Exercise 12.6

Solve the following triangles, in which

1. **a = 7 , b = 7, c = 9**
2. **a = 32, b = 40 , c = 66**
3. **a =28.3 , b = 31.7, c = 42.8**
4. **a = 31.9 , b = 56.31, c = 40.27**
5. **a = 4584, b = 5140 , c = 3624**

1. **a = 7, b = 7, c = 9**

Solution

By law of cosine

$$\text{Cos}\,\alpha = \frac{b^2+c^2-a^2}{2bc}$$

$$= \frac{(7)^2+(9)^2-(7)^2}{2(7)(9)}$$

$$= \frac{49+81-49}{126} = \frac{81}{126}$$

$$\text{Cos}\,\alpha = 0.644$$

$$\alpha = cos^{-1}(0.644)$$

$$\alpha = 49^\circ 59' = 50^\circ$$

By law of cosine

$$\text{Cos}\,\beta = \frac{a^2+c^2-b^2}{2ac}$$

$$= \frac{(7)^2+(9)^2-(7)^2}{2(7)(9)}$$

$$= \frac{49+81-49}{126} = \frac{81}{126}$$

$$\text{Cos}\,\beta = 0.644$$

$$\beta = cos^{-1}(0.644)$$

$$\beta = 49^{\circ}59' = 50^{\circ}$$

We know that

$$\alpha + \beta + \gamma = 180^{\circ}$$

$$50^{\circ} + 50^{\circ} + \gamma = 180^{\circ}$$

$$\gamma = 180^{\circ} - 100^{\circ} = 80^{\circ}$$

Hence

$\alpha = 50^{\circ}$	a=7
$\beta = 50^{\circ}$	b=7
$\gamma = 80^{\circ}$	c=9

2. a = 32, b = 40 , c = 66

Solution

By law of cosine

$$\text{Cos}\,\alpha = \frac{b^2+c^2-a^2}{2bc}$$

$$= \frac{(40)^2+(66)^2-(32)^2}{2(40)(66)}$$

$$= \frac{1600+4356-1024}{5280} = \frac{4932}{5280}$$

$$\text{Cos}\,\alpha = 0.934$$

$$\alpha = cos^{-1}(0.934)$$

$$\alpha = 20^{\circ}55'$$

By law of cosine

$$\text{Cos}\,\beta = \frac{a^2+c^2-b^2}{2ac}$$

$$= \frac{(32)^2+(66)^2-(40)^2}{2(33)(66)}$$

$$= \frac{1024+4356-1600}{4224} = \frac{3780}{4224}$$

$$\text{Cos}\,\beta = 0.89489$$

$$\beta = cos^{-1}(0.89489)$$

$$\beta = 26^{\circ}30'$$

We know that

$$\alpha + \beta + \gamma = 180^{\circ}$$

$$20^{\circ}55' + 26^{\circ}30' + \gamma = 180^{\circ}$$

$$\gamma = 180^{\circ} - 20^{\circ}55' - 26^{\circ}30' = 132^{\circ}35'$$

Hence

$\alpha = 20^{\circ}55'$ a=32

$\beta = 26^{\circ}30'$ b=40

$\gamma = 132^{\circ}35'$ c=66

3. a =28.3 , b = 31.7, c = 42.8

Solution

By law of cosine

$$\text{Cos}\,\alpha = \frac{b^2+c^2-a^2}{2bc}$$

$$= \frac{(31.7)^2+(42.8)^2-(28.3)^2}{2(31.7)(42.8)}$$

$$= \frac{1004.89+1831.84-800.89}{2713.52} = \frac{2035.84}{2713.52}$$

$$\text{Cos}\,\alpha = 0.75025$$

$$\alpha = cos^{-1}(0.75025)$$

$$\alpha = 41^{\circ}23'$$

By law of cosine

$$\text{Cos}\,\beta = \frac{a^2+c^2-b^2}{2ac}$$

$$= \frac{(28.3)^2+(42.8)^2-(31.7)^2}{2(28.3)(42.8)}$$

