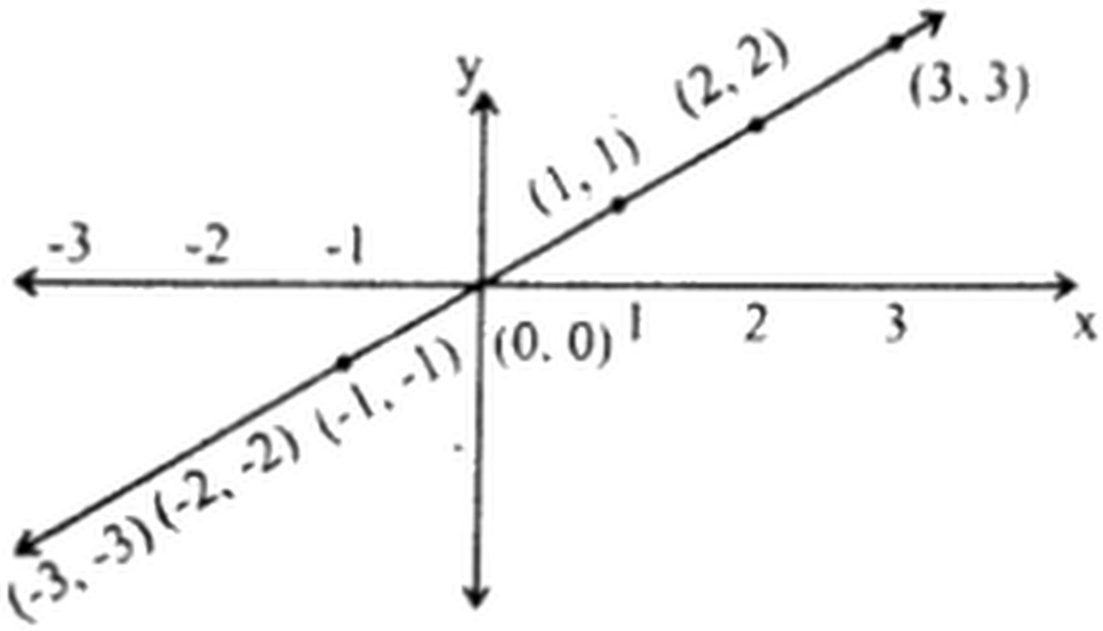
r ={(1,1), (2,2), (3,3)....} U {-1, -1}

(-2,-2)..... }

or

= {... (- 2,- 2), (-1, -1), (0,0), (1,1),

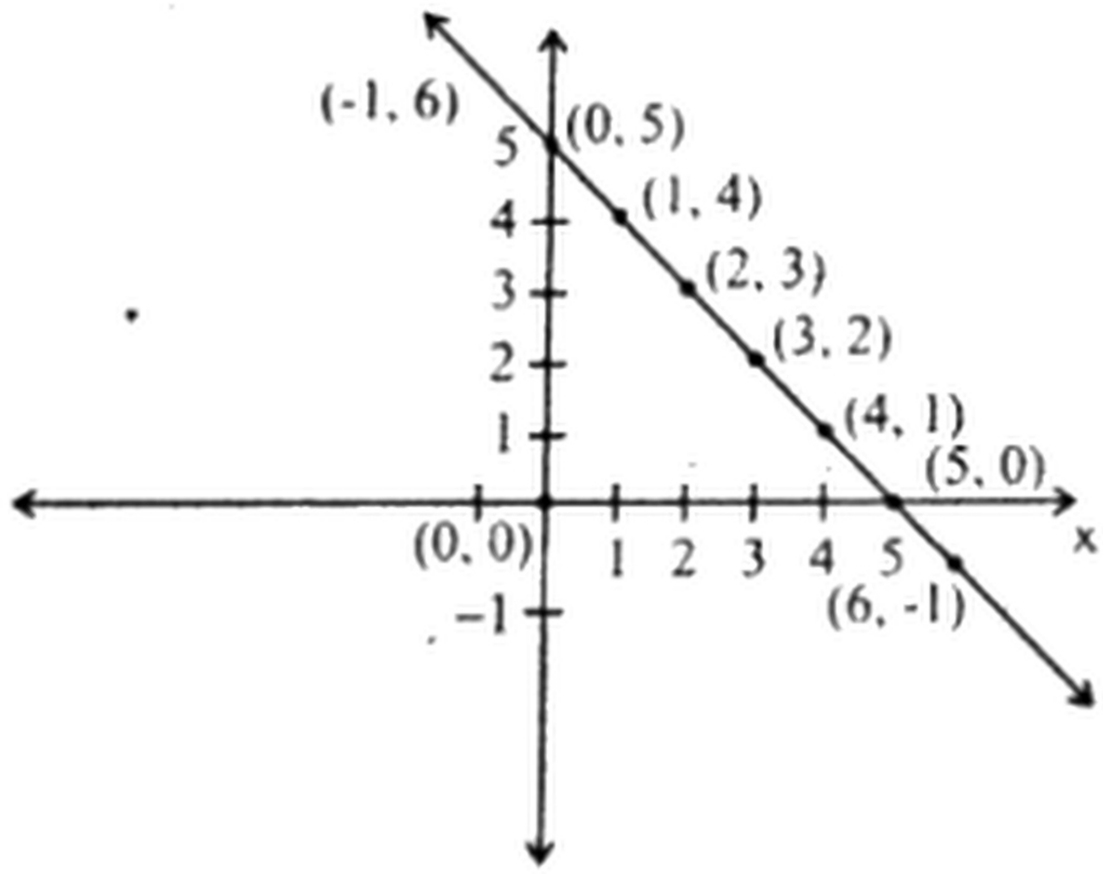
(2,2), (3,3).... }



