Exercise 13.2

Prove the following:

1. $\sin^{-1}\frac{5}{13}+\sin^{-1}\frac{7}{25}=\cos^{-1}\frac{253}{325}$

2. $\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{1}{5}=\tan^{-1}\frac{9}{19}$

3. $2\tan^{-2}\frac{2}{3}=\sin^{-1}\frac{12}{13}$ $\left[H\text{int}: Let, \tan^{-1}\frac{2}{3}=x.and, Show, \sin 2x=\frac{12}{13}\right]$

4. $\tan^{-1}\frac{120}{119}=2\cos^{-1}\frac{12}{13}$

5. $\sin^{-1}\frac{1}{\sqrt{5}}+\cot^{-1}3=\frac{\pi}{4}$

6. $\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{8}{17}=\sin^{-1}\frac{77}{85}$

7. $\sin^{-1}\frac{77}{85}-\sin^{-1}\frac{3}{5}=\cos^{-1}\frac{15}{17}$

8. $\cos^{-1}\frac{63}{65}+2\tan^{-1}\frac{1}{5}=\sin^{-1}\frac{3}{5}$

9. $\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{3}{5}-\tan^{-1}\frac{8}{9}=\frac{\pi}{4}$

$\left[H\text{int}: add, \tan^{-1}\frac{3}{5}+\tan^{-1}\frac{3}{5} then, proceed\right]$

10. $\sin^{-1}\frac{4}{5}+\sin^{-1}\frac{5}{13}+\sin^{-1}\frac{16}{65}=\frac{\pi}{2}$

11. $\tan^{-1}\frac{1}{11}+\tan^{-1}\frac{5}{6}=\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{2}$

12. $2\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}=\frac{\pi}{4}$

13. **Show that** $\cos(\sin-1x)=\sqrt{1-x^2}$

14. **Show that** $\sin(2\cos^{-1}x)=2x\sqrt{1-x^2}$

15. **Show that** $\cos(2\sin^{-1}x)=1-2x^2$

16. **Show that** $\tan^{-1}(-x)=-\tan^{-1}x$

17. **Show that** $\sin^{-1}(-x)=-\sin^{-1}x$

18. **Show that** $\cos-1(-x)=\pi-\cos^{-1}x$

19. **Show that** $\tan(\sin^{-1}x)=\frac{x}{\sqrt{1-x^2}}$

20. **Given that** $x=\sin^{-1}\frac{1}{2}$**, find the values of the following trigonometric functions:**

sinx, cosx, tanx, cotx, secx and cscx.

Solutions:

1. $\sin^{-1}\frac{5}{13}+\sin^{-1}\frac{7}{25}=\cos^{-1}\frac{253}{325}$

Let $\alpha=\sin^{-1}\frac{5}{13}$ & $\beta\sin^{-1}\frac{7}{25}$

$\Rightarrow \sin\alpha=\frac{5}{13}$ & $\sin\beta=\frac{7}{25}$

$$\Rightarrow \cos\beta=\sqrt{1-\left(5/13\right)^2}=\sqrt{1-\frac{25}{169}}=\sqrt{\frac{144}{169}}=\frac{12}{13}$$

$$\Rightarrow \cos\beta=\sqrt{1-\left(\frac{7}{25}\right)^2}=\sqrt{1-\frac{49}{625}}=\sqrt{\frac{576}{625}}=\frac{24}{25}$$

$$= \alpha + \beta \qquad \left[\therefore \theta = \cos^{-1}(\cos\theta)\right]$$

$$= \cos^{-1}\left[\cos(\alpha + \beta)\right]$$

$$= \cos^{-1}\left[\cos\alpha\cos\beta - \sin\alpha.\sin\beta\right]$$

$$= \cos^{-1}\left[\frac{12}{13} \times \frac{24}{25} - \frac{5}{13} \times \frac{7}{25}\right]$$

$$= \cos^{-1}\left[\frac{288 - 35}{325}\right]$$

$$= \cos^{-1}\left[\frac{253}{325}\right]$$

=R.H.S

Hence proved: $\sin^{-1}\frac{5}{13} + \sin^{-1}\frac{7}{25} = \cos^{-1}\frac{253}{325}$

