Chapter 13

Inverse Trigonometric Functions

Exercise 13.1

Q1. Evaluate without using tables/calculator:

i. $\sin^{-1}(1)$ **ii.** $\sin^{-1}(-1)$ **iii.** $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

iv. $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ **v.** $\cos^{-1}\left(\frac{1}{2}\right)$ **vi.** $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

vii. $\cot^{-1}(-1)$ **viii.** $\operatorname{cosec}^{-1}\left(\frac{-2}{\sqrt{3}}\right)$ **ix.** $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

Solutions:

i. $\sin^{-1}(1)$

Let $y = \sin^{-1}(1)$

$\Rightarrow \sin y = 1 \qquad y \in \left[0, \frac{\pi}{2}\right]$

We know that

$\sin\left[\frac{\pi}{2}\right] = 1$

Thus, $y = \frac{\pi}{2}$

ii. $\sin^{-1}(-1)$

Let $y = \sin^{-1}(-1)$

$$\Rightarrow \sin y = -1 \qquad y \in \left[\frac{\pi}{2}, \frac{\pi}{2}\right]$$

We know that

$$\sin\left[-\frac{\pi}{2}\right] = -1$$

Thus, y = $-\frac{\pi}{2}$

iii. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Let $y = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$$\Rightarrow \cos y = \frac{\sqrt{3}}{2} \qquad y \in \left[0, \frac{\pi}{2}\right]$$

We know that

$$\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

Thus, y = $\frac{\pi}{6}$

iv. $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

Let $y = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

$$\Rightarrow \tan y = \frac{-1}{\sqrt{3}} \qquad y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

We know that

$$\tan\left[-\frac{\pi}{6}\right] = -\frac{1}{\sqrt{3}}$$

Thus , y = $-\frac{\pi}{6}$

v. $\cos^{-1}\left(\frac{1}{2}\right)$

Let y = $\cos^{-1}\left(\frac{1}{2}\right)$

$$\Rightarrow \cos y = \frac{1}{2} \qquad y \in [0, \pi]$$

We know that

$$\cos\left[\frac{\pi}{3}\right] = \frac{1}{2}$$

Thus, y = $\frac{\pi}{3}$

vi. $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Let y = $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$$\Rightarrow \tan y = \left(\frac{1}{\sqrt{3}}\right) \qquad y \in [0, \pi]$$

$$\tan\left[\frac{\pi}{6}\right] = \frac{1}{\sqrt{3}}$$

Thus , y = $\frac{\pi}{6}$

vii. $\cot^{-1}(-1)$

Let y = $\cot^{-1}(-1)$

$\Rightarrow \cot y = (-1) \qquad y \in [0, \pi]$

We know that

$$\cot\left[\frac{3\pi}{4}\right] = (-1)$$

Thus $y = \frac{3\pi}{4}$

viii. $\operatorname{cosec}^{-1}\left(\frac{-2}{\sqrt{3}}\right)$

Let y = $\operatorname{cosec}^{-1}\left(\frac{-2}{\sqrt{3}}\right)$

$\Rightarrow \operatorname{cosec} y = \frac{-2}{\sqrt{3}} \qquad y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

We know that

$$\operatorname{cosec}\left[\frac{-\pi}{3}\right] = \frac{-2}{\sqrt{3}}$$

Thus, y = $\frac{-\pi}{3}$

ix. $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

Let $y = \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

$\Rightarrow \sin y = -\frac{1}{\sqrt{2}} \qquad y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

We know that

$\sin\left[-\frac{\pi}{4}\right] = -\frac{1}{\sqrt{2}}$

Thus, $y = -\frac{\pi}{4}$

Q2. Without using table/calculator show that:

i. $\tan^{-1}\frac{5}{12} = \sin^{-1}\frac{5}{13}$ **ii.** $2\cos^{-1}\frac{4}{5} = \sin^{-1}\frac{24}{25}$ **iii.** $\cos^{-1}\frac{4}{5} = \cot^{-1}\frac{4}{3}$

