Chapter 14

Solution of Trigonometric Equations

Exercise 14

Q.1 Find the solution of the following equations which lie in $[0, 2\pi]$

i. $\sin x = -\frac{\sqrt{3}}{2}$ **ii.** $cosec\,\theta = 2$ **iii.** $\sec x = -2$

iv. $cot\theta = \frac{1}{\sqrt{3}}$

Solution:

i. $\sin x = -\frac{\sqrt{3}}{2}$

Since, the value of $'\sin x'$ is '-ve' so, either $\sin x$ lie in III or IV quadrant

$$x = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$= \sin^{1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\pi/3$$

In III quadrant

$$x = \pi + \frac{\pi}{3} = \frac{3\pi\times\pi}{3} = \frac{4\pi}{3}$$

In IV quadrant

$$x = 2\pi - \frac{\pi}{3}$$

$$= \frac{6\pi-\pi}{3} = \frac{5\pi}{3}$$

Hence, $S.S = \{\frac{4\pi}{3}, \frac{5\pi}{3}\}$

ii. $cosec\,\theta = 2$

$$\sin\theta = \frac{1}{2}$$

Since, the value of $'sin\theta'$is '+ve' so, either $sin\theta$ lie in I or II quadrant.

$$\theta = \sin^{-1}(1/2)$$

$$\theta = \frac{\pi}{6}$$

In I quadrant

$\theta = \pi/6$

In II quadrant

$\theta = \pi - \frac{\pi}{6} = \frac{6\pi - \pi}{6} = 5.\frac{\pi}{6}$

Hence, $S.S = \left\{\frac{\pi}{6}, 5\frac{\pi}{6}\right\}$

iii. So $\sec x = -2$

$\cos x = -\frac{1}{2}$

Since, the value of $\cos x$ is '-ve' so either $\cos x$ lie in II or III quadrant

$\cos x = \frac{1}{2}$

$x = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

In II quadrant

$x = \pi - \frac{\pi}{3} = \frac{3\pi - \pi}{3} = \frac{2\pi}{3}$

In III quadrant

$x = \pi + \frac{\pi}{3} = \frac{3\pi + \pi}{3} = \frac{4\pi}{3}$

Hence, $S.S = \left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}$

Q.2 Solve the following trigonometric equations:

i. $\tan^2\theta = \frac{1}{3}$ **ii. $\operatorname{cosec}^2\theta = \frac{4}{3}$** **iii. $\sec^2\theta = \frac{4}{3}$** **iv. $\cot^2\theta = \frac{1}{3}$**

Solution:

i. $\tan^2\theta = \frac{1}{3}$

$\tan\theta = \pm\frac{1}{\sqrt{3}}$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

Since, the value of '$\tan\theta$' is +ve either $tan\theta$ lie in I or III quadrant

$$\theta = \pi/6$$

In I quadrant

$$\theta = \pi/6$$

In III quadrant

$$\theta = \pi + \frac{\pi}{6} = \frac{6\pi+\pi}{6} = \frac{7\pi}{6}$$

$$\tan\theta = -\frac{\sqrt{3}}{2}$$

$$\theta = \tan^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

Since, the value of $\tan\theta$ is '-ve' either $\tan\theta$ lie in II or IV quadrant

In II quadrant

$$\theta = \pi - \frac{\pi}{6} = \frac{6\pi-\pi}{6} = \frac{5\pi}{6}$$

In IV quadrant

$$\theta = 2\pi - \frac{\pi}{6} = \frac{12\pi-\pi}{6} = \frac{11\pi}{6}$$

Hence, $S.S = \left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

ii. $\mathbf{cosec^2\,\theta = \frac{3}{4}}$

$$cosec\,\theta = \sqrt{\frac{3}{4}}$$

$$cosec\,\theta = \pm\frac{\sqrt{3}}{2}$$

$$\sin\theta = \pm\frac{2}{\sqrt{3}}$$

$$\sin\theta = \frac{2}{\sqrt{3}}$$

Since, the value of $\sin\theta$ is '+ve' so, either $\sin\theta$ lie in I or II quadrant

$$\theta = \sin^{-1}\left(\frac{2}{\sqrt{3}}\right) = \frac{\pi}{3}$$

In I quadrant

$$\theta = \pi/3$$

In II quadrant

$$\theta = \pi - \frac{\pi}{3} = \frac{3\pi - \pi}{3} = \frac{2\pi}{3}$$

$$\sin\theta = -\frac{2}{\sqrt{3}}$$

Since, the value of $sin\theta$ is '-ve' so, either, $\sin\theta$ lie in III or IV quadrant