2. $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{1}{5} = \tan^{-1}\frac{9}{19}$

L.H.S $= \tan^{-1}\frac{1}{4} + \tan^{-1}\frac{1}{5}$

$$= \tan^{-1}\left[\tan\left(\tan^{-1}\right)\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{1}{5}\right)\right] \qquad \left[\therefore \alpha + \beta = \tan^{-1}\left(\tan\left(\alpha + \beta\right)\right)\right]$$

$$= \tan^{-1}\left[\frac{\tan^{-1}\left(\tan\left(\frac{1}{4}\right)\right) + \tan\left(\tan^{-1}\left(\frac{1}{5}\right)\right)}{1 - \tan\left(\tan^{-1}\left(\frac{1}{4}\right)\right).\tan\left(\tan^{-1}\left(\frac{1}{5}\right)\right)}\right]$$

$$= \tan^{-1}\left[\frac{1/4 + 1/5}{1 - 1/20}\right]$$

$$= \tan^{-1}\left[\frac{\frac{5+4}{20}}{\frac{20-1}{20}}\right]$$

$$= \tan^{-1}\left[\frac{9/20}{19/20}\right]$$

$$= \tan^{-1}\left(\frac{9}{19}\right)$$

=R.H.S

Hence: $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{1}{5} = \tan^{-1}\frac{9}{19}$

3. $2\tan^{-1}\frac{2}{3} = \sin^{-1}\frac{12}{13}$

L.H.S $2\tan^{-1}\frac{2}{3}$

Let y = $\tan^{-1}\frac{2}{3}$

$$\Rightarrow \tan y = \frac{2}{3}$$

$$1 + \tan^2 y = \sec^2 y$$

$$1 + \left(2/3\right)^2 = \sec^2 y$$

$$1 + \frac{4}{9} = \sec^2 y$$

$$\sec^2 y = \frac{4}{9}$$

$$\sec^2 y = \frac{9+4}{9}$$

$$\Rightarrow \sec^2 = \frac{13}{9}$$

$$\sec y = \frac{\sqrt{13}}{3}$$

$$\Rightarrow \cos y = \frac{3}{\sqrt{13}}$$

$$\Rightarrow \sin y = \tan y.\cos y$$

$$= \frac{2}{3} \times \frac{3}{\sqrt{13}} = \frac{2}{\sqrt{13}}$$

Therefore $y = \sin^{-1}\left[\sin 2(y)\right]$

$$= \sin^{-1}\left[2\sin y.\cos y\right]$$

$$= \sin^{-1}\left[2.\frac{2}{\sqrt{13}}.\frac{4}{\sqrt{13}}\right]$$

$$= \sin^{-1}\left(\frac{12}{13}\right)$$

=R.H.S

Hence proved:

$$2\tan^{-1}\frac{2}{3} = \sin^{-1}\frac{12}{13}$$

4. $\tan^{-1}\frac{120}{119} = 2\cos^{-1}\frac{12}{13}$

R.H.S = $2\cos^{-1}\frac{12}{13}$

Let $y = \cos^{-1}\frac{12}{13}$

$$\cos y = \frac{12}{13}$$

$$\Rightarrow \sin y = \sqrt{1-\left(\frac{12}{13}\right)^2} = \sqrt{1-\frac{144}{169}}$$