Solutions:

i. $\tan^{-1}\frac{5}{12} = \sin^{-1}\frac{5}{13}$

L.H.S = $\tan^{-1}\frac{5}{12}$

Let $y = \tan^{-1}\frac{5}{12}$

$\Rightarrow \tan y = \frac{5}{12}$

$1 + \tan^2 y = \sec^2 y$

$1 + \left(\frac{5}{12}\right)^2 = \sec^2 y$

$\sec^2 y = 1 + \frac{25}{144} = \frac{144+25}{144}$

$\sec^2 y = \frac{169}{144}$

$\Rightarrow \sec y = \pm \frac{13}{12}$

$\cos y = \frac{13}{12}$ (only positive term)

$\Rightarrow \frac{\sin y}{\cos y} = \tan y$

$\Rightarrow \sin y = \tan y . \cos y$

$= \frac{5}{12} \times \frac{12}{13} = \frac{5}{13}$

$\Rightarrow y = \sin^{-1}\left(\frac{5}{13}\right)$

= R.H.S

Hence proved

$\tan^{-1}\frac{5}{12} = \sin^{-1}\frac{5}{13}$

ii. $2\cos^{-1}\frac{4}{5} = \sin^{-1}\frac{24}{25}$

L.H.S = $2 \cos^{-1}\left(\frac{4}{5}\right)$

Let $y = \cos^{-1}\left(\frac{4}{5}\right)$

$\frac{y}{2} = \cos^{-1}\left(\frac{4}{5}\right)$

$$\cos\left(\frac{y}{2}\right)=\frac{4}{5}$$

$$\Rightarrow \sin^2\frac{y}{2}=1-\cos^2\frac{y}{2}$$

$$=1-\frac{16}{25}=\frac{25-16}{25}=\frac{9}{25}$$

$$\sin\frac{y}{2}=\sqrt{\frac{9}{25}}$$

$$\Rightarrow \sin\frac{y}{2}=\pm\frac{3}{5}$$

We know that

$$\sin y=2\sin\frac{y}{2}.\cos\frac{y}{2}$$

$$=2\times\frac{3}{5}\times\frac{4}{5}$$

$$=\frac{24}{25}$$

$$\Rightarrow y=\sin^{-1}\left(\frac{24}{25}\right)$$

= R.H.S

Hence proved,

$$2\cos^{-1}\frac{4}{5}=\sin^{-1}\frac{24}{25}$$

iii. $\cos^{-1}\frac{4}{5}=\cot^{-1}\frac{4}{3}$

L.H.S = $\cos^{-1}\left(\frac{4}{5}\right)$

Let y = $\cos^{-1}\left(\frac{4}{5}\right)$

$$\Rightarrow \cos y = \left(\frac{4}{5}\right)$$

We know that

$$\sin y = \sqrt{1-\cos^2 y}$$

$$= \sqrt{1-\left(\frac{4}{5}\right)^2}$$

$$= \sqrt{1-\frac{16}{25}} = \sqrt{\frac{25-16}{25}} = \sqrt{\frac{9}{25}}$$

$$\sin y = \frac{3}{5}$$

$$\Rightarrow \frac{\cos y}{\sin y} = \frac{4/5}{3/5}$$

$$\cot y = \frac{4}{3}$$

$$\Rightarrow y = \cot^{-1}\left(\frac{4}{3}\right)$$

= R.H.S

Hence proved,

$$\cos^{-1}\frac{4}{5} = \cot^{-1}\frac{4}{3}$$

Q3. Find the value of each expression:

i. $\cos\left[\sin^{-1}\frac{1}{\sqrt{2}}\right]$ **ii.** $\sec\left[\cos^{-1}\frac{1}{2}\right]$ **iii.** $\tan\left[\cos^{-1}\frac{\sqrt{3}}{2}\right]$

iv. $\sec\left[\tan^{-1}(-1)\right]$ **v.** $\sec\left[\sin^{-1}(-\frac{1}{2})\right]$ **vi.** $\tan\left[\tan^{-1}(-1)\right]$

vii. $\sin\left[\sin^{-1}(\frac{1}{2})\right]$ **viii.** $\tan\left[\sin^{-1}(-\frac{1}{2})\right]$ **ix.** $\sin\left[\tan^{-1}(-1)\right]$

