

Exercise 2.5

Q1. Convert the following theorems to logical and prove them by constructing truth tables.

- i. $(A \cap B)' = A' \cup B'$ ii. $(A \cup B) \cap C = (A \cup B) \cup C$
 iii. $(A \cap B) \cap C = A \cap (B \cap C)$ iv. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Solution:

- i. $(A \cap B)' = A' \cup B'$

L.H.S $(A \cap B)' \rightarrow \sim(p \wedge q)$

P	Q	$p \wedge q$	$\sim(p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

R.H.S $A' \cup B' \rightarrow (\sim p \vee \sim q)$

P	Q	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T

F	F	T	T	T
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ii. $(A \cup B) \cap C = (A \cup B) \cup C$

L.H.S $(A \cup B) \cap C \rightarrow (p \vee q) \wedge r$

p	q	r	$p \vee q$	$(p \vee q) \wedge r$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	T	F
F	F	F	F	F

R.H.S $(A \cup B) \cup C \rightarrow p \vee (q \vee r)$

p	q	r	$q \vee r$	$p \vee (q \vee r)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T

T	F	F	F	T
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	F	F

iii. $(A \cap B) \cap C = A \cap (B \cap C)$

L.H.S $(A \cap B) \cap C \rightarrow (p \wedge q) \wedge r$

P	Q	r	$p \wedge q$	$(p \wedge q) \wedge r$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	F
T	F	F	F	F
F	T	T	F	F
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

R.H.S $A \cap (B \cap C) \rightarrow p \wedge (q \wedge r)$

P	q	r	$q \wedge r$	$p \wedge (q \wedge r)$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	T	F
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

iv. **$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$**

L.H.S $A \cup (B \cap C) \rightarrow p \vee (q \wedge r)$

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$
T	T	T	T	T
T	T	F	F	T
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	F	F

F	F	T	F	F
F	F	F	F	F

$$\text{R.H.S } (A \cup B) \cap (A \cup C) \rightarrow (p \vee q) \wedge (p \vee r)$$

p	Q	r	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	F	F	F
F	F	F	F	F	F

L.H.S = R.H.S

Hence Proved

