Exercise 2.3

1. Verify the commutative properties of union and intersection for the following pairs of sets: -

i. **A = {1,2,3,4,5}, B = {4,6,8,10}** **ii.** **N,Z**

iii. **A = {x | x ϵ ∑ R ∧ x ≥ 0}, B = R** **iv.** **A = {1,2,3,4,5}; B = {4,6,8,10}**

Solution

i. **A = (1,2,3,4,5), B = {4,6,8,10}**

Commutative property of Union.

AUB = BUA

L.H.S = AUB = {1,2,3,4,5} U {4,6,8,10}

= {1,2,3,4,5,6,8,10}

R.H.S = BUA = {4,6,8,10} U{1,2,3,4,5}

= (1,2,3,4,5,6,8,10}

Hence, AUB = BUA

Commutative property of intersection.

A∩B = B∩A

L.H.S = A∩B = {1,2,3,4,5} ∩ {4,6,8,10}

= {4}

R.H.S = (B∩A) = {1,2,3,4,5} ∩ {4,6,8,10}

= {4}

Hence, A∩B = B∩A

ii. A= N; B = Z

Commutative property of union.

$A \cup B = B \cup A$

L.H.S $= A \cup B = N \cup Z = Z$

R.H.S $= B \cup A = Z \cup N = Z$

Hence, $A \cup B = B \cup A$

Commutative property of intersection.

$A \cap B = B \cap A$

L.H.S $= A \cap B = N \cap Z = N$

R.H.S $= B \cap A = Z \cap N = N$

Hence, $A \cap B = B \cap A$

iii. $A = \{x \mid x \in \sum R \wedge x \geq 0\}$; B = R

Commutative property of union

$A \cup B = B \cup A$

L.H.S $= A \cup B = \{x \mid x \in \sum R \wedge x \geq 0\} = R$

R.H.S $= B \cup A = \{x \mid x \in \sum R \wedge x \geq 0\} = R$

Hence, $A \cup B = B \cup A$

Commutative property of intersection

$A \cap B = B \cap A$

L.H.S = A∩B = $\{x \mid x \in \sum R \wedge x \geq 0\} \cap R$

$= \{x \mid x \in \sum R \wedge x \geq 0\}$

R.H.S = B∩A = $R \cap \{x \mid x \in \sum R \wedge x \geq 0\}$

$= \{x \mid x \in \sum R \wedge x \geq 0\}$

Hence, A∩B = B∩A

Q2. Verify the properties for the sets, A, B and C given below:

i. Associativity of Union

ii. Associativity of intersection.

iii. Distributivity of Union over intersection.

iv. Distributivity of intersection

a. A= {1,2,3,4} B = {3,4,5,6,7,8}, C={5,6,7,9.10}

b. A = { }, B = {0}, C = {0,1,2}

c. N,Z,Q

Solution:

a. A = (1,2,3,4); B = (3,4,5,6,7,8) C = (5,6,7,9,10)

i. Associativity of union

(A U B) U C = AU(BUC)

L.H.S = (AUB) UC

AUB = {1,2,3,4}U{3,4,5,6,7,8}

= {1,2,3,4,5,6,7,8}

(AUB)UC = (1,2,3,4,5,6,7,8) U (5,6,7,9,10)

= {1,2,3,4,5,6,7,8,9,10}

R.H.S = AU(BUC)

BUC = {3,4,5,6,7,8,9,10}

= {3,4,5,6,7,8,9,10}

AU(BUC) = {1,2,3,4} U {3,4,5,6,7,8.9,10}

={1,2,3,4,5,6,7,8.9,10}

Hence Proved (A U B) U C= AU (BUC)

ii. Associativity of intersection

(A∩B)∩C = A∩(B∩C)

L.H.S = (A∩B)∩C

A∩B = {1,2,3,4} ∩ {3,4,5,6,7,8}

= {3,4}

(A∩B)∩C = {3,4} ∩ {5,6,7,8,9,10}

= { }

R.H.S = A∩(B∩C)

B∩C = {3,4,5,6,7,8} ∩ {5,6,7,9,10}

= {5,6,7}

A∩ (B∩C) = {1,2,3,4} ∩ {1,2,3,4}∩(5,6,7) = { }

Hence Proved (A∩B)∩C = A ∩ (B∩C)

iii. Distributivity of union over intersection.

AU(B∩C) = (AUB) ∩ (AUC)

L.H.S = A U (B∩C)

B∩C = {3,4,5,6,7,8} ∩ {5,6,7,9,10}

= {5,6,7}

AU(B∩C) = {1,2,3,4}U{5,6,7}

= {1,2,3,4,5,6,7}

R.H.S AUB = {1,2,3,4}U{3,4,5,6,7,8}

= {1,2,3,4,5,6,7,8}

R.H.S = (AUB) ∩ (AUC)

AUB = {1,2,3,4} U {3,4,5,6,7,8}

= {1,2,3,4,5,6,7,8}

AUC = {1,2,3,4}U{5,6,7,9,10}

= {1,2,3,4,5,6,7,9,10}

(AUB) ∩ (AUC) = {1,2,3,4,5,6,7,8} ∩ {1,2,3,4,5,6,7,8,9,10}

Hence Proved, A U (B∩C) = (AUB) ∩ (AUC)

b. A= { }; B = {0); C = {0,1,2}

i. Associativity of union

(AUB) UC = AU (BUC)

R.H.S = (AUB)UC

AUB = { } U {0} = {0}

(AUB) UC = {0}U {0,1,2} = {0,1,2}

R.H.S = AU(BUC)

BUC = {0)U{0,1,2} = {0,1,2}

AU(BUC) = { } U {0,1,2} = {0,1,2}

Hence Proved, (AUB)UC=AU (BUC)

ii. Associativity of intersection.

(A∩B)∩C = A∩ (B∩C)

L.H.S = (A∩B)∩C

A∩B = { } ∩ {0} = { }

(A∩B) ∩C = { }∩{0,1,2} = { }

R.H.S = A∩(B∩C)

B∩C = {0}∩{0,1,2} = {0}

A∩(B∩C) = { } U {0} = { }

Hence Proved, (A∩B)∩C=A∩ (B∩C)

iii. Distributivity of union over intersection.

AU(B∩C) = (AUB) ∩ (AUC)

L.H.S = A U (B∩C)

B∩C = {0} ∩ {0,1,2}

= {0}

AU(B∩C) = { }U{0}

= {0}

R.H.S AUB = { }U{0}

= {0}

AU = { } U {0,1,2}

= {0}

Hence Proved, A U (B∩C) = (AUB) ∩ (AUC)

iii. Distributivity of intersection over union.

A∩(BUC) = (A∩B) U (A∩C)

L.H.S = A ∩ (BUC)

BUC = {0} U {0,1,2}

= {0,1,2}

A∩(BUC) = { }∩{0,1,2}

= { }

R.H.S = (A∩B) U (A∩C)

A∩C = { } ∩ {0,1,2}

= { }

A∩B = { }∩{0}

= {1,2,3,4,5,6,7,9,10}

(A ∩B) U (A∩C) = { } U{ }

Hence Proved, A ∩ (BUC) = (A∩B)U (A∩C)

c. A=N; B = Z, C = Q

(AUB) UC = AU (BUC)

R.H.S = (AUB)UC

AUB = N UZ = Z

(AUB) UC = ZUQ= Q

R.H.S = AU(BUC)

BUC = ZUQ = Q

Hence Proved, (AUB)UC = AU(BUC)

ii. Associativity of intersection

(A∩B)∩C = A∩(B∩C)

L.H.S = (A∩B)∩C

(A∩B∩C = N∩Q = N

R.H.S = A∩(B∩C)

(B∩C) = Z∩Q =Z

A∩(B∩C) = N∩Z = Z

Hence Proved, (A∩B)∩C = A∩ (B∩C)

iii. Distributivity of union over intersection.

AU(B∩C) = (AUB) ∩ (AUC)

L.H.S = A U (B∩C)

B∩C = Z ∩ Q = Z

AU(B∩C) = NUZ = Z

R.H.S = (AUB) ∩ (AUC)

AUB = NUZ=Z

AUC = N U Q = Q

(AUB) ∩ (AUC) = Z ∩ Q = Z

Hence Proved, A U (B∩C) = (AUB) ∩ (AUC)

iii. Distributivity of intersection over union.