In III quadrant

$$\theta = \pi + \frac{\pi}{3} = \frac{3\pi + \pi}{3} = \frac{4\pi}{3}$$

In IV quadrant

$$\theta = 2\pi - \frac{\pi}{3} = \frac{6\pi - \pi}{3} = \frac{5\pi}{3}$$

Hence, $S.S = \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$

iii. $\boldsymbol{\sec^2\theta = \frac{3}{4}}$

$$\sec\theta = \sqrt{\frac{3}{4}} = \pm\frac{3}{\sqrt{2}}$$

$$\sec\theta = \pm\frac{2}{\sqrt{3}}$$

$$\cos\theta = \frac{2}{\sqrt{3}}$$

Since, the value of $\cos\theta$ $is' + ve'$ either $\cos\theta$ lie in I or IV quadrant

$$\theta = \cos^{-1}\left(\frac{2}{\sqrt{3}}\right) = \frac{\pi}{6}$$

In I quadrant

$$\theta = \frac{\pi}{6}$$

In IV quadrant

$$\theta = 2\pi - \frac{\pi}{6} = \frac{12\pi - \pi}{6} = \frac{11\pi}{6}$$

$$\cos\theta = -\frac{\sqrt{3}}{2}$$

Since, the value of $\cos\theta$ is '-ve' either $\cos\theta$ lie in III or IV quadrant

$$\theta = \cos^{-1}\left(\frac{3}{2}\right) = \frac{\pi}{6}$$

In II quadrant

$$\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

In III quadrant

$$\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$$

Hence, $S.S = \left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

iv. $\cot^2\theta = \frac{1}{3}$

$$\cot\theta = \pm\frac{1}{\sqrt{3}}$$

$$\tan\theta = \pm\sqrt{3}$$

$$\tan\theta = \sqrt{3}$$

Since, the value of $\tan\theta$ is '+ve' either $\tan\theta$ lie in I or III quadrant

$$\theta = \tan^{-1}(\sqrt{3})$$

$$\theta = \pi/3$$

In I quadrant

$$\theta = \pi/3$$

In III quadrant

$$\theta = \pi + \frac{\pi}{3} = \frac{3\pi + \pi}{3} = \frac{4\pi}{3}$$

$$\tan\theta = -\sqrt{3}$$

Since, the value of $\tan\theta$ is '-ve' either $\tan\theta$ lie in II or IV quadrant

$\theta = \tan^{-1}(\sqrt{3}) = \pi/3$

In II quadrant

$\theta = \pi - \frac{\pi}{3} = \frac{3\pi-\pi}{3} = \frac{2\pi}{3}$

In IV quadrant

$\theta = 2\pi - \frac{\pi}{3} = \frac{6\pi-\pi}{3} = \frac{5\pi}{3}$

Hence, $S.S = \left\{\frac{\pi}{3}, \frac{4\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{3}\right\}$

Find the values of θ satisfying the following equations:

Solution:

3. $3\tan^2\theta + 2\sqrt{3}\tan\theta + 1 = \theta$

Put $\tan\theta = x$

$3x^2 + 2\sqrt{3}x + 1 = 0$

$\left(\sqrt{3x}\right)^2 + 2\left(\sqrt{3x}\right)(1) + (1)^2 = 0$

$\left(\sqrt{3x} + 1\right)^2 = 0$

$\sqrt{3x} + 1 = 0$

$x = -1/\sqrt{3}$

Therefore $\tan\theta = -1/\sqrt{3}$

Since, the value of $\tan\theta$ is '-ve' so, either $\tan\theta$ lie in II or IV quadrant

$Tan\ \theta = 1/\sqrt{3}$

$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \pi/6$

In II quadrant

$\theta = \pi - \frac{\pi}{6} = \frac{6\pi-\pi}{6} = \frac{5\pi}{6}$

In IV quadrant

$\theta = 2\pi - \frac{\pi}{6} = \frac{12\pi - \pi}{6} = \frac{11\pi}{6}$

Hence, $S.S = \left\{\frac{5\pi}{6}, \frac{11\pi}{6}\right\}$

4. $\tan^2 \theta - \sec \theta - 1 = 0$

Solution:

$(\sec^2 \theta - 1) - sec\theta - 1 = 0$

$\sec^2 \theta - sec\theta - 2 = 0$

$\sec \theta = x$

$x^2 - x - 2 = 0$

$x^2 - 2x + x - 2 = 0$

$x(x-2) + 1(x-2) = 0$

$x + 1 = 0$ or $x - 2 = 0$

$x = -1$ $x = 2$

Therefore

$\sec x = -1$ and $\sec x = 2$

$\sec x = -1$

Since, the value of $\sec x$ is –ve, so either $\sec x$ lie in II or III quadrant