$$= \frac{800.89+1831.84-1004.89}{2422.48} = \frac{1627.84}{2422.48}$$

$$\text{Cos}\,\beta = 0.67197$$

$$\beta = cos^{-1}(0.67197)$$

$$\beta = 47^{\circ}47'$$

We know that

$$\alpha + \beta + \gamma = 180^{\circ}$$

$$41^{\circ}23' + 47^{\circ}47' + \gamma = 180^{\circ}$$

$$\gamma = 180^{\circ} - 41^{\circ}23' - 47^{\circ}47' = 90^{\circ}50'$$

Hence

$\alpha = 41^{\circ}23'$ a=28.3

$\beta = 47^{\circ}47'$ b=31.7

$\gamma = 90^{\circ}50'$ c=42.8

4. a = 31.9 , b = 56.31, c = 40.27

Solution

By law of cosine

$$\text{Cos}\,\alpha = \frac{b^2+c^2-a^2}{2bc}$$

$$= \frac{(56.31)^2+(40.27)^2-(31.9)^2}{2(56.31)(40.27)}$$

$$= \frac{3170.8161+1621.6729-1017.61}{4535.2074} = \frac{3774.879}{4535.2074}$$

$$\text{Cos}\,\alpha = 0.8323$$

$$\alpha = cos^{-1}(0.8323)$$

$$\alpha = 33^{\circ}39'$$

By law of cosine

$$\text{Cos}\,\beta = \frac{a^2+c^2-b^2}{2ac}$$

$$=\frac{(31.9)^2+(40.27)^2-(56.31)^2}{2(31.9)(40.27)}$$

$$=\frac{1017.61+1621.6729-3170.8161}{2569.226} = \frac{-532.5332}{2569.226}$$

$$\text{Cos}\,\beta = -0.20688$$

$$\beta = cos^{-1}(-0.20688)$$

$$\beta = 101^\circ 56'$$

We know that

$$\alpha+\beta+\gamma = 180^\circ$$

$$33^\circ 39' + 101^\circ 56' + \gamma = 180^\circ$$

$$\gamma = 180^\circ - 33^\circ 39' - 101^\circ 56' = 44^\circ 24'$$

Hence

$\alpha = 33^\circ 39'$ a=28.3

$\beta = 101^\circ 56'$ b=31.7

$\gamma = 44^\circ 25'$ c=42.8

5. a = 4584, b = 5140 , c = 3624

Solution

By law of cosine

$$\text{Cos}\,\alpha = \frac{b^2+c^2-a^2}{2bc}$$

$$=\frac{(5140)^2+(3624)^2-(4584)^2}{2(5140)(3624)}$$

$$=\frac{26419600+13133376-21013056}{37254720} = \frac{18539920}{37254720}$$

$$\text{Cos}\,\alpha = 0.4976$$

$$\alpha = cos^{-1}(0.4976)$$

$$\alpha = 60^\circ 9'$$

By law of cosine

$$\text{Cos}\,\beta = \frac{a^2+c^2-b^2}{2ac}$$

$$= \frac{(4584)^2+(3624)^2-(5140)^2}{2(4584)(3624)}$$

$$= \frac{21013056+13133376-26419600}{33224832}$$

…

a= 4584

b= 5140

c = 3624

… the triangle ABC, when s = 37.34 , b = 3.32 …

… c = 25.036

… smallest side is smallest

$$\ldots = \frac{a^2+b^2+c^2}{2ac}$$

$$= \frac{(37.34)^2+(35.06)^2+(3.24)^2}{2(37.34)(35.06)}$$

$$= \frac{2612.98}{2618.28} = 0.998$$

$$\text{Cos}\,\beta = 0.998$$

$$\beta = cos^{-1}(0.998)$$

$$\beta = 3^\circ 37'$$

Hence the smallest angle = $3^\circ 37'$

6. **Find the measure of the greatest angle, if sides of the triangle are 16,20 ,33.**