$A\cap(B\cup C) = (A\cap B) \cup (A\cap C)$

L.H.S $= A \cap (B\cup C)$

$B\cup C = Z \cup Q = Q$

$A\cap(B\cup C) = N\cap Q = N$

R.H.S $= (A\cap B) \cup (A\cap C)$

$A\cap C = N \cap Q = N$

$A\cap B = N\cap Z = N$

$(A \cap B) \cup (A\cap C) = N \cup N = N$

Hence Proved, $A \cap (B\cup C) = (A\cap B)\cup (A\cap C)$

Q3. Verify De Morgan's Laws for the following sets:

U = {1,2,3,....,20}, A = {2,4,6,....20} and B = {1,3,5,19}

Solution:

i. $(A\cup B)' = A'\cap B'$

L.H.S $= A\cup B = \{2,4,6,\ldots,20\}\cup\{1,3,5,\ldots,19\}$

$= \{1,2,3,\ldots,20\}$

$(A\cup B)' = U - (A\cup A) = \{1,2,3,\ldots,20\} - \{1,2,3,\ldots,20\}$

$=\varnothing$

R.H.S $A'\cap B'$

$A' = U - A = \{1,2,3,\ldots,20\} - \{1,3,5,\ldots,19\}$

$= \{2,4,6,\ldots,20\}$

$B' = U - B = \{1,2,3,\ldots,20\} - \{2,4,6,\ldots,20\}$

= {1,3,5,.....,19}

A'∩B' = {1,3,5,.....,19} ∩ {2,4,6,.....,20}

= ∅

Hence Proved

(A∩B)' =

L.H.S = (A∩B)'

A∩B ={2,4,6,....20} ∩ {1,3,5,.....19} = ∅

(A∩B)' = U - (A∩B) = {1,2,3,....,20} = { }

={1,2,3....,20)

A' = U - A = {1,2,3,4,5,....20} – {2,4,6,,.....,20}

= {1,2,3,......,19}

B' = U-B = {1,2,3,....20} – {1,2,3,......,19}

= {2,4,6,,.....,20}

R.H.S =(A∩B)' = {1,2,3,......,19} - {2,4,6,....,20}

={1,2,3,4,5,....,20}

Hence Proved.

Q4. Let U = The set of the English alphabet.

A = {x | x is vowel}, B = {y | y is a consonant},

Verify De Morgan's Law for these sets.

Solution:

i. (A∪B)' = A'∩B'

L.H.S = (A∪B)'

AUB = {xIx is a vowel} U {y | y is a consonant}

= The set of the English alphabet

(AUB)' = U - (AUB) =The set of the English alphabet - The set of the English alphabet

= ø

R.H.S = A'∩B'

A' = U - A = The set of the English alphabet

= {xIx is a vowel}

B' = U -B = The set of the English alphabet - {y | y is a consonant}

= {x | x is a vowel}

A'∩B' = (yIy is consonant) ∩ {xIx is a vowel)

= ø

Hence proved, (AUB)' = A'∩B'

ii. (A∩B)' = A'∪B'

L.H.S = (A∩B)'

ArB = {xIx is a vowel) ∩ {Ay is a consonant}

= ø

(A∩B)' = U - (A∩B) = The set of the English alphabet – ø

(A∩B)' = The set of the English alphbet

R.H.S = A'UB'

A' = U - A = The set of the English alphabet - {x | x is a vowel)

= {y | y is a consonant}

B' = U - B = The set of the English alphabet - {x|x is a consonant}

= {x | x is a vowel}

A'UB' = {y|y is a consonant} U {x | x is a vowel}

= The set of the English alphabet

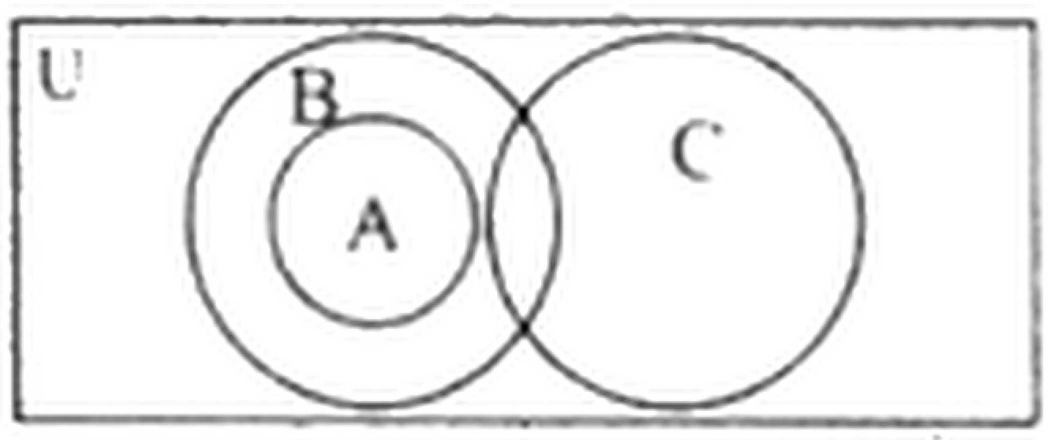
Hence proved: (A∩B)' = A'UB'

Q5. With the help of Venn diagrams, verify the two distributive properties in the following cases w.r.t union and intersection.

i. A⊆B, A∩C = ø and B and C are overlapping.

ii. A and B are overlapping; B and C are overlapping but A and C are disjoint

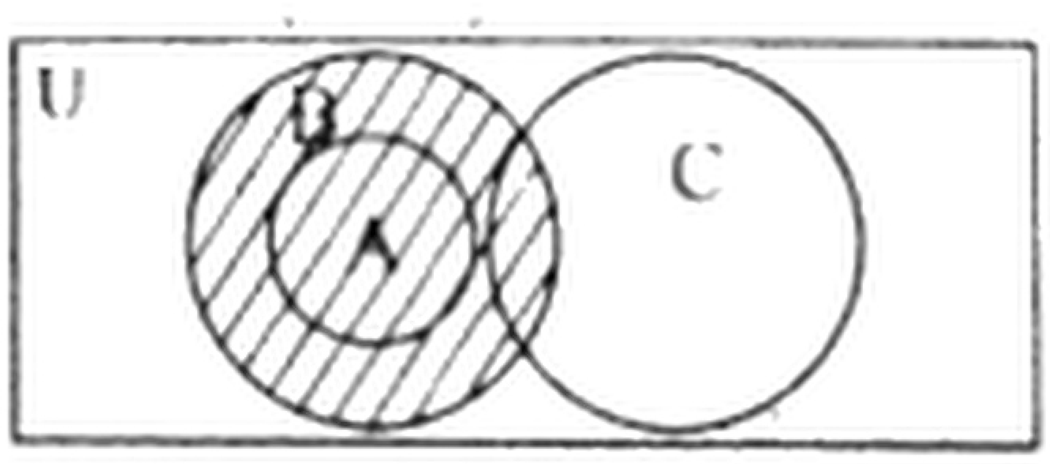
Solution:



Prove, Distributive of union over intersection

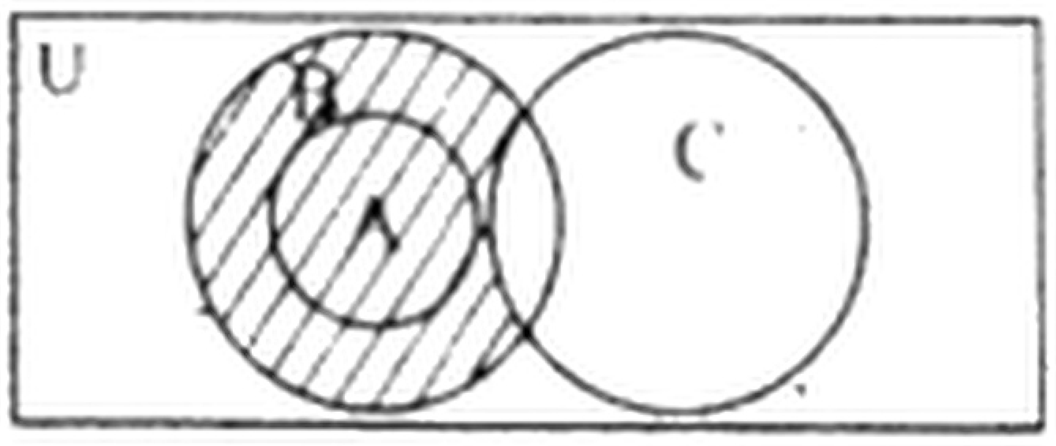
AU(B∩C) = (AUB) ∩ (AUC)

L.H.S = A U (B∩C)