$x = (\sec^{-1})(1) = 0,$

In II quadrant

$= 0 + \pi = \pi$

In III quadrant

$= \pi - 0 = \pi$

$\sec x = 2$

Since, the value of $\sec x$ is +ve, so either $\sec x$ lie in I or IV quadrant

$x = \sec(2) = \pi/3$

In I quadrant

$$x = 0 + \frac{\pi}{3} = \frac{\pi}{3}$$

In IV quadrant

$$x = 2\pi - \frac{\pi}{3} = \frac{6\pi - \pi}{3} = \frac{5\pi}{3}$$

Hence, $S.S = \left\{\frac{\pi}{3}, \pi, \frac{5\pi}{3}\right\}$

5. $2\sin\theta - \cos^2\theta - 1 = 0$

$$2\sin\theta + (-\sin^2\theta + 1) - 1 = 0$$

$$2\sin\theta - \sin^2\theta = 0$$

$$\sin^2\theta - 2\sin\theta = 0$$

Let $\sin\theta = x$

$$x^2 - 2x = 0$$

$$x(x - 2) = 0$$

$x = 0$ or $x - 2 = 0$

$$x = 2$$

Therefore,

$$\sin\theta = 0$$

$$\theta = \sin^{-1}(0) = 0, \pi$$

and

$$\sin\theta = 2$$

not possible since of value of $\sin\theta$ lie $-1 \le \theta \le -1$

Hence, $S.S = \{0, \pi\}$

6. $2\sin^2 - sin = 0$

$\sin\theta(2\sin\theta - 1) = 0$

$\sin\theta = 0$ or $2\sin\theta - 1 = 0$

$\theta = \sin^{-1}(0)$

$\theta = 0, \pi$

and

$2\sin\theta - 1 = 0$

$\sin\theta = 1/2$

$\theta = \sin^{-1}\left(\frac{1}{2}\right)$

$= \pi/6$

Since, the value of $\sin\theta$ is +ve, so it lie in I or II quadrant

In II quadrant

$\theta = \pi - \frac{\pi}{6} = 5\pi/6$

Hence, $S.S = \left\{0, \frac{\pi}{6}, \pi, \frac{5\pi}{6}\right\}$

7. $3\cos^2\theta - 2\sqrt{3}\sin\theta\cos\theta - 3\sin^2\theta = 0$

Divide the equation by $\sin^2\theta$

$\frac{3\cos^2\theta}{\sin^2\theta} - 2\sqrt{3}\frac{\sin\theta\, cos\theta}{\sin^2\theta} - \frac{3\sin^2\theta}{\sin^2\theta} = 0$

$3\cot^2\theta - 2\sqrt{3}\cot\theta - 3 = 0$

Let $\cot\theta = x$

$3x^2 - 2\sqrt{3}\,x - 3 = 0$

By quadrant formula

$a = 3; b = -2\sqrt{3}; c = -3$

$x = \frac{2\sqrt{3} \pm \sqrt{12+36}}{6}$

$= \frac{2\sqrt{3} \pm 4\sqrt{3}}{6}$

either

$x = \frac{2\sqrt{3}+4\sqrt{3}}{6}$, $x = \frac{2\sqrt{3}-4\sqrt{3}}{6}$

$= \frac{6\sqrt{3}}{6} = \sqrt{3}$, $= \frac{-2\sqrt{3}}{6} = \frac{-\sqrt{3}}{6}$

$= \sqrt{3}$, $= \frac{-\sqrt{3}}{3} = \frac{-1}{\sqrt{3}}$

so , $\cot\theta = \sqrt{3}$, $\cot\theta = {}^{-1}/_{\sqrt{3}}$

$\cot\theta = \sqrt{3}$

since the value of $\cot\theta$ is +ve, so, either $\cot\theta$ lies in I or III quadrant

$\theta = \cot^{-1}(\sqrt{3})$

$\theta = {}^{\pi}/_{6}$

In I quadrant

$\theta = {}^{\pi}/_{6}$

In III quadrant

$\theta = \pi + {}^{\pi}/_{6} = 7\,{}^{\pi}/_{6}$

and $\cot\theta = -\frac{1}{\sqrt{3}}$

Since, the value of $\cot\theta$ is –ve, so either $\cot\theta$ lies in II or IV quadrant