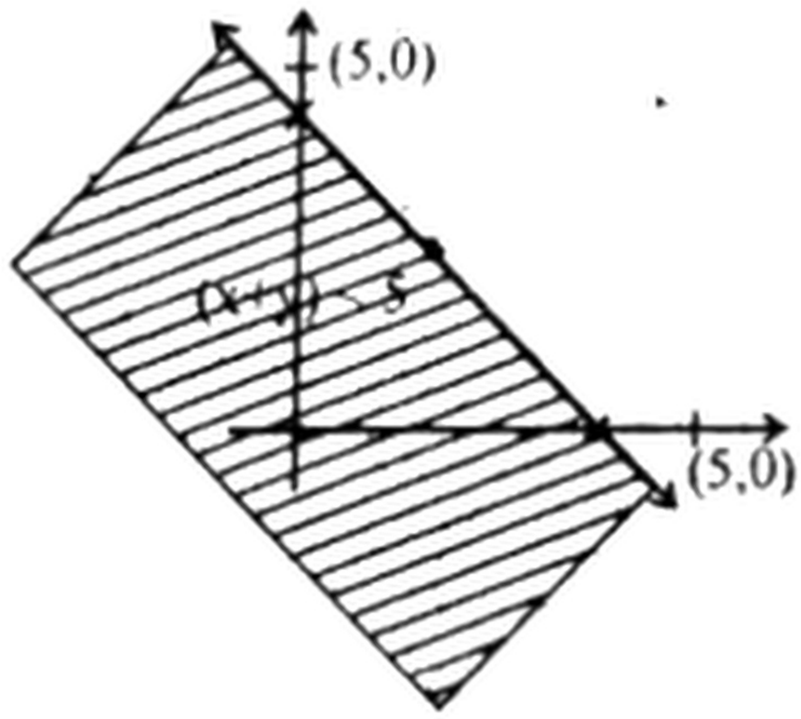
ii. **A ϵ R**

r = {(x+y) | (x+y = 5)}

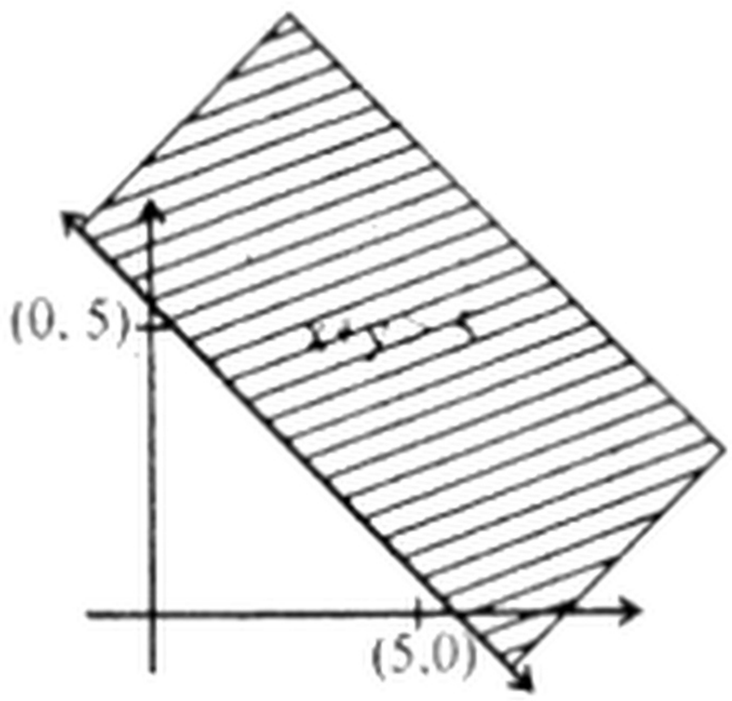
r = { ....,(-1,6) (0,5), (1,5), (2,3), (3,2), (4,1) (5,0), (6,-1)...}



iii. **{(x,y) I x+y<5)**