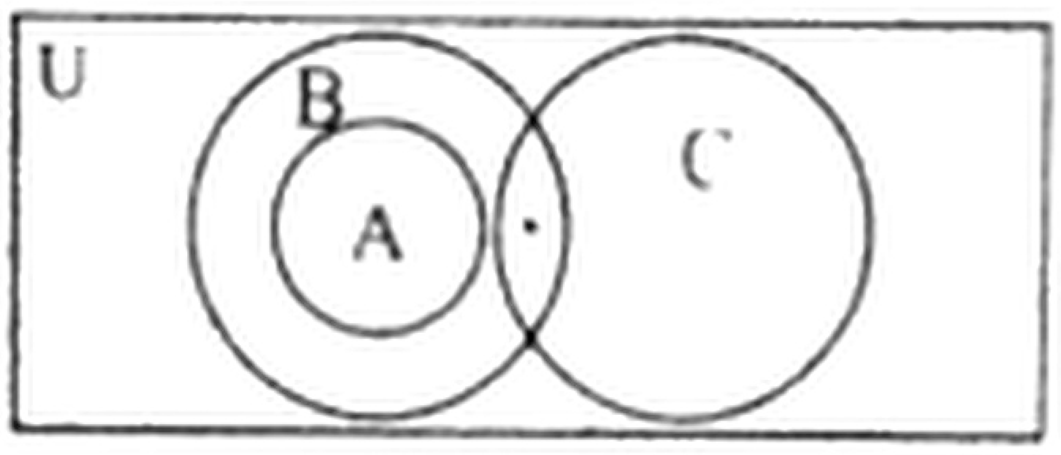


R.H.S = (AUB) ∩ (AUC)

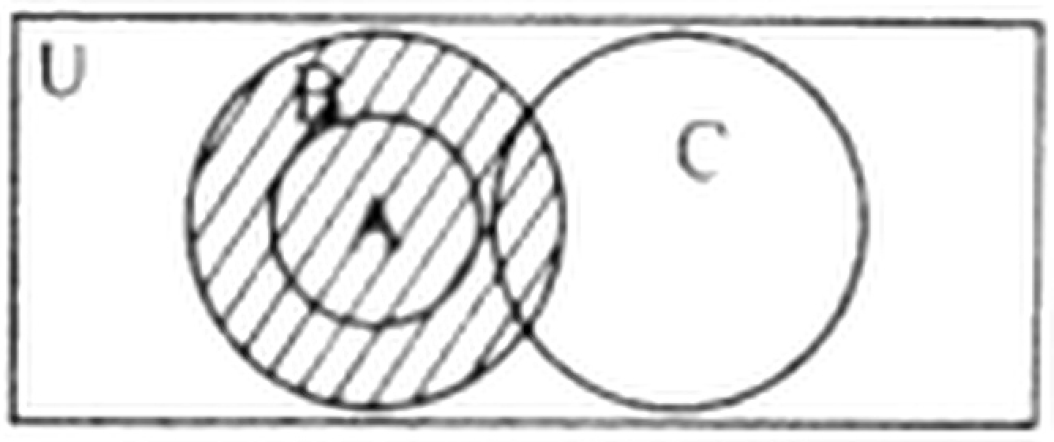
AUB



AUC



(AUB) ∩ (AUC)



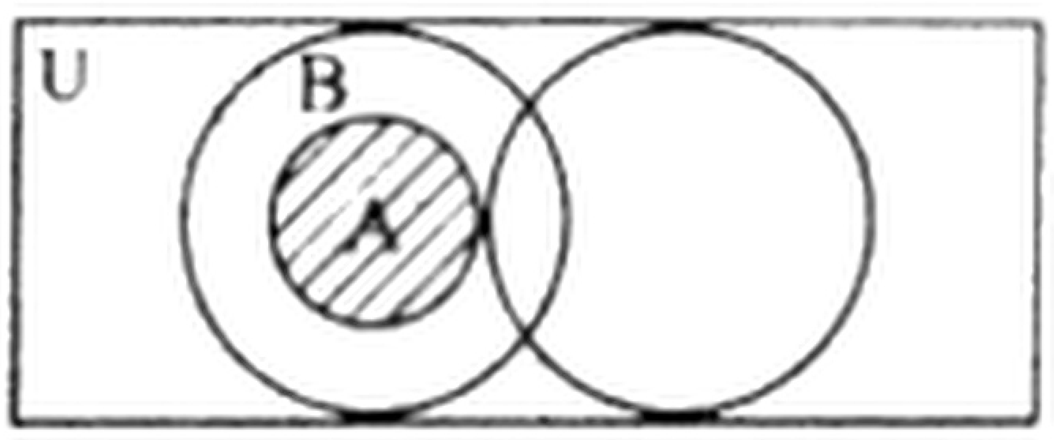
Hence Proved, A U (B∩C) = (AUB) ∩ (AUC)

Prove, Distributive of intersection over union

A∩(BUC) = (A∩B) U (A∩C)

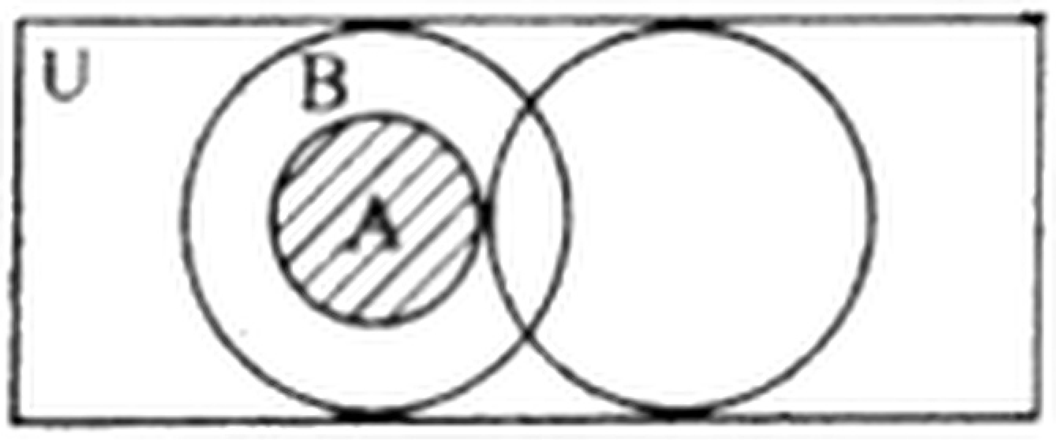
L.H.S = A ∩ (BUC)

A∩(BUC)

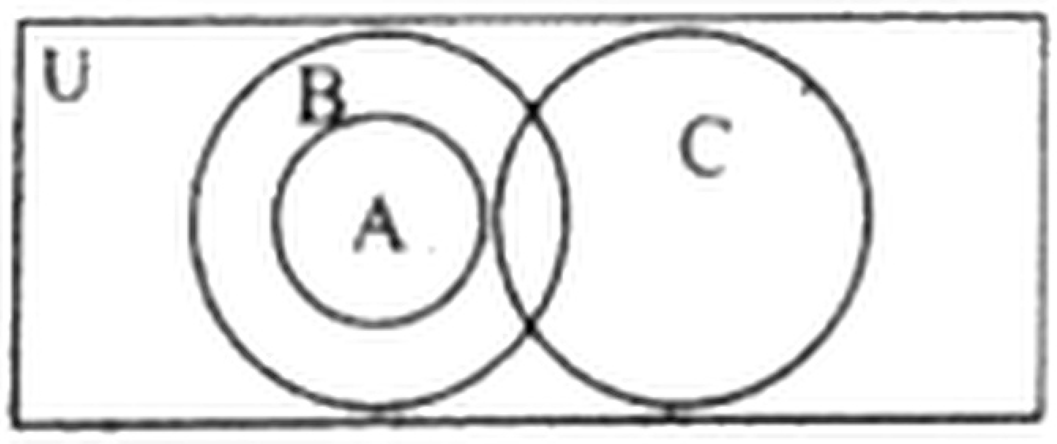


R.H.S = (A∩B) U (A∩C)