$\theta = \cot\left(\frac{1}{\sqrt{3}}\right)$

$\theta = {}^{\pi}/_{3}$

In II quadrant

$\theta = \pi - {}^{\pi}/_{3} = {}^{2\pi}/_{3}$

In IV quadrant

$\theta = 2\pi - {}^{\pi}/_{3} = \frac{6\pi - \pi}{3} = {}^{5\pi}/_{3}$

Hence S.S = $\left\{{}^{\pi}/_{6}, {}^{2\pi}/_{3}, {}^{7\pi}/_{6}, {}^{5\pi}/_{3}\right\}$

Q8. $\mathbf{4\sin^2\theta - 8\cos\theta + 1 = 0}$

Solution

$$4(1-\cos^2\theta) - 8\cos\theta + 1 = 0$$

$$4 - 4\cos^2\theta - 8\cos\theta + 1 = 0$$

$$-4\cos^2\theta - 8\cos\theta + 5 = 0$$

$$4\cos^2\theta + 8\cos\theta - 5 = 0$$

Let $\cos\theta = x$

then $4x^2 + 8x - 5 = 0$

$$4x^2 + 10x - 2x - 5 = 0$$

$$2x(2x+5) - 1(2x+5) = 0$$

$$(2x-1)(2x+1) = 0$$

$2x - 1 = 0$, $2x + 5 = 0$

$2x = 1$, $2x = {}^{-5}/_2$

$x = {}^{1}/_2$, $x = {}^{-5}/_2$

Therefore $\cos\theta = {}^{1}/_2$

Since, the value of $\cos\theta$ is +ve either cos θ lies in I or IV quadrant

$$\theta = \cos^{-1}({}^{1}/_2) = {}^{\pi}/_3$$

In I quadrant

$$\theta = {}^{\pi}/_3$$

In IV quadrant

$$\theta = 2\pi - {}^{\pi}/_3 = \frac{6\pi-\pi}{3} = {}^{5\pi}/_3$$

And $x = {}^{-5}/_2$ (not possible)

Since the value of $\cos\theta$ lies between $-1 \le x \le -1$

Hence $S.S = \{ {}^{\pi}/_{3}, {}^{5\pi}/_{3} \}$

Q9. $\sqrt{3}\tan x - \sec x - 1 = 0$

Solution

$$\sqrt{3}\tan x - \sec x + 1$$

squaring both sides

$$\left(\sqrt{3\tan x}\right)^2 = (\sec x + 1)^2$$

$$3\tan^2 x = \sec^2 x + 2\sec x + 1$$

$$-3 + 3\sec^2 x = \sec^2 x + 2\sec x + 1$$

$$3\sec^2 x - \sec^2 x - 2x\sec - 3 - 1 = 0$$

$$2\sec^2 x - 2\sec x - 4 = 0$$

$$2(\sec^2 x - \sec x - 2) = 0$$

$$\sec^2 - x - \sec x - 2 = 0$$

Let $\sec x = y$

$$y^2 - y - 2 = 0$$

$$y^2 - 2y + y - 2 = 0$$

$$y(y-2) + 1(y-2) = 0$$

$$(y-2)(y+1) = 0$$

either, $y - 2 = 0$, $y + 1 = 0$

$y = 2$; $y = -1$

$$\sec x = 2$$

$$\cos x = \frac{1}{2}$$

$$x = \cos^{-1}\left(\frac{1}{2}\right) = {}^{\pi}/_{3} \,\&\left(2\pi - {}^{\pi}/_{3}\right)$$

$$x = {}^{\pi}/_{3}, {}^{5\pi}/_{3}$$

Since; they have a period of 2π

So, $x = {}^{x}/_{3} + 2n\pi$; & ${}^{5\pi}/_{3} + 2n\pi$

and

$$\sec x = -1$$

$$\cos x = -1$$

$$x = \cos^{-1}(-1) = \pi$$

Since, they have a period of 2π

$$x = \pi + 2n\pi$$

Hence S.S= $\{\pi + 2n\pi\} \cup \left\{{}^{\pi}/_{3} + 2n\pi\right\} \cup \left\{{}^{5\pi}/_{3} + 2n\pi\right\}$

Q10.$\cos 2x = \sin 3x$

Solution

We know that

$$\cos 2x = 1 - 2\sin 3x$$

and $\sin 3x = 3\sin x - 4\sin^3 x$

By putting the value in equation

$$1 - 2\sin^2 x = 3\sin x - 4\sin^3 x$$

$$4\sin^3 x - 2\sin^2 x - 3\sin x + 1 = 0$$

Let $\sin x = y$

Then $4y^3 - 2y^2 - 3y + 1 = 0$

$$P(y) = 4y^3 - 2y^2 - 3y + 1$$

$$P(1) = 4(1)^3 - 2(1)^2 - 3(1) + 1$$

$$= 4 - 2 - 3 + 1 = 0$$

$$= 5 - 5 = 0$$

Therefore;

