

Exercise 12.7

Q1. Find the area of the triangle ABC, given two sides and their included angle:

i. $a = 200, b = 120, \gamma = 150^\circ$ ii. $b = 37, c = 45, \alpha = 30^\circ 50'$

iii. $a = 4.33, b = 9.25, \gamma = 56^\circ 44'$

i. $a = 200, b = 120, \gamma = 150^\circ$

Solution

$$a = 200, b = 120, \gamma = 150^\circ$$

$$\Delta = \frac{1}{2} a \cdot b \cdot \sin \gamma$$

$$= \frac{1}{2} 200 \times 120 \times \sin 150^\circ$$

$$= 12000 \times \frac{1}{2} = 6000 \text{ sq. units}$$

Hence area of triangle = 6000 sq. units

ii. $b = 37, c = 45, \alpha = 30^\circ 50'$

Solution

$$\Delta = \frac{1}{2} a \cdot b \cdot \sin \alpha$$

$$= \frac{1}{2} \times 37 \times 45 \times \sin(30^\circ 50')$$

$$= 832.5 \times 0.5126 = 426.69 \text{ sq. units}$$

Hence area of triangle = 426.69 sq. units

iii. $a = 4.33, b = 9.25, \gamma = 56^\circ 44'$

Solution

$$a = 4.33, b = 9.25, \gamma = 56^\circ 44'$$

$$\Delta = \frac{1}{2} a \cdot b \cdot \sin \gamma$$

$$= \frac{1}{2} \times 4.33 \times 9.25 \times \sin (56^\circ 44')$$

$$= 20.026 \times 0.8361 = 16.74 \text{ sq. units}$$

Hence area of triangle = 16.74 sq. units

Q2. Find the area of the triangle ABC, given two sides and their included angle:

i. $b = 25.4, \gamma = 36^\circ 41', \alpha = 45^\circ 17'$

ii. $c = 32, \alpha = 47^\circ 41', \alpha = 70^\circ 16'$

iii. $a = 4.8, \alpha = 83^\circ 42', \gamma = 37^\circ 12'$

i. $b = 25.4, \gamma = 36^\circ 41', \alpha = 45^\circ 17'$

Solution

We know that

$$\alpha + \beta + \gamma = 180^\circ$$

$$\beta = 180^\circ - \alpha - \gamma$$

$$= 180^\circ - (45^\circ 17') - (36^\circ 41') = 98^\circ 2'$$

$$\Delta = \frac{1}{2} b^2 \cdot \frac{\sin \alpha \cdot \sin \gamma}{\sin \beta}$$

$$\begin{aligned}
 &= \frac{1}{2} (25.4)^2 \cdot \frac{\sin(45^\circ 17') \sin(36^\circ 41')}{\sin(98^\circ 2')} \\
 &= \frac{322.58(0.60)(0.71)}{0.99} = 138.29 \text{ sq. units}
 \end{aligned}$$

$$\Delta = 138.29 \text{ sq. units}$$

Hence area of triangle = 138.29 sq. units

ii. $c = 32, \alpha = 47^\circ 41', \beta = 70^\circ 16'$

We know that

$$\alpha + \beta + \gamma = 180^\circ$$

$$\gamma = 180^\circ - \alpha - \beta$$

$$= 180^\circ - (47^\circ 25') - (70^\circ 16') = 60^\circ 20'$$

$$\Delta = \frac{1}{2} c^2 \cdot \frac{\sin \alpha \sin \beta}{\sin \gamma}$$

$$= \frac{1}{2} (32)^2 \cdot \frac{\sin(47^\circ 25') \sin(70^\circ 16')}{\sin(60^\circ 20')}$$

$$= \frac{512(0.7361)(0.9443)}{0.8857} = 400.54 \text{ sq. units}$$

$$\Delta = 400.54 \text{ sq. units}$$

Hence area of triangle = 400.54 sq. units

iii. $a = 4.8, \alpha = 83^\circ 42', \gamma = 37^\circ 12'$

We know that

$$\alpha + \beta + \gamma = 180^\circ$$

$$\beta = 180^\circ - \alpha - \gamma$$

$$= 180^\circ - (83^\circ 42') - (37^\circ 12') = 59^\circ 6'$$

$$\begin{aligned}\Delta &= \frac{1}{2} c^2 \frac{\sin \beta \sin \gamma}{\sin \alpha} \\ &= \frac{1}{2} (4.8)^2 \frac{\sin(59^\circ 6') \sin(37^\circ 12')}{\sin(83^\circ 42')} \\ &= \frac{11.52(0.8581)(0.6046)}{0.99396} = 6.01 \text{ sq. units}\end{aligned}$$

$$\Delta = 6.01 \text{ sq. units}$$

Hence area of triangle = 6.01 sq. units

Q3. Find the area of the triangle ABC given three sides:

- i. $a = 18, b = 24, c = 30$
- ii. $a = 524, b = 276, c = 315$
- iii. $a = 32.65, b = 42.81, c = 64.92$