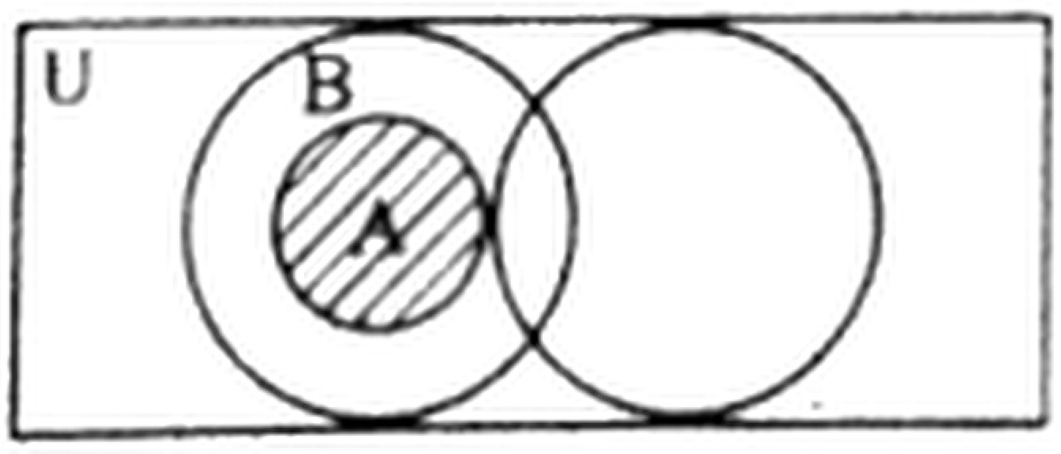
A∩B



A∩C

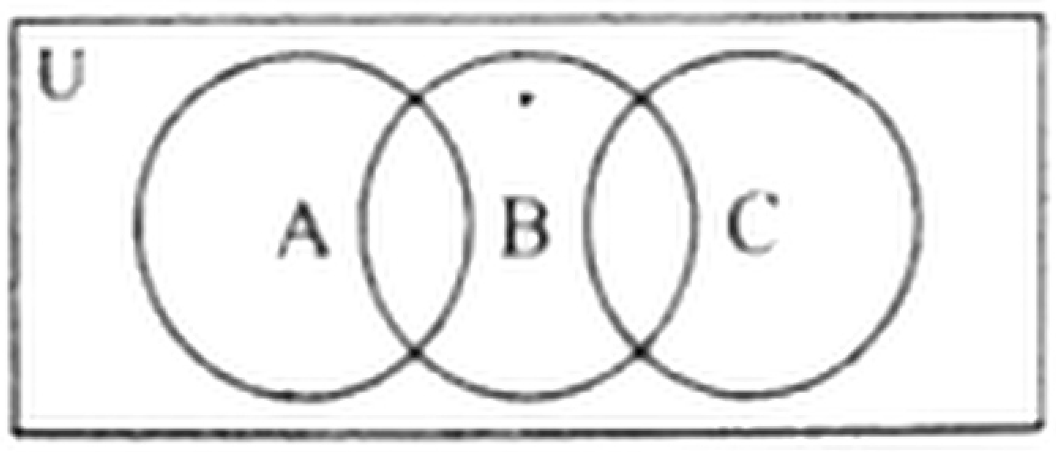


(A ∩B) U (A∩C)



Hence Proved, A ∩ (BUC) = (A∩B)U (A∩C)

ii. A and B are overlapping; B and C are overlapping but A and C are disjoint

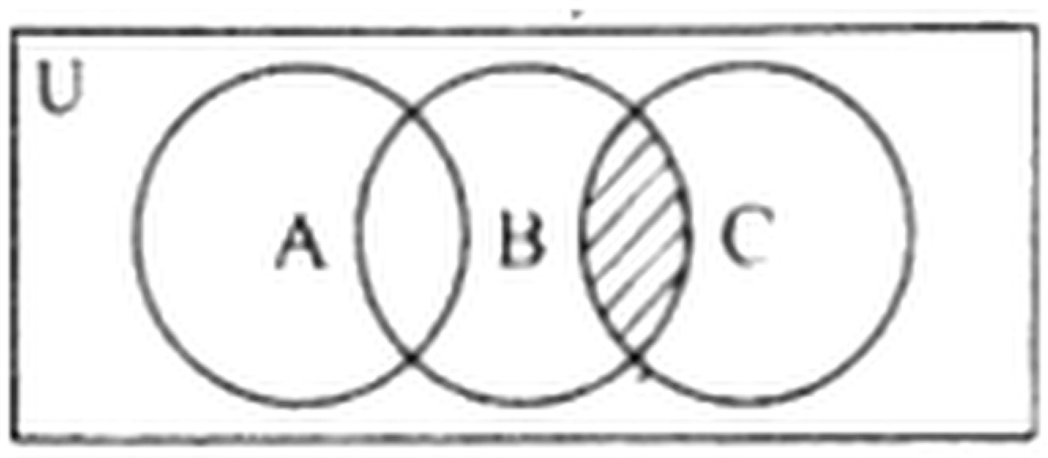


Prove, Distributive of union over intersection

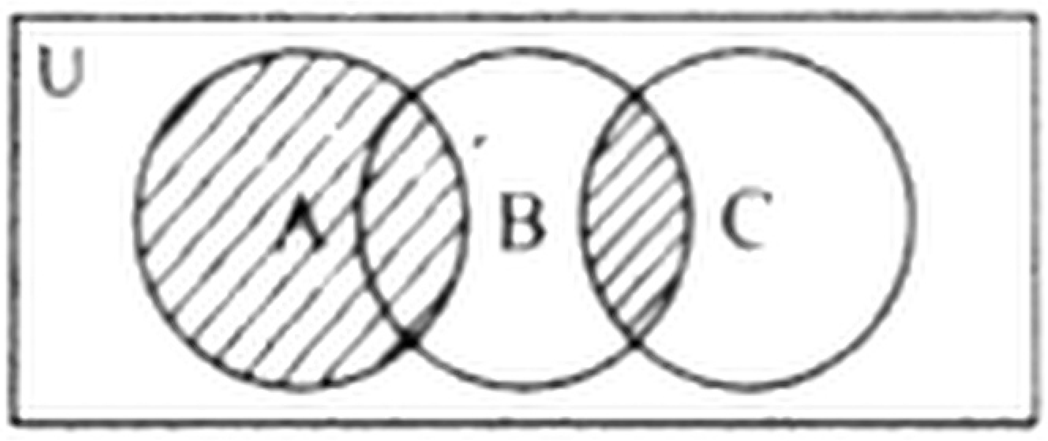
AU(B∩C) = (AUB) ∩ (AUC)

L.H.S = A U (B∩C)

B∩C

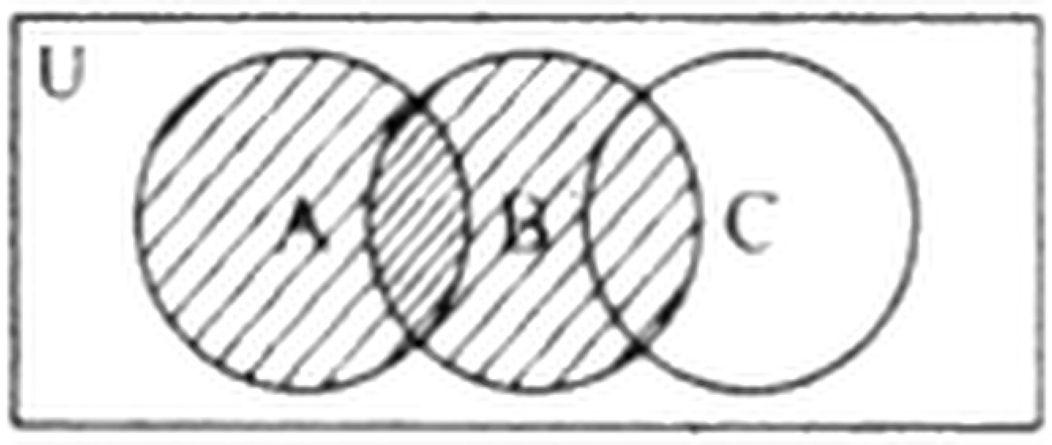


A U (B∩C)

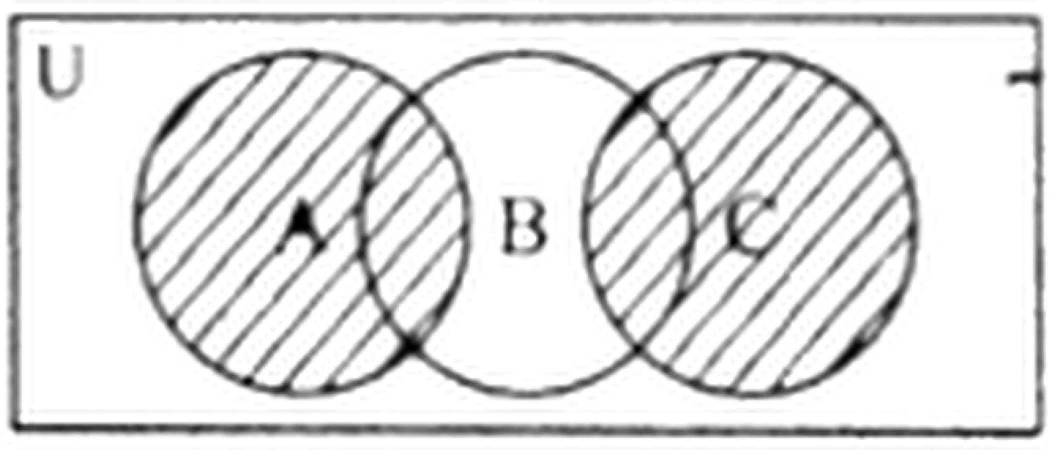


R.H.S = (AUB) ∩ (AUC)