$$= \sqrt{\frac{169-144}{169}} = \sqrt{\frac{25}{169}} = \frac{5}{13}$$

$$\tan y = \frac{\sin y}{\cos y} = \frac{5/13}{12/13} = \frac{5}{12}$$

Therefore $\tan^{-1}\left[\tan 2(y)\right]$

$$= \tan^{-1}\left[\frac{2\tan y}{1-\tan^2 y}\right]$$

$$= \tan^{-1}\left[\frac{2\left(5/12\right)}{1-\left(5/12\right)^2}\right]$$

$$= \tan^{-1}\left[\frac{\left(10/12\right)}{1-\left(25/144\right)}\right]$$

$$= \tan^{-1}\left[\frac{\left(10/12\right)}{\left(144-25/144\right)}\right]$$

$$= \tan^{-1}\left[\frac{10}{12}\times\frac{144}{119}\right]$$

$$= \tan^{-1}\left[\frac{10\times 12}{119}\right]$$

$= \tan^{-1}\left[\frac{120}{119}\right]$

= L.H.S

Hence; $\tan^{-1}\frac{120}{119} = 2\cos^{-1}\frac{12}{13}$

5. $\sin^{1}\frac{1}{\sqrt{5}} + \cot^{-1}3 = \frac{\pi}{4}$

L.H.S = $\sin^{1}\frac{1}{\sqrt{5}} + \cot^{-1}3$

Let $\sin^{1}\frac{1}{\sqrt{5}} = \alpha$; $\cot^{-1}3 = \beta$

$\frac{1}{\sqrt{5}} = \sin\alpha$; $\cot\beta = 3$

$\cos\alpha = \sqrt{1-\sin^2\alpha}$; $1+\cot^2\beta = \cos ec^2\beta$

$= \sqrt{1-\left(\frac{1}{\sqrt{5}}\right)^2}$; $1+(3)^2 = \cos ec^2\beta$

$= \sqrt{1-\frac{1}{5}}$; $1+9 = \cos ec^2\beta$

$= \sqrt{\frac{5-1}{5}} = \sqrt{\frac{4}{5}}$; $10 = \cos ec^2\beta$

$\cos\alpha = \frac{2}{\sqrt{5}}$; $\cos ec\beta = \sqrt{10}$

; $\sin\beta = \frac{1}{\sqrt{10}}$

; $\cos\beta = \cot\beta.\sin\beta$

; $= 3.\frac{1}{\sqrt{10}}$

; $\cos\beta = \frac{3}{\sqrt{10}}$

$\Rightarrow \alpha + \beta = \cos^{-1}\left[\cos(\alpha+\beta)\right]$

$= \cos^{-1}\left[\cos\alpha.\cos\beta - \sin\alpha.\sin\beta\right]$

$= \cos^{-1}\left[\frac{2}{\sqrt{5}}.\frac{3}{\sqrt{10}} - \frac{1}{\sqrt{5}}.\frac{1}{\sqrt{10}}\right]$

$= \cos^{-1}\left[\frac{6}{\sqrt{50}} - \frac{1}{\sqrt{50}}\right]$

$= \cos^{-1}\left[\frac{5}{\sqrt{50}}\right]$

$= \cos^{-1}\left[\frac{5}{5\sqrt{2}}\right]$

$= \cos^{-1}\left[\frac{1}{\sqrt{2}}\right] = \frac{\pi}{4}$

= R.H.S

Hence proved;

$\sin^{1}\frac{1}{\sqrt{5}} + \cot^{-1}3 = \frac{\pi}{4}$

6. $\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{8}{17} = \sin^{-1}\frac{77}{85}$

L.H.S $\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{8}{17}$

Let $\alpha = \sin^{-1}\frac{3}{5}$; $\beta = \sin^{-1}\frac{8}{17}$

$\sin\alpha = \sqrt{1-\sin^2\alpha}$; $\cos\beta = \sqrt{1-\sin^2\beta}$

$= \sqrt{1 - 9/25}$; $= \sqrt{1-\left(8/17\right)^2}$

$= \sqrt{\frac{25-9}{25}}$; $= \sqrt{1-\frac{64}{289}}$

$= \sqrt{\frac{16}{25}}$; $= \sqrt{\frac{289-64}{289}}$

$= \frac{4}{5}$; $= \sqrt{\frac{225}{289}}$

$\cos\alpha = \frac{4}{5}$; $\cos\beta = \frac{3}{\sqrt{10}}$

; $\cos\beta = \frac{15}{17}$

Therefore = $\alpha + \beta$

$$\alpha + \beta = \sin^{-1}\left(\sin\left(\alpha+\beta\right)\right)$$

$$= \sin^{-1}\left[\sin\alpha.\cos\beta + \cos.\alpha\sin\beta\right]$$

$$= \sin^{-1}\left[3/5 \times 15/17 + 4/5 \times 8/17\right]$$

$$= \sin^{-1}\left[\frac{45}{85} + \frac{32}{85}\right]$$

$$= \sin^{-1}\left[\frac{45+32}{85}\right]$$

$$= \sin^{-1}\left[\frac{77}{85}\right]$$

$= \text{R.H.S}$

Hence proved;

$$\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{8}{17}=\sin^{-1}\frac{77}{85}$$

