Exercise 2.4

Q1. Write the converse, inverse and contra positive of the following conditional:

i) $\sim p \to q$ **ii)** $q \to p$ **iii)** $\sim p \to \sim q$ **iv)** $\sim q \to \sim p$

Solution:

i) $\sim p \to q$

		Converse	Inverse	Contra Positive
i)	$\sim p \to q$	$q \to \sim p$	$p \to \sim q$	$\sim p \to p$

ii) $q \to p$

		Converse	Inverse	Contra Positive
ii)	$q \to p$	$p \to q$	$\sim q \to \sim p$	$\sim p \to \sim q$

iii) $\sim p \to \sim q$

		Converse	Inverse	Contra Positive
iii)	$\sim p \to \sim q$	$\sim p \to \sim q$	$p \to q$	$q \to p$

iv) $\sim q \to \sim p$

		Converse	Inverse	Contra Positive
iv)	$\sim q \to \sim p$	$\sim p \to \sim q$	$q \to p$	$p \to q$

Q2. Construct truth tables for the following statements.

i. $(p \rightarrow \sim p) \vee (p \rightarrow q)$

ii. $(p \wedge \sim p) \rightarrow q$

iii. $(\sim(p \rightarrow q) \leftrightarrow (p \wedge \sim q)$

Solution:

i. $(p \rightarrow \sim p) \vee (p \rightarrow q)$

p	q	$\sim p$	$\sim q$	$p \rightarrow \sim p$	$p \rightarrow q$	$(p \rightarrow \sim p) \vee (p \rightarrow q)$
T	T	F	F	F	T	T
T	F	F	T	F	F	F
F	T	T	F	T	T	T
F	F	T	T	T	T	T

ii. $(p \wedge \sim p) \rightarrow q$

p	q	$\sim p$	$p \wedge \sim p$	$(p \wedge \sim p) \rightarrow q$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	F	T

iii. $(\sim(p \rightarrow q) \leftrightarrow (p \wedge \sim q)$

p	q	$\sim p$	$(p \rightarrow q$	$\sim(p \rightarrow q)$	$p \wedge \sim q$	$(\sim(p \rightarrow q)$ $\rightarrow \sim(p \wedge q)$	$(\sim(p \rightarrow q)$ $\rightarrow \sim(p \wedge q)$	$(\sim(p \rightarrow q)$ $\leftrightarrow \sim(p \wedge q)$

T	T	F	T	F	F	T	T	T
T	F	F	F	T	T	T	T	T
F	T	T	T	F	F	T	T	T
F	F	T	T	F	F	T	T	T

Q3. Show that each of the following statements is a tautology.

i. $(p \wedge q) \rightarrow p$

ii. $p \rightarrow (p \vee q)$

iii. $\sim(p \rightarrow q) \rightarrow p)$

iv. $\sim q \wedge (p \rightarrow q) \rightarrow \sim p$

Solution:

i. $(p \wedge q) \rightarrow p$

P	q	$p \wedge q$	$(p \wedge q) \rightarrow p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

ii. $p \rightarrow (p \vee q)$

P	q	$p \vee q$	$p \rightarrow (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

iii. $\sim(p \rightarrow q) \rightarrow p$

P	q	$\sim(p \rightarrow q)$	$\sim(p \rightarrow q) \rightarrow p$
T	T	F	T
T	F	T	T
F	T	F	T
F	F	F	T

iv. $\sim q \wedge (p \rightarrow q) \rightarrow \sim p$

p	q	$\sim p$	$\sim q$	$p \rightarrow q$	$\sim q \wedge (p \rightarrow q)$	$\sim q \wedge (p \rightarrow q) \rightarrow \sim p$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

Q4. Determine whether each of the following is a tautology, a contingency or an absurdity.

i. $p \wedge \sim p$

ii. $p \rightarrow (p \rightarrow q)$

iii. $q \vee (\sim q \vee p))$

Solution:

i. $p \wedge \sim p$

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

ii. $p \rightarrow (q \rightarrow p)$

P	q	$q \rightarrow p$	$p \rightarrow (q \rightarrow p)$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

iii. $q \vee (\sim q \vee p))$

P	Q	$\sim q$	$\sim q \vee p$	$q \vee (\sim q \vee p)$
T	T	F	T	T
T	F	T	T	T
F	T	F	F	T
F	F	T	T	T

Q5. Prove that $p \vee (\sim p \wedge \sim q) \vee (p \wedge q) = p \vee (\sim p \wedge \sim q)$

Solution:

L.H.S = $p \vee (\sim p \wedge \sim q) \vee (p \wedge q)$

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$p \wedge q$	$p \vee (\sim p \wedge \sim q)$	$p \vee (\sim p \wedge \sim q)$
T	T	F	F	F	T	T	T
T	F	F	T	F	F	T	T
F	T	T	F	F	F	F	F
F	F	T	T	T	F	T	T

R.H.S = $p \vee (\sim p \wedge \sim q)$

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$p \vee (\sim p \wedge \sim q)$
T	T	F	F	F	T
T	F	F	T	F	T
F	T	T	F	F	F

F	F	T	T	T	T

So, L.H.S = R.H.S

Hence Prove.