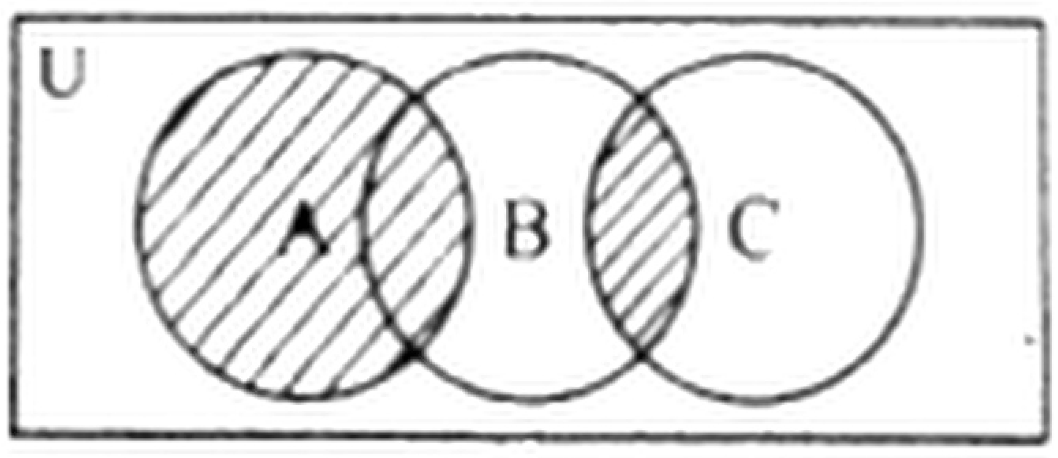
AUB



AUC



(AUB) ∩ (AUC)



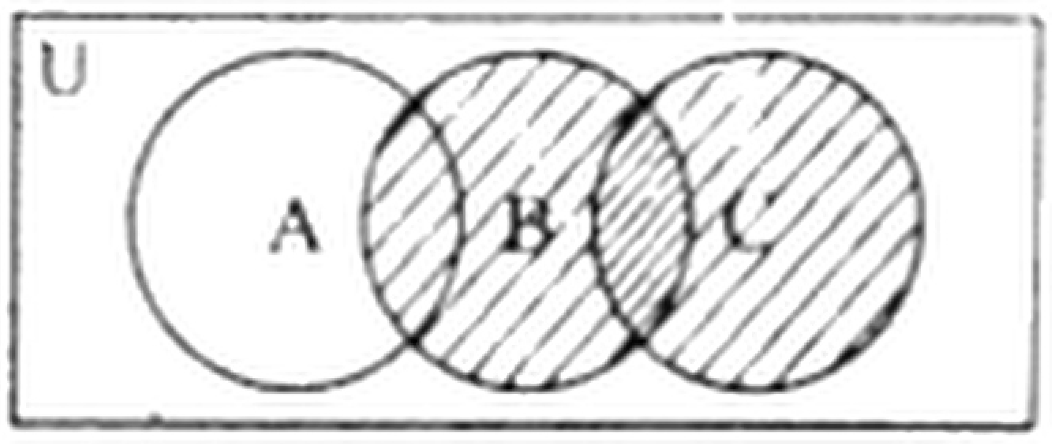
Hence Proved, A U (B∩C) = (AUB) ∩ (AUC)

Prove, Distributive of intersection over union

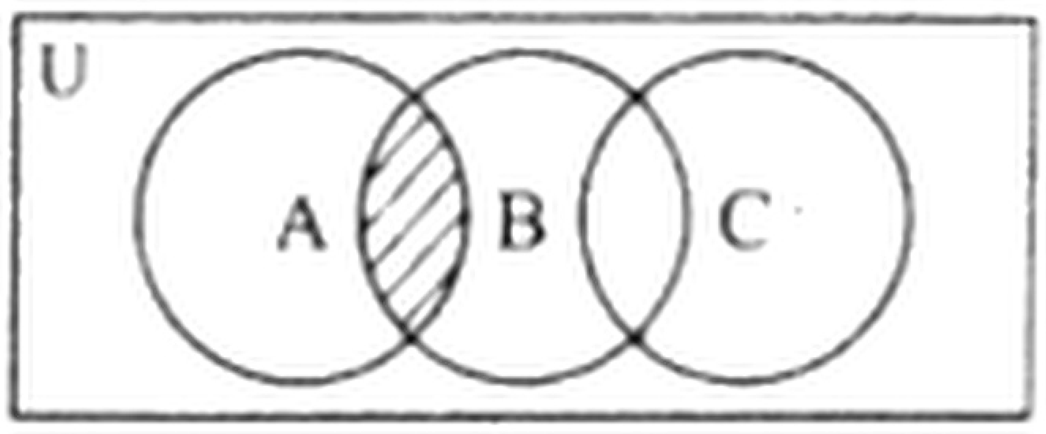
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

L.H.S $= A \cap (B \cup C)$

$B \cup C$

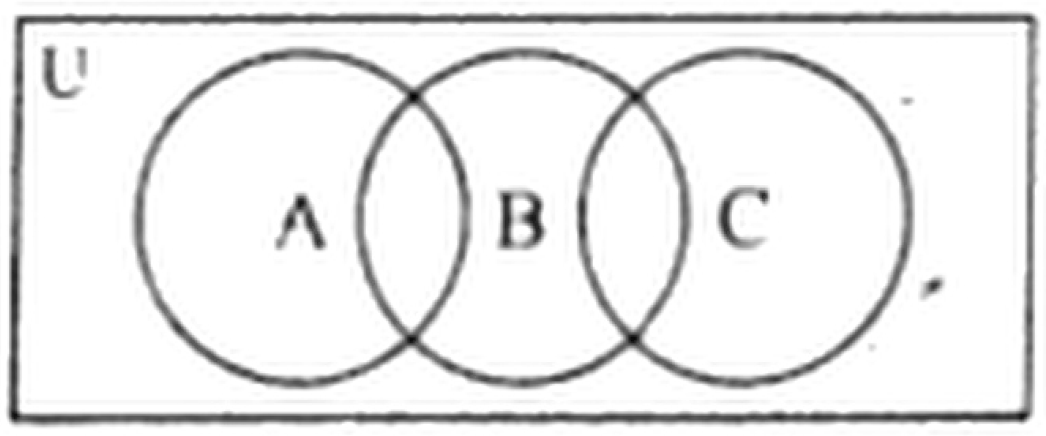


$A \cap (B \cup C)$

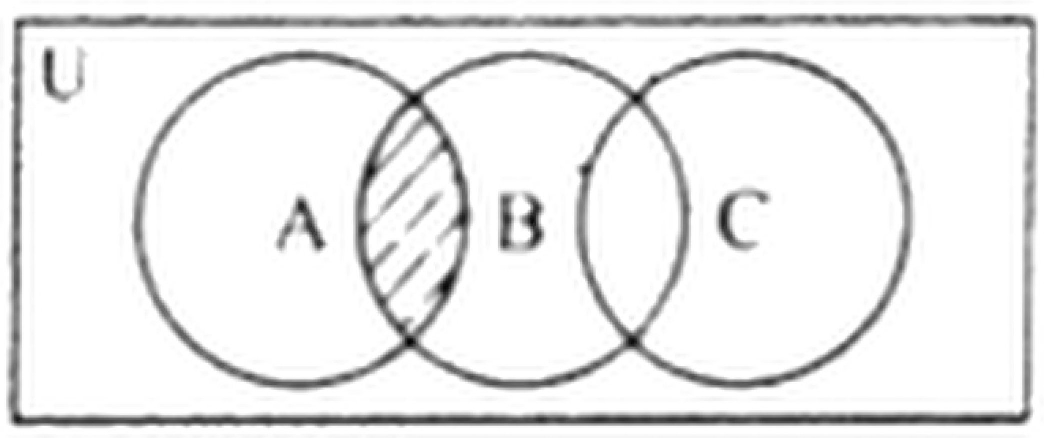


R.H.S $= (A \cap B) \cup (A \cap C)$

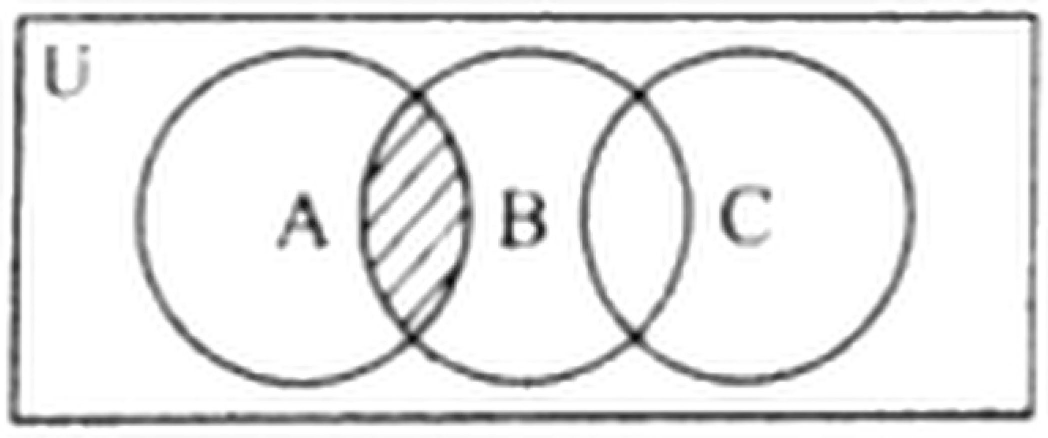
$A \cap C$



$A \cap B$



$(A \cap B) \cup (A \cap C)$



Hence Proved, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Q6. Taking any set say A = (1,2,3,4,5) verify the following:-

i. $A \cup ø = A$ ii. $A \cap A = A$

Solution:

i. $A \cup ø = A$

R.H.S = $A \cup ø = A$ = R.H.S

Hence, Proved

ii. $A \cup A = A$

R.H.S $A \cup A = A \cup A = A$ = R.H.S

Hence, Proved

iii. $A \cap A = A$

R.H.S $A \cap A = A$ = R.H.S

Hence, Proved

Q7. If U = (1,2,3,4,…20) and A = (1,3,5,.....,19), verify the following.