7. $\sin^{-1}\frac{77}{85}-\sin^{-1}\frac{3}{5}=\cos^{-1}\frac{15}{17}$

L.H.S $= \sin^{-1}\frac{77}{85}-\sin^{-1}\frac{3}{5}$

Let $\alpha=\sin^{-1}\frac{77}{85}$; $\beta=\sin^{-1}\frac{3}{5}$

$\sin\alpha=\frac{77}{85}$; $\sin\beta=\frac{3}{5}$

$\cos\alpha=\sqrt{1-\sin^2\alpha}$; $\cos\beta=\sqrt{1-\sin^2\beta}$

$=\sqrt{1-\left(\frac{77}{85}\right)^2}$; $=\sqrt{1-\left(\frac{3}{5}\right)^2}$

$=\sqrt{\frac{(85)^2-(77)^2}{(85)^2}}$; $=\sqrt{1-9/25}$

$\cos\alpha=\frac{36}{85}$; $=\sqrt{\frac{25-9}{25}}=\frac{16}{25}$

; $\cos\beta=\frac{4}{5}$

Therefore, $=\alpha-\beta$

$$=\cos^{-1}\left[\cos(\alpha-\beta)\right]$$

$$=\cos^{-1}\left[\cos\alpha.\cos\beta+\sin\alpha.\sin\beta\right]$$

$$= \cos^{-1}\left[\frac{36}{85}.\frac{4}{5}+\frac{77}{85}.\frac{3}{5}\right]$$

$$= \cos^{-1}\left[\frac{144}{455}+\frac{231}{455}\right]$$

$$= \cos^{-1}\left[\frac{375}{455}\right]$$

$$= \cos^{-1}\left[\frac{15}{17}\right]$$

= R.H.S

Hence proved;

$$\sin^{-1}\frac{77}{85}-\sin^{-1}\frac{3}{5}=\cos^{-1}\frac{15}{17}$$

8. $\cos^{-1}\frac{63}{65}+2\tan^{-1}\frac{1}{5}=\sin^{-1}\frac{3}{5}$

L.H.S = $\cos^{-1}\frac{63}{65}+2\tan^{-1}\frac{1}{5}$

Let $\alpha=\cos^{-1}\frac{63}{65}$; $\beta=\tan^{-1}\frac{1}{5}$

$\cos\alpha=\frac{63}{65}$; $\tan\beta=\frac{1}{5}$

$\sin\alpha=\sqrt{1-\cos^2\alpha}$; $1+\tan^2\beta=\sec^2\beta$

$=\sqrt{1-\left(\frac{63}{65}\right)^2}$; $1+\frac{1}{25}=\sec^2\beta$

$=\sqrt{1-\frac{3969}{4225}}$; $\frac{26}{25}=\sec^2\beta$

$= \sqrt{\frac{4225-3969}{4225}}$; $\sec^2 \beta = \frac{\sqrt{26}}{5}$

$= \frac{256}{4225}$; $\Rightarrow \cos\beta = \frac{5}{\sqrt{26}}$

$\sin\alpha = \frac{16}{65}$; $\sin\beta = \tan\beta \times \cos\beta$

; $= \frac{1}{5} \times \frac{5}{\sqrt{26}}$

; $\sin\beta = \frac{1}{\sqrt{26}}$

Therefore $\alpha + 2\beta$

$$\alpha + 2\beta = \sin^{-1}\left[\sin\left(\alpha + 2\beta\right)\right]$$

$$= \sin^{-1}\left[\sin\alpha.\cos 2\beta + \sin 2\beta.\cos\alpha\right]$$

$$= \sin^{-1}\left[\frac{16}{65} \times \left[\cos^2\beta - \sin^2\beta\right] + (2\sin\beta\cos\beta)\frac{63}{65}\right]$$

$$= \sin^{-1}\left[\frac{16}{65} \times \left[\frac{5}{26} - \frac{1}{26}\right] + \left[2.\frac{5}{\sqrt{26}}.\frac{1}{\sqrt{26}}\right]\frac{63}{65}\right]$$

$$= \sin^{-1}\left[\frac{16}{65} \times \frac{4}{26} + \frac{10}{26} \times \frac{63}{65}\right]$$