$y = 1$ is the root of equation by synthetic division

1	4	−2	−3	+1
	0	4	2	−1
	4	2	−1	0 (Remainder)

$$(y-1)(4y^2 = 2y - 1) = 0$$

Either $y - 1 = 0$ or $4y^2 + y - 1 = 0$

$y = 1;$

$4y^2 + 2y - 1 = 0$

$a = 4;$ $b = 2;$ $c = 1$

By quadratic formula

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{4} - 4(4)(-1)}{2(4)}$$

$$= \frac{-2 \pm \sqrt{20}}{8}$$

$$= \frac{-2 \pm \sqrt{5}}{8}$$

$$= \frac{-2 \pm 2\sqrt{5}}{8} = \frac{-2 \pm \sqrt{5}}{4}$$

either $\sin x = and$ $\sin x = \frac{-1 \pm \sqrt{5}}{4}$

$x = \sin^{-1}(1)$; $x = \sin^{-1}\left(\frac{-2 \pm \sqrt{5}}{2}\right)$

$x = {}^{\pi}/_{2}$

It has a period of 2π

$x = 2n\pi + {}^{\pi}/_{2}$

and $x = \sin^{-1}\left(\frac{2 \pm \sqrt{5}}{2}\right)$ $= \frac{\pi}{10}$ or $\frac{9\pi}{10}$

It has a period of 2π

$x = \frac{\pi}{10} + 2n\pi$; $\frac{9\pi}{10} + 2n\pi$

Hence: S.S= $\left\{2n\pi + \frac{\pi}{2}\right\} \cup \left\{\frac{\pi}{10} + 2n\pi\right\} \cup \left\{\frac{7\pi}{10} + 2n\pi\right\}$ where nϵz.

Q11. $\mathbf{\sec 3\theta = \sec\theta}$

Solution:

$$\frac{1}{\cos 3\theta} = \frac{1}{\cos\theta}$$

$$\cos 3\theta = \cos\theta$$

$$\cos 3\theta - \cos\theta = 0$$

$$-2\sin\frac{3\theta+0}{2}\sin\frac{3\theta-0}{2} = 0$$

$$-2\sin 2\theta \sin\theta = 0$$

$$\sin\theta \quad \sin\theta = 0$$

Either $\sin 2\theta = 0$ or $\sin\theta = 0$

$$2\theta = \sin^{-1} 0$$

$$2\theta = 0, \pi$$

It has period of 2π.

$$2\theta = 2n\pi + \pi$$

$$\theta = n\pi + \frac{\pi}{2}$$

And $\sin\theta = 0$

$$\theta = \sin^{-1}(0)$$

$$\theta = \pi$$

It has period of 2π $\theta = \pi + 2n\pi$

Hence, $S.S = \left\{n\pi + \frac{\pi}{2}\right\} \cup \{2n\pi + \pi\}, n \in Z$

Q.12 $\mathbf{\tan 2\theta + \cot\theta = 0}$

Solution:

$$\tan 2\theta + \frac{-}{\tan\theta} = 0$$

$$\tan 2\theta \tan\theta + 1 = 0$$

$$\frac{\sin 2\theta}{\cos 2\theta}\frac{\sin\theta}{\cos\theta} + 1 = 0$$

$$\frac{\sin 2\theta \sin\theta + \cos 2\theta \cos\theta}{\cos 2\theta \cos\theta} = 0$$

$$\frac{\cos(2\theta - \theta)}{\cos 2\theta\ cos\theta} = 0$$

$$\cos\theta = 0$$

$$\theta = \cos^{-1}(0)$$

$$= \frac{\pi}{2}, 3\frac{\pi}{2}$$

It has period of 2π

$$\theta = \frac{\pi}{2} + 2n\pi, 3\frac{\pi}{2} + 2n\pi$$

Hence, $S.S = \left\{\frac{\pi}{2} + 2n\pi\right\} \cup \left\{3\frac{\pi}{2} + 2n\pi\right\}, where\ n \in Z$