i. $A \cup A' = U$ ii. $A \cap U = A$ iii. $A \cap A' = ø$

i. **$A \cup A' = U$**

L.H.S = $A \cup A'$

$A' = U - A = \{1,2,3,.....,20\} - \{1,3,5,.....,19\}$

$= \{2,4,6,.....,20\}$

$A \cup A' = \{1,3,5,.....,19\} \cup \{2,4,6,....,20\}$

$= \{1,2,3,4,.....,20\}$

Hence, Proved

ii. **$A \cap U = A$**

L.H.S = $A \cap U$

$A \cap U = \{1,2,3,5,.....,19\} \cap \{1,2,3,.....,20\}$

$=\{1,3,5,.....,19\} = A =$ R.H.S

Hence, Proved

iii. **$A \cap A' = \varnothing$**

L.H.S = $A \cap A'$

$A' = U - A = \{1,2,3,.....,20\} - \{1,3,5,.....,19\} = \{2, 4, 6,....., 20\}$

$A \cap A' = \{1,3,5,.....,19\} \cap \{2,4,6,....,20\} = \{\ \} =$ R.H.S

Hence, Proved

Q8. From suitable properties of union intersection deduce the following results:

i. **$A \cap (A \cup B) = A \cup (A \cap B)$**

ii. **$A \cup (A \cap B) = A \cap (A \cup B)$**

Solution:

i. **$A \cap (A \cup B) = A \cup (A \cap B)$**

L.H.S. = $A \cap (A \cup B)$

= $(A \cap A) \cup (A \cap B)$

(Distributive property of intersection over union)

= $A \cup (A \cap B)'$ **$\{\therefore A \cap A = A\}$**

= R.H.S

Hence, Proved $A \cap (A \cup B) = A \cup (A \cap B)$

ii. **$A \cup (A \cap B) = A \cap (A \cup B)$**

L.H.S = $A \cup (A \cup B)$

$A \cup (A \cap B) = (A \cap A) \cup \{A \cap B\}$

(Distributive property of intersection over union)

= $A \cap (A \cap B)$ $\{\because A \cup A = A)$

= R.H.S

Hence Proved: $A \cup (A \cap B) = (A \cap A) \cup \{A \cap B\}$

Q9. Using Venn diagrams, verify the following results.

i. **$A \cap B' = A$ iff $A \cap B = \varnothing$**

ii. **$(A - B) \cup B = A \cup B$**

iii. **$(A - B) \cap B = \varnothing$**

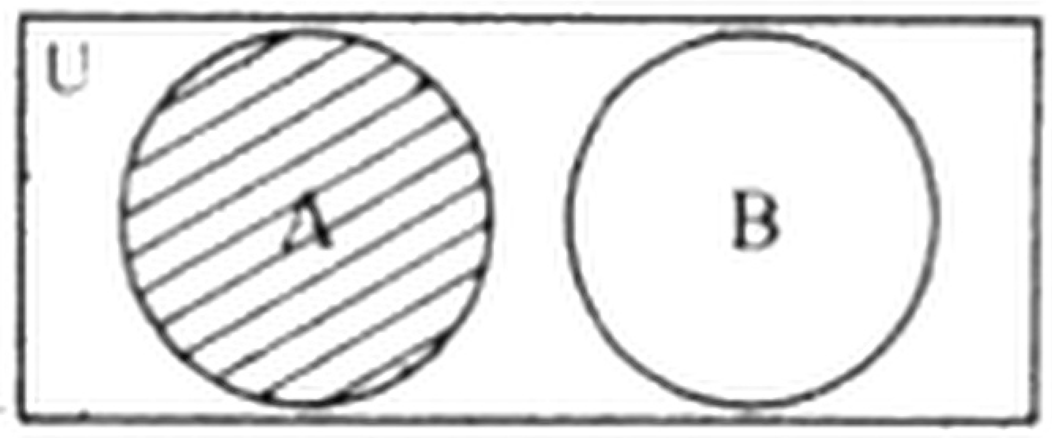
iv. **$A \cup B' = A \cup (A' \cap B')$**

Solution:

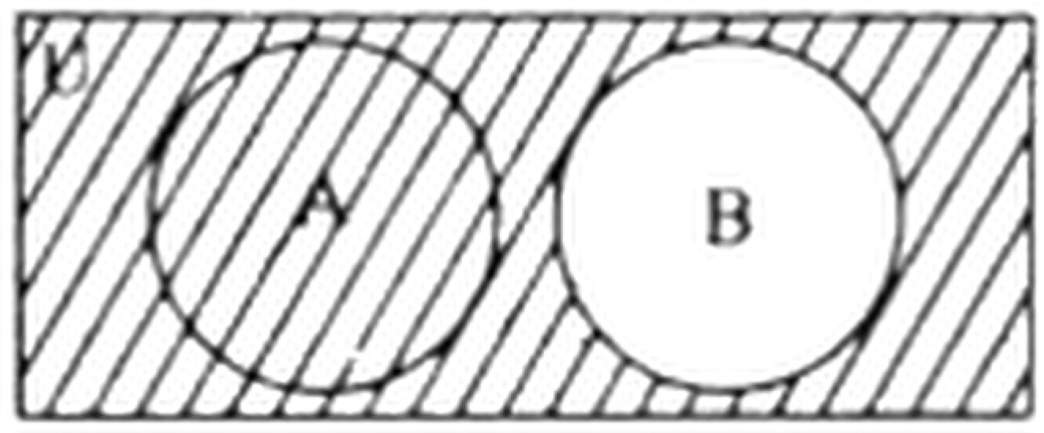
i. A∩B' = A iff A∩B= ø

if A∩B= ø

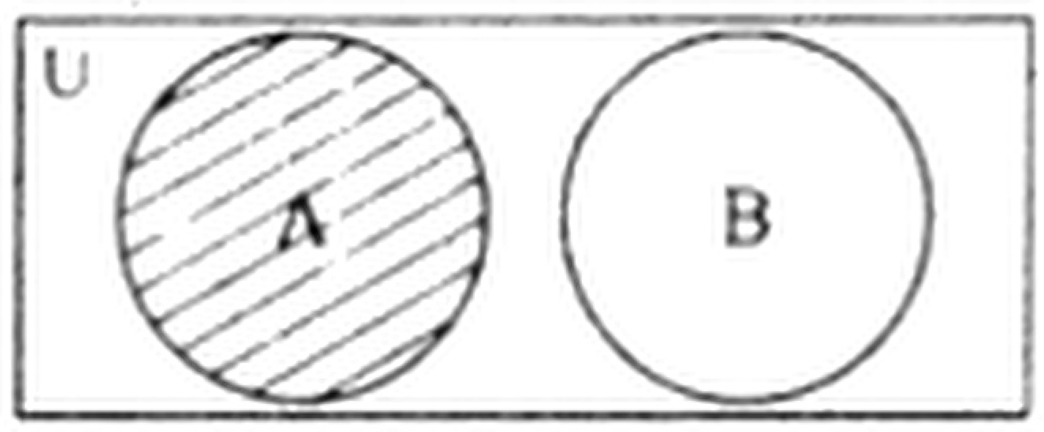
Then L.H.S = A∩B'



B' = U-B



A∩B'