$$= \sin^{-1}\left[\frac{32}{845} + \frac{315}{845}\right]$$

$$= \sin^{-1}\left[\frac{32+315}{845}\right]$$

$$= \sin^{-1}\left[\frac{347}{845}\right]$$

$$= \sin^{-1}\left[\frac{77}{85}\right]$$

= R.H.S

Hence proved

$$\cos^{-1}\frac{63}{65} + 2\tan^{-1}\frac{1}{5} = \sin^{-1}\frac{3}{5}$$

9. $\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{3}{5} - \tan^{-1}\frac{8}{9} = \frac{\pi}{4}$

L.H.S $= \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{3}{5} - \tan^{-1}\frac{8}{9}$

$$= \tan^{-1}\left[\frac{3/4 + 3/5}{1 - 3/4 \cdot 3/5}\right] - \tan^{-1}\frac{8}{9}$$

$$\left[\therefore \tan^{-1}\left[\tan\left(\tan^{-1}\alpha + \tan^{-1}\beta\right)\right] = \tan\left[\frac{\alpha+\beta}{1-\alpha\beta}\right]\right]$$

$$= \tan^{-1}\left[\frac{27/20}{11/20}\right] - \tan^{-1}\frac{8}{19}$$

$$= \tan^{-1}\left[\frac{27}{11}\right] - \tan^{-1}\frac{8}{19}$$

$$= \tan^{-1}\left[\frac{27/11 - 8/19}{1 - 27/11 \cdot 8/19}\right]$$

$$= \tan^{-1}\left[\frac{\frac{513-88}{209}}{\frac{209+216}{209}}\right]$$

$$= \tan^{-1}\left[\frac{425}{425}\right]$$

$$= \tan^{-1}(1)$$

$$= \pi/4$$

= R.H.S

10. $\sin^{-1}\frac{4}{5} + \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{16}{65} = \frac{\pi}{2}$

L.H.S $= \sin^{-1}\frac{4}{5} + \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{16}{65}$

$$= \sin^{-1}\left[\frac{4}{5}\sqrt{1-\left(\frac{5}{13}\right)^2} + \frac{5}{13}\sqrt{1-\left(\frac{4}{5}\right)^2}\right] + \sin^{-1}\frac{16}{65}$$

$$= \sin^{-1}\left[\frac{4}{5}\sqrt{\frac{169-25}{169}} + \frac{5}{13}\sqrt{\frac{25-16}{25}}\right] + \sin^{-1}\frac{16}{65}$$

$$= \sin^{-1}\left[\frac{4}{5}\times\frac{12}{13} + \frac{5}{13}\times\frac{3}{5}\right] + \sin^{-1}\frac{16}{65}$$

$$= \sin^{-1}\left[\frac{48}{65} + \frac{15}{65}\right] + \sin^{-1}\frac{16}{65}$$

$$= \sin^{-1}\left[\frac{63}{65}\right] + \sin^{-1}\frac{16}{65}$$

$$= \sin^{-1}\left[\frac{63}{65}\sqrt{1-\left(\frac{16}{65}\right)^2} + \frac{16}{65}\sqrt{1-\left(\frac{63}{65}\right)^2}\right]$$

$$= \sin^{-1}\left[\frac{63}{65}\sqrt{1-\frac{256}{4225}} + \frac{16}{65}\sqrt{\frac{4225-3969}{4225}}\right]$$

$$= \sin^{-1}\left[\frac{63}{65}\sqrt{\frac{4225-256}{4225}} + \frac{16}{65}\sqrt{\frac{4225-3969}{4225}}\right]$$

$$= \sin^{-1}\left[\frac{63}{65}\sqrt{\frac{4169}{4225}} + \frac{16}{65}\sqrt{\frac{256}{4225}}\right]$$

$$= \sin^{-1}\left[\frac{63}{65}\times\frac{63}{65} + \frac{16}{65}\times\frac{16}{65}\right]$$

$$= \sin^{-1}\left[\frac{4169}{4225} + \frac{256}{4225}\right]$$

$$= \sin^{-1}\left[\frac{4169+256}{4225}\right]$$

$$= \sin^{-1}\left[\frac{4225}{4225}\right]$$

$$= \sin^{-1}(1)$$

$$= \frac{\pi}{2}$$

= R.H.S

Hence proved; $\sin^{-1}\frac{4}{5} + \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{16}{65} = \frac{\pi}{2}$