Q.13 $\sin 2x + \sin x = 0$

Solution:

$$2\sin x\cos x + \sin x = 0$$

$$\sin x(2\cos + 1) = 0$$

Either, $\sin x = 0$ or $2\cos x + 1 = 0$

$$x = \sin -1(0)$$

$$x = 0, \pi$$

It has period of 2π

And $2\cos + 1 = 0$

$$\cos x = -1/2$$

$$x = \cos^{-1}\left(-\frac{1}{2}\right)$$

$x = \frac{}{6}$

$x = \pi - \pi/6$ and $\pi + \pi/6$

$= 5\pi/6$ and $7\pi/6$

It has period of 2π

$= 2n\pi \frac{5\pi}{6}; 2n\pi + 7\pi/6$

Hence, $S.S = \{2n\pi\} \cup \{\pi + 2n\} \cup \left\{2n\pi \frac{5\pi}{6}\right\} \cup \left\{2n\pi + \frac{7\pi}{6}\right\}$

Q.14 $\sin 4x - \sin 2x = \cos 3x$

Solution:

$2\cos\frac{4x+2x}{2}\sin\frac{4x-2x}{2} = \cos 3x$

$2\cos 3x \sin x = \cos 3x$

$2\cos 3x \sin x - \cos 3x = 0$

$\cos 3x\,(2\sin x - 1) = 0$

Either $\cos 3x = 0$ or $2\sin x - 1 = 0$

$3x = \cos^{-1}(0)$

$3x = \frac{\pi}{2}, 3\frac{\pi}{2}$

It has period of 2π

$3x = \frac{\pi}{2} + 2n\pi, 3\frac{\pi}{2} + 2n\pi$

$x = \frac{\pi}{6} + \frac{2}{3}2n\pi, \frac{\pi}{2} + \frac{2}{3}n\pi$

And $2\sin x - 1 = 0$

$sin\ x = \frac{1}{2}$

$x = \sin^{-1}\left(\frac{1}{2}\right)$

$x = \frac{\pi}{6}, 5\frac{\pi}{6}$

It has period of 2π

$$x = \frac{\pi}{6} + 2n\pi, 5\frac{\pi}{6} + 2n\pi$$

Hence, $S.S = \left\{\frac{\pi}{6} + \frac{2}{3}n\pi\right\} \cup \left\{\frac{\pi}{2} + \frac{2}{3}n\pi\right\} \cup \left\{\frac{\pi}{6} + 2n\pi\right\} \cup \left\{5\frac{\pi}{6} + 2n\pi\right\}, n \in Z$

Q.15 $\sin x + \cos 3x = \cos 5x$

Solution:

$$\cos 5x - \cos 3x = \sin x$$

$$-2\sin\frac{5x+3x}{2}\sin\frac{5x-3x}{2} = \sin x$$

$$-2\sin 4x \sin x = \sin x$$

$$-2\sin 4x \sin x - \sin x = 0$$

$$-\sin x(2\sin 4x + 1) = 0$$

$\sin x = 0$ or $2\sin 4x + 1 = 0$

$\sin x = 0$

$$x = \sin^{-1}(0)$$

$$= 0, \pi$$

It has period of 2π

$$= 2n\pi, \pi + 2n\pi$$

and $2\sin 4x = 1 = 0$

$$\sin 4x = -\frac{1}{2}$$

$$4x = \sin^{-1}\left(\frac{-1}{2}\right)$$

$$4x = 7\frac{\pi}{6}, 11\frac{\pi}{6}$$

It has a period of 2π

$$4x = 7\frac{\pi}{6} + 2n\pi, 11\frac{\pi}{6} + 2n\pi$$

$$x = 7\frac{\pi}{24} + \frac{n\pi}{2}, 11\frac{\pi}{24} + \frac{n\pi}{2}$$

Hence, $S.S = \{2n\pi\} \cup \{\pi + 2n\pi\} \cup \left\{7\frac{\pi}{24} + \frac{n\pi}{2}\right\} \cup \left\{11\frac{\pi}{24} + \frac{n\pi}{2}\right\}, n \in Z$

Q.16 $\sin 3x + \cos 2x = \sin x$

Solution:

$$\text{Sin } 3x - \sin x + \sin 2x = 0$$

$$2 \sin \frac{3x + x}{2} \cos \frac{3x - x}{2} + \sin 2x = 0$$

$$2 \sin 2x \cos x + \sin 2x = 0$$

$$\sin 2x(2 \cos x + 1) = 0$$

Either $\sin 2x = 0$ or $2 \cos x + 1 = 0$

$$2x = \sin^{-1}(0)$$

$$2x = 0, \pi$$

It has a period of 2π

$$2x = 0 + 2n\pi, \pi + 2n\pi$$

$$x = n\pi, \frac{\pi}{2} + n\pi$$

And $2 \cos x + 1 = 0$

$$\cos x = -\frac{1}{2}$$

$$x = \cos^{-1}\left(\frac{-1}{2}\right)$$

$$x = 4\frac{\pi}{3}, 7\frac{\pi}{3}$$

It has a period of 2π

$$x = 4\frac{\pi}{3} + 2n\pi, 7\frac{\pi}{3} + 2n\pi$$

Hence, $S.S = \{n\pi\} \cup \left\{\frac{\pi}{2} + n\pi\right\} \cup \left\{4\frac{\pi}{3} + 2n\pi\right\} \cup \left\{7\frac{\pi}{3} + 2n\pi\right\}, where\ n \in Z$