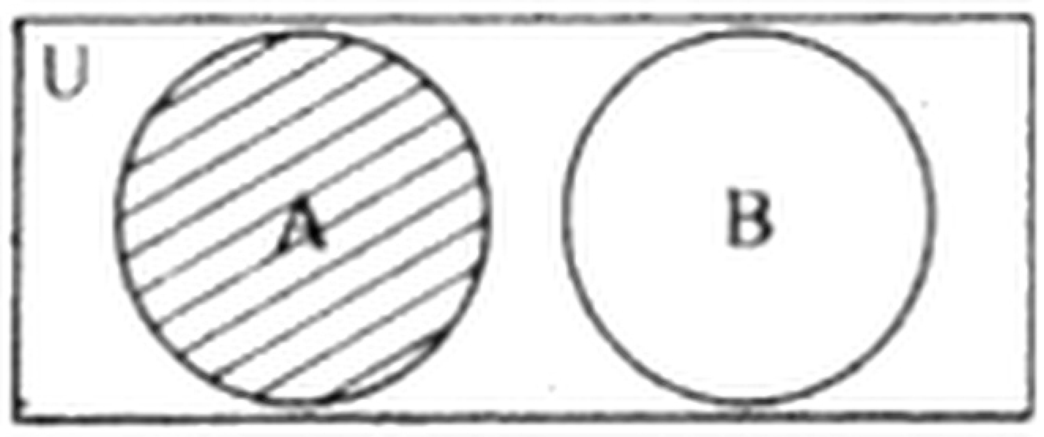
Hence, Proved

A∩B' = A iff A∩B= ø

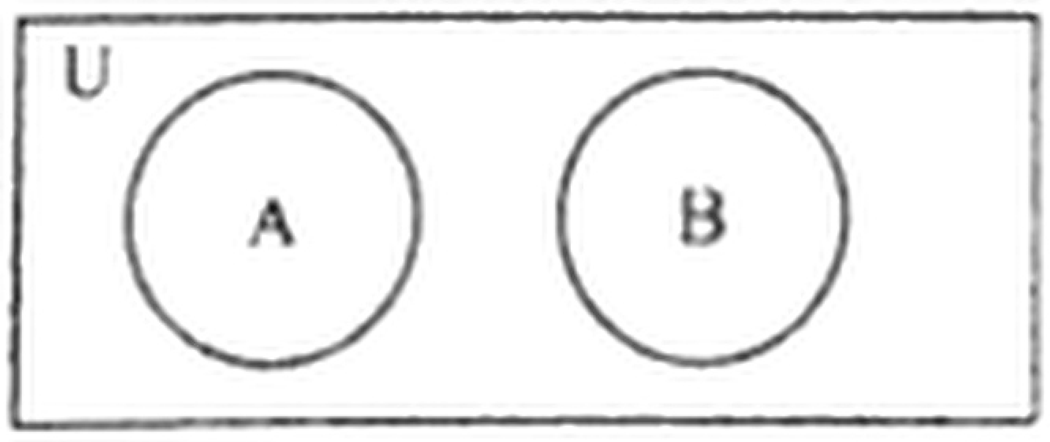
Conversely

A∩B= ø iff A∩B' = A

A∩B' = A



Then A∩B

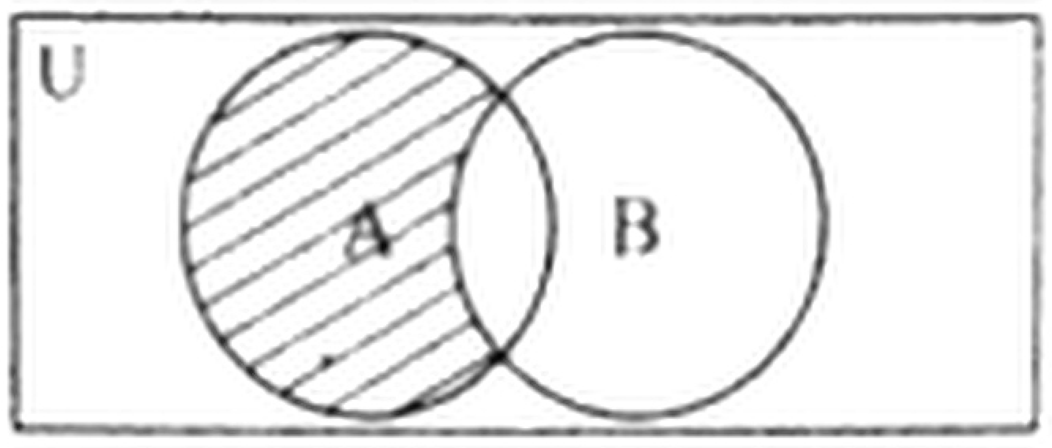


Hence Proved, $A \cap B = \emptyset$ iff $A \cap B' = A$

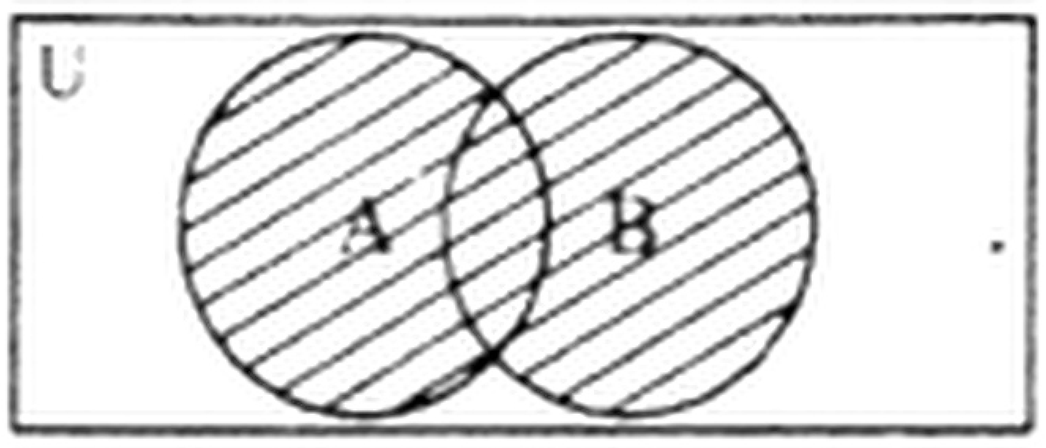
ii. **(A-B) ∪ B = A∪B**

L.H.S = (A-B) ∪ B

A-B



(A-B) ∪ B



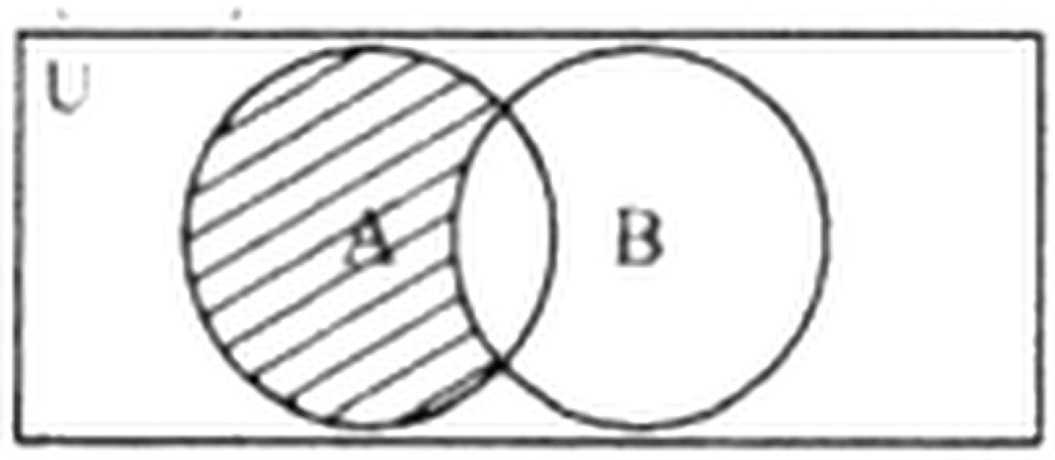
R.H.S =A∪B

Hence Proved (A-B) ∪ B = A∪B