11. $\tan^{1}\frac{1}{11} + \tan^{-1}\frac{5}{6} = \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2}$

L.H.S $\tan^{1}\frac{1}{11} + \tan^{-1}\frac{5}{6}$

$$= \tan^{-1}\left[\frac{\frac{1}{11}+\frac{5}{6}}{1-\frac{1}{11}.\frac{5}{6}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{6+55}{66}}{1-\frac{5}{66}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{61}{66}}{\frac{61}{66}}\right]$$

$$= \tan^{-1}(1)$$

$$= \frac{\pi}{4}$$

R.H.S $= \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2}$

$$= \tan^{-1}\left[\frac{\frac{1}{3}+\frac{1}{2}}{1-\frac{1}{3}.\frac{1}{2}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{2+3}{6}}{\frac{6-1}{6}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{5}{6}}{\frac{5}{6}}\right]$$

$$= \tan^{-1}[1]$$

$$= \frac{\pi}{4}$$

= L.H.S

Hence proved;

$$\tan^{-1}\frac{1}{11}+\tan^{-1}\frac{5}{6}=\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{2}$$

12. $2\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}=\frac{\pi}{4}$

L.H.S $= 2\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}$

$$=\tan^{-1}\left[\frac{2\left(\frac{1}{3}\right)}{1-\left(\frac{1}{3}\right)^2}\right]+\tan^{-1}\frac{1}{7}$$

$$=\tan^{-1}\left[\frac{\frac{2}{3}}{1-\frac{1}{9}}\right]+\tan^{-1}\frac{1}{7}$$

$$=\tan^{-1}\left[\frac{\frac{2}{3}}{\frac{9-1}{9}}\right]+\tan^{-1}\frac{1}{7}$$

$$=\tan^{-1}\left[\frac{\frac{2}{3}}{\frac{8}{9}}\right]+\tan^{-1}\frac{1}{7}$$

$$=\tan^{-1}\left[\frac{3}{4}\right]+\tan^{-1}\frac{1}{7}$$

$$=\tan^{-1}\left[\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\cdot\frac{1}{7}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{21+4}{28}}{\frac{28-5}{28}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{25}{28}}{\frac{25}{28}}\right]$$