iv. {(x,y) I x+y>5}



Q3. Which of the following diagrams represent functions and of which type?

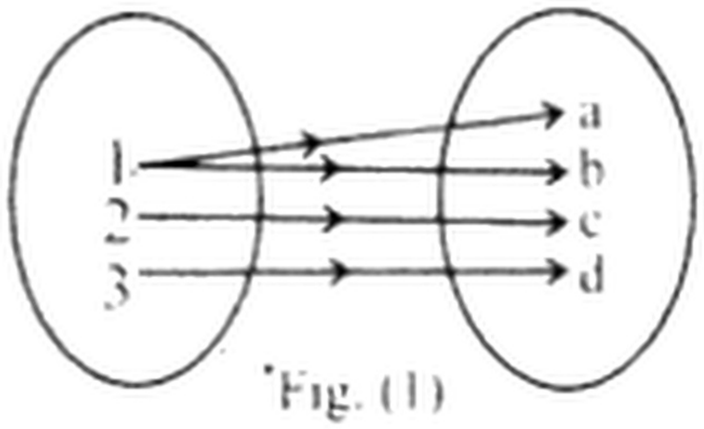


Fig. (1)

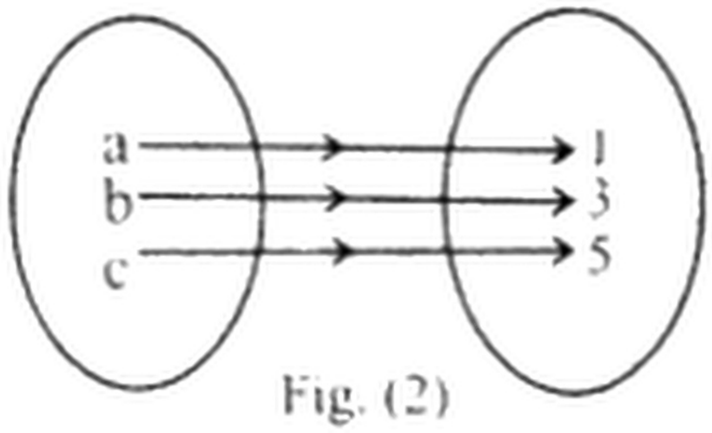


Fig. (2)