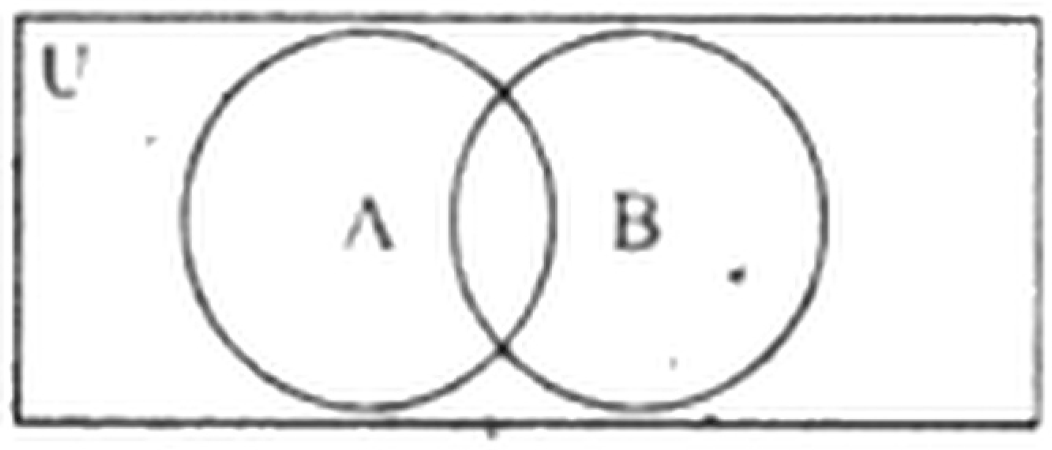
iii. **(A-B) ∩ B = ø**

L.H.S = (A-B) ∩ B

A-B



(A-B) ∩ B

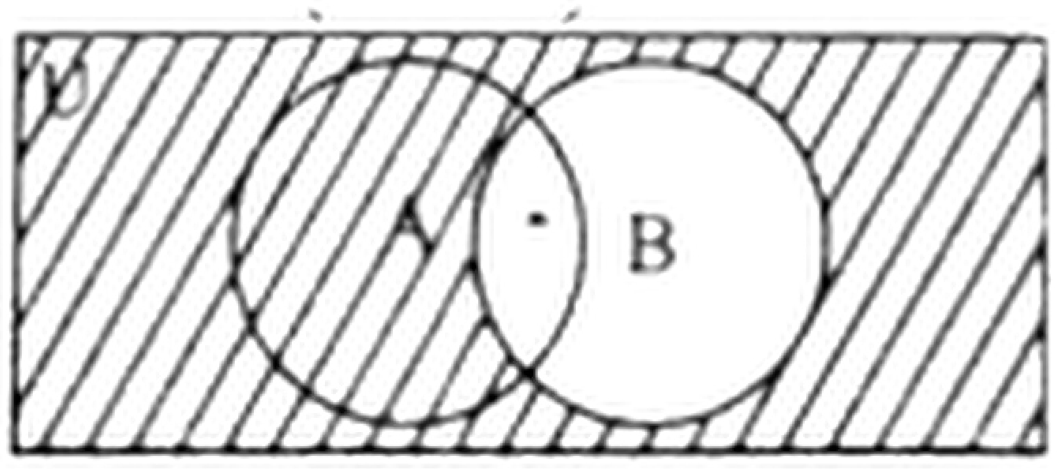


R.H.S = ø

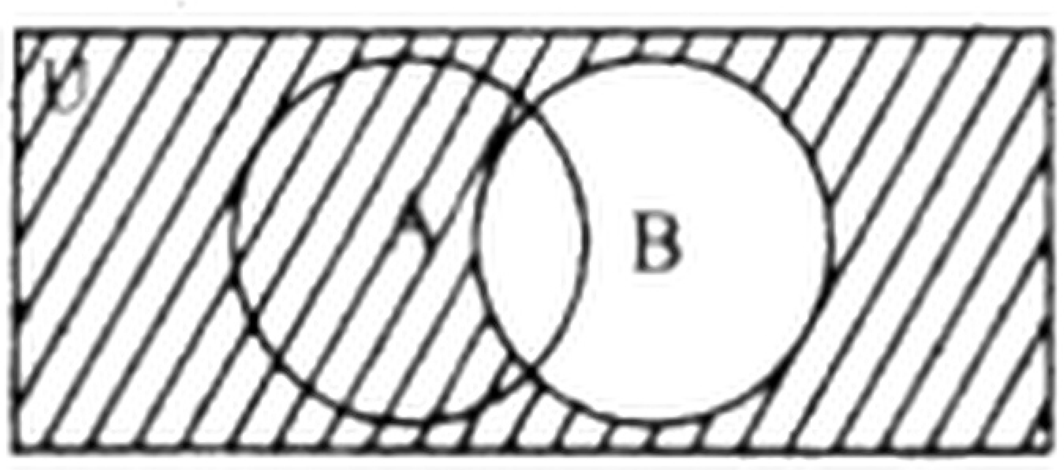
Hence Proved (A-B) ∩ B = ø

iv. A∪B' = A ∪(A'∩B')

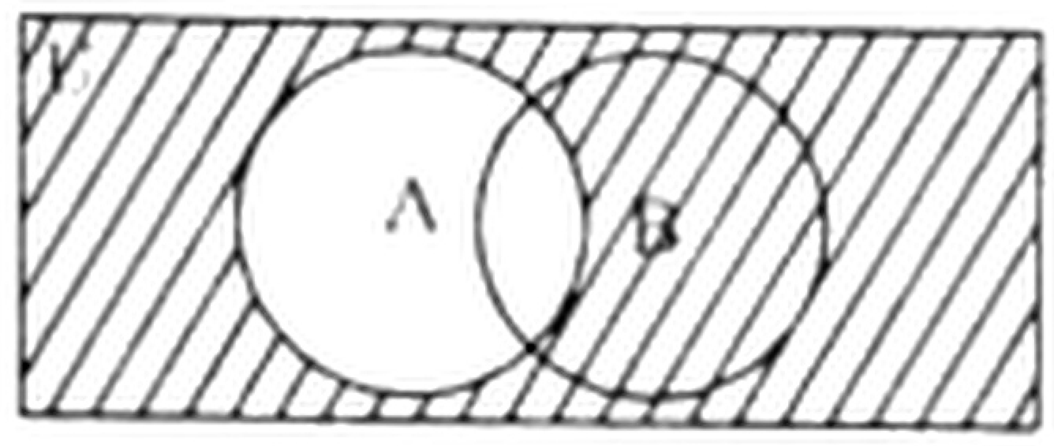
R.H.S = A ∪(A'∩B')



B' = U-B

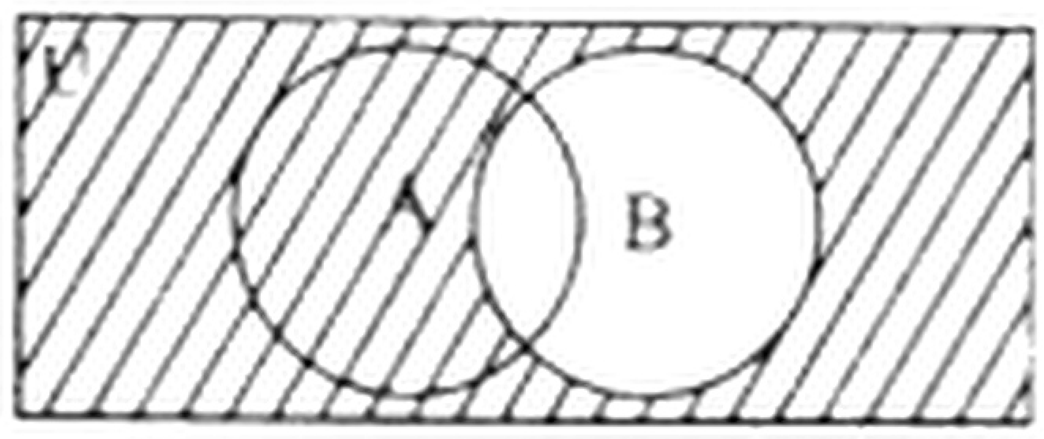


A' = U-A



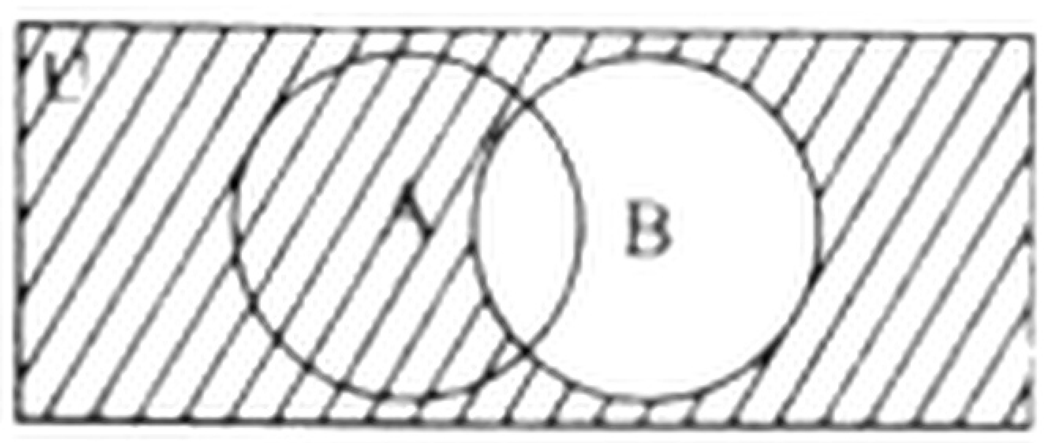
A'∩B'

A U(A'∩B')



L.H.S

AUB'



Hence Proved AUB' = A U(A'∩B')