Solutions:

i. $\cos\left[\sin^{-1}\frac{1}{\sqrt{2}}\right]$

$\Rightarrow \cos\left[\frac{\pi}{4}\right]$ $\left[\therefore \sin^{-1}\frac{1}{\sqrt{2}} = \frac{\pi}{4}\right]$

$\Rightarrow \frac{1}{\sqrt{2}}$

Hence; $\cos\left[\sin^{-1}\frac{1}{\sqrt{2}}\right] = \frac{1}{\sqrt{2}}$

ii. $\sec\left[\cos^{-1}\frac{1}{2}\right]$

$\Rightarrow \frac{1}{\cos\left[\cos^{-1}\frac{1}{2}\right]}$ $\left[\therefore \cos(\cos^{-1}\theta) = \theta\right]$

$\Rightarrow \frac{1}{1/2}$

$\Rightarrow 2$

Hence; $\sec\left[\cos^{-1}\frac{1}{2}\right] = 2$

iii. $\tan\left[\cos^{-1}\frac{\sqrt{3}}{2}\right]$

$\Rightarrow \tan\frac{\pi}{6}$ $\left[\therefore \cos^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{6}\right]$

$\Rightarrow \frac{1}{\sqrt{3}}$

Hence; $\tan\left[\cos^{-1}\frac{\sqrt{3}}{2}\right] = \frac{1}{\sqrt{3}} = \frac{\pi}{6}$

iv. $\sec\left[\tan^{-1}(-1)\right]$

$\Rightarrow \cos ec\left(\frac{3\pi}{4}\right)$ $\left[\therefore \tan^{-1}(-1) = \frac{3\pi}{4}\right]$

$\Rightarrow \frac{1}{\sin\left(\frac{3\pi}{4}\right)}$

$\Rightarrow \frac{1}{1/\sqrt{2}} = \sqrt{2}$

Hence; $\cos ec\left[\tan^{-1}(-1)\right] = -\sqrt{2}$

v. $\sec\left[\sin^{-1}(-\frac{1}{2})\right]$

$= \sec\left[-\left(\sin^{-1}\left\langle\frac{1}{2}\right\rangle\right)\right]$

$= \sec\left[-\left(\frac{\pi}{6}\right)\right]$ $\left[\therefore \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}\right]$

$$= \sec\frac{\pi}{6}$$

$$= \frac{1}{\cos\left(\frac{\pi}{6}\right)}$$

$$= \frac{1}{\sqrt{3}/2}$$

$$= \frac{2}{\sqrt{3}}$$

Hence; $\sec\left[\sin^{-1}\left(-\frac{1}{2}\right)\right] = \frac{2}{\sqrt{3}}$

vi. $\tan\left[\tan^{-1}(-1)\right]$

= (-1) $\left[\therefore \tan(\tan^{-1}\theta) = \theta\right]$

Hence $\tan\left[(\tan^{-1}(-1)\right] = -1$

vii. $\sin\left[\sin^{-1}(\frac{1}{2})\right]$

$= \frac{1}{2}$ $\left[\therefore \sin(\sin^{-1}\theta) = \theta\right]$

Hence $\sin\left[(\sin^{-1}\left(\frac{1}{2}\right))\right] = \frac{1}{2}$

viii. $\tan\left[\sin^{-1}(-\frac{1}{2})\right]$

$$= \tan\left[-\left(\sin^{-1}\left(\frac{1}{2}\right)\right)\right]$$

$$= -\tan\left[\sin^{-1}\left(\frac{1}{2}\right)\right] \qquad \left[\therefore \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}\right]$$

$$= -\tan\left(\frac{\pi}{6}\right)$$

$$= -\frac{1}{\sqrt{3}}$$

Hence; $\tan\left[\sin^{-1}\left(-\frac{1}{2}\right)\right] = -\frac{1}{\sqrt{3}}$

ix. $\sin\left[\tan^{-1}(-1)\right]$

$$= \sin\left[\tan^{-1}(1)\right]$$

$$= -\sin\left(\frac{\pi}{4}\right) \qquad \left[\therefore \tan^{-1} 1 = \frac{\pi}{4}\right]$$

$$= -\frac{1}{\sqrt{2}}$$

Hence; $\sin\left[\tan^{-1}(-1)\right] = -\frac{1}{\sqrt{2}}$