$$= \tan^{-1}(1)$$

$$= \frac{\pi}{4}$$

= R.H.S

Hence proved

$$2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} = \frac{\pi}{4}$$

13. Show that $\cos(\sin^{-1}(x)) = \sqrt{1-x^2}$

L.H.S $= \cos(\sin^{-1}(x))$

Let y = $\sin^{-1}(x)$

$\Rightarrow \sin y = x$

$\cos y = \sqrt{1-\sin^2 y}$

$= \sqrt{1-x^2}$

$y = \cos^{-1}\left[\sqrt{1-x^2}\right]$

Therefore

$$\cos y = \cos\left[\cos^{-1}\left[\sqrt{1-x^2}\right]\right]$$

$$= \sqrt{1-x^2} \text{ = R.H.S}$$

Hence proved

$$\cos(\sin^{-1}(x)) = \sqrt{1-x^2}$$

14. Show that $\sin(2\cos^{-1} x) = 2x\sqrt{1-x^2}$

L.H.S $= \sin(2\cos^{-1} x)$

Let $\cos^{-1} x = y$

$$\Rightarrow \cos y = x$$

$$\sin y = \sqrt{1-\cos^2 y} = \sqrt{1-x^2}$$

$$\Rightarrow \sin(2y) = 2\sin y.\cos y$$

$$= 2x.\sqrt{1-x^2}$$

$$= 2x\sqrt{1-x^2} = R.H.S$$

Hence proved

$$\sin(2\cos^{-1} x) = 2x\sqrt{1-x^2}$$

15. Show that $\cos(2\sin^{-1} x) = 1-2x^2$

L.H.S $= \cos(2\sin^{-1} x)$

Let $y = \sin^{-1} x$

$$\Rightarrow \sin y = x$$

$$\cos y = \sqrt{1-\sin^2 y}$$

$$= \sqrt{1-x^2}$$

$$\cos(2y) = 1-2\sin^2 y$$

$$= 1-2x^2$$

= R.H.S

Hence proved

$$\cos(2\sin^{-1} x) = 1-2x^2$$

16. Show that $\tan^{-1}(-x) = -\tan x$

L.H.S $= \tan^{-1}(-x)$

$$= \tan^{-1}(0-x)$$

$$= \frac{\tan(0)-\tan(x)}{1+\tan(0)(\tan x)}$$

$$= \frac{0-\tan(x)}{1+0}$$

$$= -\tan(x)$$

= R.H.S

Hence proved

$$\tan^{-1}(-x) = -\tan x$$

17. Show that $\sin^{-1}(-x) = -\sin^{-1}(x)$

L.H.S $= \sin^{-1}(-x)$

Let $\sin^{-1}(-x) = y$

$\Rightarrow \sin y = -x$

$\Rightarrow \sin(-y) = x$

$-\sin y = x$

$\sin y = -x$

$y = -\sin^{-1}(x)$

$\sin^{-1}(-x) = -\sin^{-1}(x)$

$=$ R.H.S

Hence proved $\sin^{-1}(-x) = -\sin^{-1}(x)$

18. Show that $\cos^{-1}(-x) = \pi - \cos^{-1} x$

$\cos^{-1}(-x) = \pi - \cos^{-1} x$

$-x = \cos(\pi - \cos^{-1} x)$

$= \cos\pi.\cos.\cos^{-1} x + \sin\pi.\sin\cos^{-1} x$

$= (-1)(x) + (0).\sin(\cos^{-1} x)$

$-x = -x$

Hence proved

$\cos^{-1}(-x) = \pi - \cos^{-1} x$

19. Show that $\tan(\sin^{-1} x) = \dfrac{x}{\sqrt{1-x^2}}$

L.H.S $= \tan(\sin^{-1} x)$

Let $y = \sin^{-1} x$

$\Rightarrow \sin y = x$

And $\cos y = \sqrt{1 - x^2}$

$$\tan y = \frac{\sin y}{\cos y} = \frac{x}{\sqrt{1-x^2}}$$

$$y = \tan^{-1} \frac{x}{\sqrt{1-x^2}}$$

$$\tan(\sin^{-1} x) = \tan y$$

$$= \tan\left[\tan^{-1} \frac{x}{\sqrt{1-x^2}}\right]$$

$$= \frac{x}{\sqrt{1-x^2}}$$

Hence proved

$$\tan(\sin^{-1} x) = \frac{x}{\sqrt{1-x^2}}$$

20. Given that $x = \sin^{-1} \frac{1}{2}$, find the values of following trigonometric functions

1. sin x; 2. cos x; 3. tan x; 4. cot x; 5. Sec x;

6. cosec x

Solutions

$$x = \sin^{-1} \frac{1}{2}$$

$$\Rightarrow \sin x = \frac{1}{2}$$

Hence, $\sin x = \frac{1}{2}$

$$\cos ecx = \frac{1}{\sin x} = \frac{1}{\frac{1}{2}} = 2$$

Hence, $\cos ecx = 2$

$$\cos x = \sqrt{1-\sin^2 x}$$

$$= \sqrt{1-\left(\frac{1}{2}\right)^2}$$

$$= \sqrt{1-\frac{1}{4}} = \sqrt{\frac{4-1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

$$\cos x = \frac{\sqrt{3}}{2}$$

Hence $\cos x = \frac{\sqrt{3}}{2}$

$$\sec x = \frac{1}{\cos x} = \frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

Hence $\sec x = \frac{2}{\sqrt{3}}$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

Hence $\tan x = \frac{1}{\sqrt{3}}$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\frac{1}{\sqrt{3}}} = \sqrt{3}$$

Hence $\cot x = \sqrt{3}$

So,

$$\sin x = \frac{1}{2} \qquad \operatorname{cosec} x = 2$$

$$\cos x = \frac{\sqrt{3}}{2} \qquad \sec x = \frac{2}{\sqrt{3}}$$

$$\tan x = \frac{1}{\sqrt{3}} \qquad \cot x = \sqrt{3}$$