Solution

a = 16, b= 20 , c= 33

the angle opposite is greatest side is greatest

$$\text{Cos}\,\gamma = \frac{a^2+b^2-c^2}{2ab}$$

$$= \frac{(16)^2+(20)^2-(33)^2}{2(16)(20)}$$

$$= \frac{256+400-1089}{640}$$

$$= -\frac{433}{640} = -0.68$$

$$\gamma = cos^{-1}(-0.68) = 132^{\circ}34'$$

Hence the greatest angle of triangle = $132^{\circ}34'$

7. **The sides of a triangle are $x^2 + x + 1, 2x + 1$ *and* $x^2 - 1$. Prove that the greatest angle of the triangle is 120°.**

Solution

a = x^2+x+1

b = 2x+1

c = $x^2 - 1$

the greatest angle opposite to side is greatest

by law of cosine

$$\text{Cos } \alpha = \frac{b^2+c^2-a^2}{2bc} \; ; \; a>b>c$$

$$= \frac{(2x+1)^2+(x^2-1)^2-(x^2+x+1)^2}{2(2x+1)(x^2-1)}$$

$$= \frac{4x^2+1+4x+x^4+1-2x^2-[x^4+x^2+1+2x^3+2x^2+2x]}{2[2x^3-2x+x^2-1]}$$

$$= \frac{x^4+2x^2+4x+2-x^4-x^2-1-2x^3-2x^2-2x}{4x^3-4x+2x^2-2}$$

$$= \frac{-2x^3-x^2+2x+1}{2(2x^3)-2x+x^2-1} = \frac{2x^3+x^2-2x-1}{2(2x^3-2x+x^2-1)}$$