Q.17 $\sin 7x - \sin x = \sin 3x$

Solution:

$$2\cos\frac{7x+x}{2}\sin\frac{7x-x}{2} = \sin 3x$$

$$2\cos 4x \sin 3x - \sin 3x = 0$$

$$\sin 3x(2\cos 4x - 1) = 0$$

Either $\sin 3x = 0$ or $2\cos 4x - 1 = 0$

$$3x = \sin^{-1}(0)$$

$$3x = 0, \pi$$

It has period of 2π

$$3x = 2n\pi, \pi + 2n\pi$$

$$x = \frac{2}{3}n\pi, \frac{\pi}{3} + \frac{2}{3}2\pi$$

And $2\cos 4x - 1 = 0$

$$\cos 4x = \frac{1}{2}$$

$$4x = \cos^{-1}\left(\frac{1}{2}\right)$$

$$4x = \frac{\pi}{3}, 5\frac{\pi}{3}$$

It has a period of 2π

$$4x = 2n\pi + \frac{\pi}{3}, 2n\pi + 5\frac{\pi}{3}$$

$$x = \frac{n\pi}{2} + \frac{\pi}{12}, \frac{n\pi}{2} + \frac{5\pi}{12}$$

Hence, $S.S = \left\{\frac{2}{3}n\pi\right\} \cup \left\{\frac{\pi}{3} + 2n\pi\right\} \cup \left\{n\frac{\pi}{2} + \frac{\pi}{12}\right\} \cup \left\{n\frac{\pi}{2} + 5\frac{\pi}{12}\right\}, where\ n \in Z$

Q.18 $\sin x + \sin 3x + \sin 5x = 0$

Solution:

$$\sin 5x + \sin x - \sin 3x = 0$$

$$2\sin\frac{5x+x}{2}\cos\frac{5x-x}{2} + \sin 3x = 0$$

$2\sin 3x\cos 2x + \sin 3x = 0$

$\sin 3x(2\cos 2x + 1) = 0$

$\sin 3x = 0$ or $2\cos 2x + 1 = 0$

Therefore

$3x = \sin^{-1}(0)$

$= 0, \pi$

It has period of 2π

$3x = 2n\pi, \pi + 2n\pi$

$x = \frac{2n\pi}{3}, \frac{\pi}{3} + 2n\frac{\pi}{3}$

And $2\cos 2x + 1 = 0$

$\cos 2x = -\frac{1}{2}$

$2x = \cos^{-1}\left(-\frac{1}{2}\right)$

$2x = \frac{2\pi}{3}, \frac{4\pi}{3}$

It has a period of 2π

$2x = 2\frac{\pi}{3} + 2n\pi, 4\frac{\pi}{3} + 2n\pi$

$x = \frac{\pi}{3} + n\pi, \frac{\pi}{6} + n\pi$

Hence, $S.S = \left\{2n\frac{\pi}{3}\right\} \cup \left\{\frac{\pi}{3} + 2n\frac{\pi}{3}\right\} \cup \left\{\frac{\pi}{3} + n\pi\right\} \cup \left\{\frac{\pi}{6} + n\pi\right\}, where\ n \in Z$

Q.19 $\sin\theta + \sin 3\theta + \sin 5\theta + \sin 7\theta = 0$

Solution:

$\text{Sin}\ 7\theta + \sin\theta + \sin 5\theta + \sin 3\theta = 0$

$2\sin\frac{7\theta+\theta}{2}\cos\frac{7\theta-\theta}{2}$

$+2\sin\frac{5\theta+3\theta}{2}\cos\frac{5\theta-3\theta}{2} = 0$

$$2\sin 4\theta\cos 3\theta + 2\sin 4\theta\cos\theta = 0$$

$$2\sin 4\theta(\cos 3\theta + \cos\theta) = 0$$

$$2\sin 4\theta(2\cos\tfrac{3\theta+\theta}{2}\cos\tfrac{3\theta-\theta}{2}) = 0$$