Solution

- i. $a = 18, b = 24, c = 30$

$$s = \frac{a+b+c}{2} = \frac{18+24+30}{2} = \frac{72}{2} = 36$$

$$\begin{aligned}\Delta &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{36(36-18)(36-24)(36-30)} \\ &= \sqrt{36(18)(12)(6)} = \sqrt{46656}\end{aligned}$$

$$\Delta = 216 \text{ sq. units}$$

Hence area of triangle = 216 sq. unit s

- ii. $a=524, b = 276, c = 315$

Solution

$$s = \frac{a+b+c}{2} = \frac{524+276+315}{2} = \frac{1115}{2} = 557.5$$

$$\begin{aligned}\Delta &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{557.5(557.5 - 524)(557.5 - 276)(557.5 - 315)} \\ &= \sqrt{557.5(33.5)(281.5)(242.5)} = \sqrt{1274910861}\end{aligned}$$

$$\Delta = 35705.89 \text{ sq. units}$$

Hence area of triangle = 35705.89 sq. unit s

iii. **a = 32.65, b = 42.81, c = 64.92**

Solution

$$s = \frac{a+b+c}{2} = \frac{32.65+42.81+64.92}{2} = \frac{140.38}{2} = 70.19$$

$$\begin{aligned}\Delta &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{70.19(70.19 - 32.65)(70.19 - 42.81)(70.19 - 64.92)} \\ &= \sqrt{70.19(37.54)(27.38)(5.27)} = \sqrt{3802012.275}\end{aligned}$$

$$\Delta = 616.605 \text{ sq. units}$$

Hence area of triangle = 616.605 sq. unit s

Q4. The area of triangle is 2437. If a = 79, and c = 97, then find angle β .

Solution

$$\Delta = 2437 \quad a=79, c=97$$

We know that

$$\Delta = \frac{1}{2} a. b. \sin \beta$$

$$\begin{aligned}\sin \beta &= \frac{2\Delta}{ac} \\ &= \frac{2 \times 2437}{97 \times 79} = 0.6361\end{aligned}$$

$$\begin{aligned}\beta &= \sin^{-1}(0.6363) \\ &= 39^{\circ}29'50'' \cong 39^{\circ}30'\end{aligned}$$

$$\beta = 39^{\circ}30'$$

Hence area of triangle = 216 sq. units

Q5. The area of triangle is 121.34. if $\alpha = 32^{\circ}15'$, $\beta = 65^{\circ}37'$ then find γ

Solution

$$\Delta = 121.32, \alpha = 32^{\circ}15', \beta = 65^{\circ}37'$$

We know that

$$\alpha + \beta + \gamma = 180^{\circ}$$

$$\begin{aligned}\gamma &= 180^{\circ} - \alpha - \beta \\ &= 180^{\circ} - (32^{\circ}) - (65^{\circ}37') = 82^{\circ}8'\end{aligned}$$

$$\Delta = \frac{1}{2} c^2 \frac{\sin \alpha \sin \beta}{\sin \gamma}$$

$$\begin{aligned}c^2 &= \frac{2\Delta \sin \gamma}{\sin \alpha \sin \beta} \\ &= \frac{2(121.34) \sin(82^{\circ}8')}{\sin(32^{\circ}) \sin(65^{\circ}37')} \\ &= \frac{2 \times 121.34 \times 0.9906}{0.5336 \times 0.9108}\end{aligned}$$

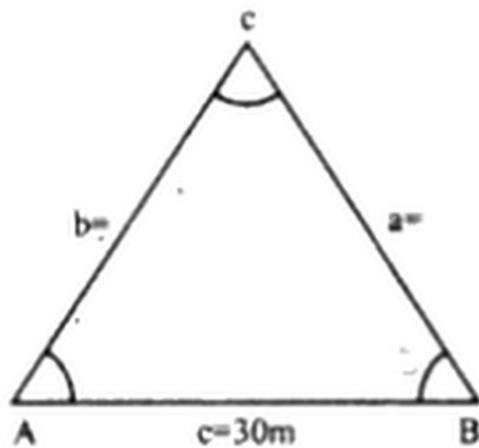
$$c^2 = 494.623$$

$$c = \sqrt{494.623} = 22.24$$

hence $c = 22.24$ and $\gamma = 82^{\circ}8'$

Q6. One side of triangle garden is 30m. If its two corner angles are $22^{\circ}\frac{1}{2}'$ and $112^{\circ}\frac{1}{2}'$, find the cost of planning the grass at the rate of Rs. 5 per square meter.

Solution



$$c = 30 \text{ m}$$

$$\alpha = [22^{\circ}\frac{1}{2}]' = 22^{\circ}30'$$

$$\beta = [112^{\circ}\frac{1}{2}]' = 112^{\circ}30'$$

$$\gamma = ?$$

We know that

$$\alpha + \beta + \gamma = 180^{\circ}$$

$$\gamma = 180^{\circ} - \alpha - \beta$$

$$= 180^{\circ} - (22^{\circ}30') - (112^{\circ}30') = 45$$

$$\Delta = \frac{1}{2} c^2 \cdot \frac{\sin \alpha \cdot \sin \beta}{\sin \gamma}$$

$$\Delta = \frac{1}{2} (30)^2 \cdot \frac{\sin (22.5) \cdot \sin (112.5)}{\sin 45}$$

$$\Delta = \frac{1}{2} \times 900 \times \frac{(0.38268)(0.923879)}{(0.707)}$$

$$\Delta = 224.9978 \cong 225 \text{ sqm}^2$$

The cost of 1 sq meter = Rs. 5

So, the cost of 225 sq meters = 5×225

$$= \text{Rs. } 1125$$

Hence the cost of planning the grass = Rs. 1125