$$= -\frac{1}{2}$$

$$\text{Cos } \alpha = -\frac{1}{2}$$

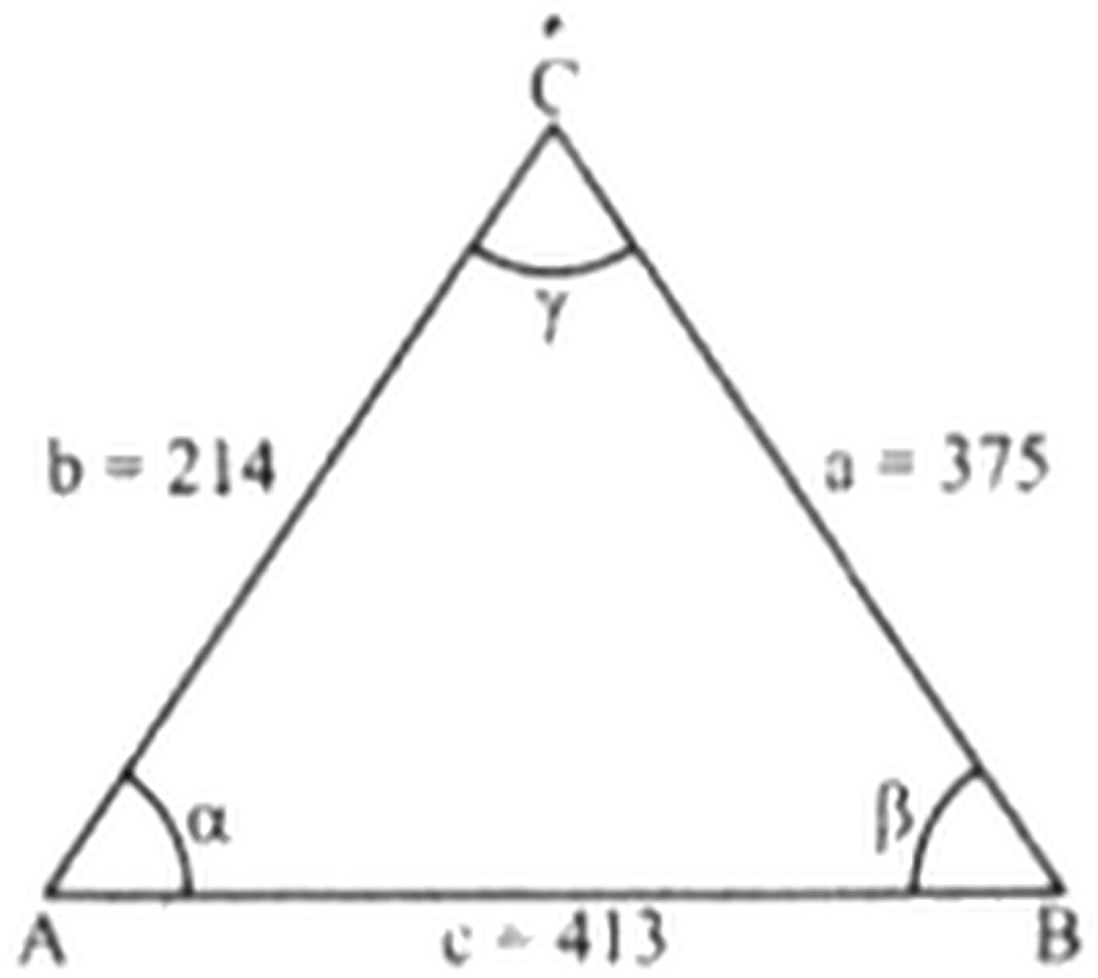
$$\alpha = cos^{-2}(-\frac{1}{2})$$

$$\alpha = 120^{\circ}$$

Hence proved the greatest angle in the triangle is 120°

8. The measure of side of a triangle plot are 413, 214 and 375 meters. Find the measure of the corner angles of the plot.

Solution



By the law of cosine

$$\text{Cos}\,\alpha = \frac{b^2+c^2-a^2}{2bc}$$

$$= \frac{(214)^2+(413)^2-(375)^2}{2(214)(413)}$$

$$= \frac{45796+170569-140625}{176764} = \frac{75740}{176764}$$

$$\text{Cos}\,\alpha = 0.4284$$

$$\alpha = cos^{-1}(0.4284)$$

$$\alpha = 64^\circ 38'$$

By law of cosine

$$\text{Cos}\,\beta = \frac{a^2+c^2-b^2}{2ac}$$

$$= \frac{(375)^2+(413)^2-(214)^2}{2(375)(413)}$$

$$= \frac{140625+170569-45796}{309750} = \frac{265398}{309750}$$

$$\text{Cos}\,\beta = 0.8568$$

$\beta = cos^{-1}(0.8568)$

$\beta = 31^{\circ}2'$

We know that

$\alpha + \beta + \gamma = 180^{\circ}$

$64^{\circ}38' + 31^{\circ}2' + \gamma = 180^{\circ}$

$\gamma = 180^{\circ} - 64^{\circ}38' - 31^{\circ}2' = 84^{\circ}20'$

Hence the measurement of angles of corner = $64^{\circ}38', 31^{\circ}2'$ *and* $84^{\circ}20'$.

9. Three villages A, B , C are connected by straight .... Km. what angles these roads make with each other?

Solution

a= 6 km

b = 9 km

c = 13 km

By the law of cosine

$\text{Cos}\,\alpha = \frac{b^2+c^2-a^2}{2bc}$

$= \frac{(9)^2+(13)^2-(6)^2}{2(9)(13)}$

$= \frac{81+169-36}{234} = \frac{214}{234}$

$\text{Cos}\,\alpha = 0.914$

$\alpha = cos^{-1}(0.914)$

$\alpha = 23^{\circ}57'$

By law of cosine

$\text{Cos}\,\beta = \frac{a^2+c^2-b^2}{2ac}$

$= \frac{(6)^2+(13)^2-(9)^2}{2(375)(413)}$

$$= \frac{36+169+81}{156} = \frac{124}{156}$$

$$\text{Cos}\,\beta = 0.7948$$

$$\beta = cos^{-1}(0.7948)$$

$$\beta = 37^{\circ}21'$$

We know that

$$\alpha + \beta + \gamma = 180^{\circ}$$

$$23^{\circ}57' + 37^{\circ}21' + \gamma = 180^{\circ}$$

$$\gamma = 180^{\circ} - 23^{\circ}57' - 37^{\circ}21' = 118^{\circ}42'$$

Hence the measurement of angles of corner = $23^{\circ}57'$, $37^{\circ}21'$ and $118^{\circ}42'$.