$$2\sin 4\theta(2\cos 2\theta\cos\theta) = 0$$

$$4\sin 4\theta\cos 2\theta\cos\theta = 0$$

Or $\sin 4\theta\cos 2\theta\cos\theta = 0$

Either $\sin 4\theta = 0$ or $\cos 2\theta = 0$ or $\cos\theta = 0$

$$\sin 4\theta = 0$$

$$4\theta = \sin^{-1}(0)$$

$$4\theta = 0, \pi$$

It has period of 2π

$$4\theta = 2n\pi, \pi + 2n\pi$$

$$\theta = \frac{n\pi}{2}, \frac{\pi}{4} + n\frac{\pi}{2}$$

$$\cos 2\theta = 0$$

$$2\theta = \cos^{-1} 0$$

$$2\theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

It has a period of 2π

$$2\theta = \frac{\pi}{2} + 2n\pi + \frac{3\pi}{2} + 2n\pi$$

$$\cos\theta = 0$$

$$\theta = \cos^{-1}(0)$$

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

It has a period of 2π

$$2\theta = \frac{\pi}{2} + 2n\pi + \frac{3\pi}{2} + 2n\pi$$

$$\theta = \frac{\pi}{4} + n\pi, 3\frac{\pi}{4} + n\pi$$

$\cos\theta = 0$

$\theta = \cos^{-1}(0)$

$\theta = \frac{\pi}{2}.\frac{3\pi}{2}$

It has a period of 2π

$\theta = \frac{\pi}{2} + 2n\pi, 3\frac{\pi}{2} + 2n\pi$

Hence, $S.S = \left\{n\frac{\pi}{2}\right\} \cup \left\{\frac{\pi}{4} + n\frac{\pi}{2}\right\} \cup \left\{\frac{\pi}{4} + n\pi\right\} \cup \left\{3\frac{\pi}{4} + n\pi\right\} \cup \left\{\frac{\pi}{2} + 2n\pi\right\} \cup \left\{3\frac{\pi}{2} + 2n\pi\right\}$

Q.20 $\cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta = 0$

Solution:

$\cos 7\theta + \cos\theta + \cos 5\theta + \cos 3\theta = 0$

$2\cos\frac{7\theta+\theta}{2}\cos\frac{7\theta-\theta}{2} + 2\cos\frac{5\theta+3\theta}{2}\sin\frac{5\theta-3\theta}{2} = 0$

$2\cos 4\theta\cos 3\theta + 2\cos 4\theta\cos\theta = 0$

$2\cos 4\theta(\cos 3\theta + \cos\theta) = 0$

$2\cos 4\theta(2\cos\frac{3\theta+\theta}{2}\cos\frac{3\theta-\theta}{2}) = 0$

$2\cos 4\theta(2\cos 2\theta\cos\theta) = 0$

$4\cos 4\theta\cos 2\theta\cos\theta = 0$

$\cos 4\theta\cos 2\theta\cos\theta = 0$

Either $\cos 4\theta = 0$ or $\cos 2\theta = 0$ or $\cos\theta = 0$

$\cos 4\theta = 0$

$4\theta = \cos^{-1}(0)$

$= \frac{\pi}{2}, 3\frac{\pi}{2}$

It has period of 2π

$4\theta = \frac{\pi}{2} + 2n\pi, 3\frac{\pi}{2} + 2n\pi$

$\theta = \frac{\pi}{8} + \frac{n\pi}{2}, 3\frac{\pi}{8} + n\frac{\pi}{2}$

And $\cos 2\theta = 0$

$2\theta = \cos^{-1}(0)$

$2\theta = \frac{\pi}{2}, 3\frac{\pi}{2}$

It has period of 2π

$2\theta = \frac{\pi}{4} + n\pi, 3\frac{\pi}{4} + n\pi$

$\theta = \frac{\pi}{4} + n\pi, 3\frac{\pi}{4} + n\pi$

And $\cos\theta = 0$

$\theta = \cos^{-1}(0)$

$\theta = \frac{\pi}{2}, 3\frac{\pi}{2}$

It has a period of 2π

$\theta = \frac{\pi}{2} + 2n\pi, 3\frac{\pi}{2} + 2n\pi$

Hence, $S.S = \left\{\frac{\pi}{8} + n\frac{\pi}{2}\right\} \cup \left\{3\frac{\pi}{8} + n\frac{\pi}{2}\right\} \cup \left\{\frac{\pi}{4} + n\pi\right\} \cup \left\{3\frac{\pi}{4} + n\pi\right\} \cup \left\{\frac{\pi}{2} + 2n\pi\right\} \cup \left\{3\frac{\pi}{2} + 2n\pi\right\}$,

$n \in Z$