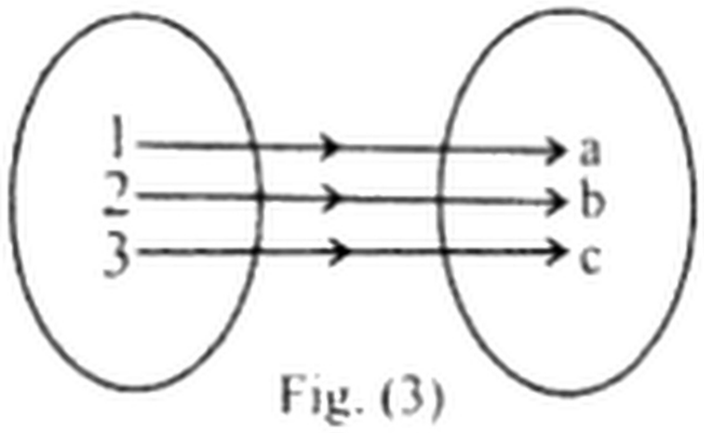


Fig. (3)

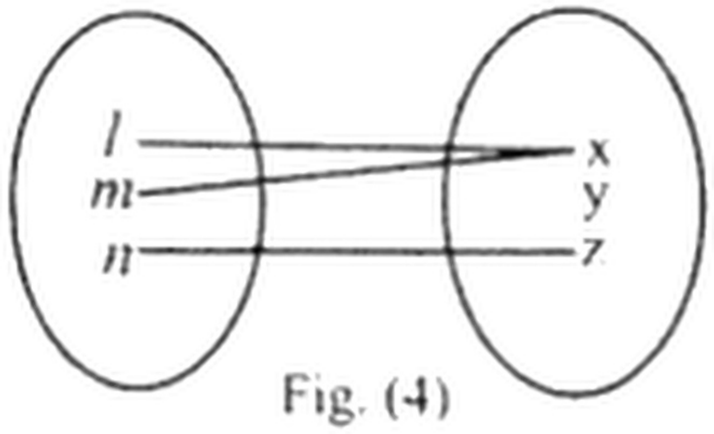


Fig. (4)

Solution:

i. Fig (1), does not represent a function because element '1' has not unique image.

ii. Fig (2), represent a (1-1) one—to—one function. So , it is on to function.

iii. Fig. (3), represent a (1-1) one-to-one function, so, it is onto (injunction) function.

iv. Fig (4), represents a function which in onto function

Q4. Find the inverse of each of the following relations. Tell whether each relation and its inverse is a function or not.

i. **{(2,1), (3,2), (4,3), (5,4), (6,5)}**

ii. **{(1,3), (2,5), (3,7), (4,9), (5,11)}**

iii. **$\{(x,y) \mid y = 2x+3 \in R\}$**

iv **$\{(x,y) \mid y^2 = 4ax, x \geq 0\}$**

v. $\{(x,y)\ x^2+y^2 = 9,\ |x, y \mid x,y\ | \leq 3\}$

Solution:

i. $r_1 = \{(2,1), (3,2), (4,3), (5,4), (6,5)\}$

$r_1^{-1} = \{(2,1), (3,2), (4,3), (5,4), (6,5\}$

dom $(r_1^{-1}) = \{1,2,3,4,5\}$

So, it is a function

ii. $r_2 = \{(1,3), (2,5), (3,7), (4,9), (5,11)\}$

$r_2^{-1} = \{(1,3), (2,5), (3,7), (4,9), (5,11)\}$

dom $(r_2^{-1}) = \{3,5,7,9,11)\}$

So, it is a function

iii. $r_3 = ((x,y)\ I\ y = 2x+3, \epsilon R)$

$r_3^{-1} = \{(x,y)\ \mid\ y = 2x+3, \epsilon R\}$

It is a function

iv. $x_4 = \{(x,y)\ \mid\ y^2 = 4ax, x \geq 0\}$

$r_4^{-1} = \{(x,y) \mid x = \frac{y^2}{49}, y \epsilon R\}$

It is not a function

v. $r_5 = \{(x,y)\ x^2+y^2 = 9,\ \mid x,y\ \mid \leq |3|\}$

$r_5^{-1} = \{(x,y)\ x^2+y^2 = 9,\ |x, y \mid \leq |3|\}$

It is not a function