# Exercise 12.5

**Solve the triangle ABC in which**

1. **b = 95 ,c =34 and $\alpha = 52^\circ$**
2. **b=12.5 , c= 23 and $\alpha = 38^\circ 20'$**
3. **a= $\sqrt{3} - 1$ , b = $\sqrt{3} + 1$ and $\gamma = 60^\circ$**
4. **a=3 , c=6 and $\beta = 36^\circ 20'$**
5. **a=7, b=3 and $\gamma = 38^\circ 13'$**

**1. b = 95 ,c =34 and $\alpha = 52^\circ$**

**Solution**

We know that by law of cosines

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$a^2 = (95)^2 + (34)^2 - 2(95)(34) \cos (52^\circ)$$

$$a^2 = 6203.8269$$

$$a = \sqrt{6203.8269} = 78.76$$

by law of sine

$$\frac{\sin \beta}{a} = \frac{\sin \alpha}{a}$$

$$\sin \beta = \frac{95}{78.76} . \sin (52^\circ)$$

$$= \frac{95 \times 0.788}{78.796} = 0.9505$$

$$\beta = \sin^{-1}(0.9505) = 71.89$$

$$\beta = 71^\circ 54'$$

$$\alpha + \beta + \gamma = 180^\circ$$

$$\gamma = 180^\circ - 52^\circ - 71^\circ 54'$$

$$\gamma = 55^\circ 6'$$

Hence

| | |
|---|---|
| a = 78.76 | $\alpha = 52^\circ$ |
| b = 95 | $\beta = 71^\circ 54'$ |
| c= 34 | $\gamma = 55^\circ 6'$ |

**2. b=12.5 , c= 23 and $\alpha = 38^\circ 20'$**

**Solution**

We know that by law of cosines

$$a^2 = b^2 + c^2 - 2bc\ cos\ \alpha$$

$$a^2 = (12.5)^2 + (23)^2 - 2(12.5)(23)\ cos\ (38^\circ 20')$$

$$a^2 = 234.211$$

$$a = \sqrt{234.211} = 15.30$$

by law of sine

$$\frac{b}{sin\ \beta} = \frac{a}{sin\ \alpha}$$

$$sin\ \beta = \frac{125}{15.3}.\ sin\ (38^\circ 20')$$

$$= 0.507$$

$$\beta = sin^{-1}(0.507)$$

$$\beta = 30^\circ 26'$$

$$\alpha + \beta + \gamma = 180^\circ$$

$$\gamma = 180^\circ - 38^\circ 20' - 30^\circ 26'$$

$$\gamma = 111^\circ 14'$$

Hence

a = 15.30 $\alpha = 38^\circ 20'$

b = 12.5 $\beta = 30^\circ 26'$

c= 23 $\gamma = 111^\circ 14'$

**3. a= $\sqrt{3} - 1$ , b = $\sqrt{3} + 1$ and $\gamma = 60^\circ$**

**Solution**

We know that by law of cosines

$$c^2 = a^2 + b^2 - 2bc\cos\gamma$$

$$c^2 = (0.732)^2 + (2.732)^2 - 2(0.732)(2.732)\cos(60^\circ)$$

$$c^2 = 6$$

$$c = \sqrt{6} = 2.45$$

by law of sine

$$\frac{\sin\alpha}{a} = \frac{\sin\gamma}{c}$$

$$\sin\alpha = \frac{0.732}{2.45}.\sin(60^\circ)$$

$$= 0.2587$$

$$\alpha = \sin^{-1}(0.2587)$$

$$\alpha = 15^\circ$$

We know that

$$\alpha + \beta + \gamma = 180^\circ$$

$$\beta = 180^\circ - 60^\circ - 15^\circ$$

$$\beta = 75^\circ$$

Hence

a = $\sqrt{3} - 1$ $\alpha = 15^\circ$

b = $\sqrt{3} + 1$ $\beta = 75^\circ$

c= 2.45 $\gamma = 60^\circ$

**4. a=3 , c=6 and $\beta = 36^\circ 20'$**

**Solution**

We know that by law of cosines

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

$$b^2 = (3)^2 + (6)^2 - 2(3)(6)\cos(36^\circ 20')$$

$$b^2 = 15.89$$

$$b = \sqrt{15.89} = 4$$

by law of sine

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b}$$

$$\sin\alpha = \frac{3}{4}.\sin(36^\circ 20')$$

$$= 0.4412$$

$$\alpha = \sin^{-1}(0.4412)$$

$$\alpha = 26^\circ 10'$$

We know that

$$\alpha + \beta + \gamma = 180^\circ$$

$$\gamma = 180^\circ - 26^\circ 10' - 36^\circ 20'$$

$$\gamma = 117^\circ 30'$$

Hence

| | |
|---|---|
| a = 3 | $\alpha = 26^\circ 10'$ |
| b = 4 | $\beta = 36^\circ 20'$ |
| c= 6 | $\gamma = 117^\circ 30'$ |

**5. a=7, b=3 and $\gamma = 38^\circ 13'$**

**Solution**

We know that by law of cosines

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

$$c^2 = (7)^2 + (3)^2 - 2(7)(3)\cos(38^\circ 13')$$

$$c^2 = 25$$

$$c = \sqrt{25} = 5$$

by law of sine

$$\frac{\sin\alpha}{a} = \frac{\sin\gamma}{c}$$

$$\sin\alpha = \frac{7}{5}.\sin(38^\circ 13')$$

$$= 0.866$$

$$\alpha = \sin^{-1}(0.866)$$

$$\alpha = 60^\circ$$

We know that

$$\alpha + \beta + \gamma = 180^\circ$$

$$\beta = 180^\circ - 60^\circ - 38^\circ 47'$$

$$\gamma = 81^\circ 47'$$

Hence

| | |
|---|---|
| a = 7 | $\alpha = 60^\circ$ |
| b = 3 | $\beta = 81^\circ 47'$ |
| c= 5 | $\gamma = 38^\circ 13'$ |

**Solve the following triangles, using first law of tangent and law of sines:**

**6. a = 36.21 b = 42. 09 and** $\gamma = 44^\circ 29'$

**7. a = 93 , b = 101, and** $\beta = 80^\circ$

**8. b = 14.8 , c = 16,** $\alpha = 42^\circ 45'$

**9. a= 319, b = 168 and** $\gamma = 110^\circ 22'$

**10. b= 61, a = 32 and** $\alpha = 59^\circ 30'$

**6. a = 36.21 b = 42. 09 and $\gamma = 44^\circ 29'$**

**Solution**

We know that

$$\alpha + \beta + \gamma = 180^\circ$$

$$\alpha + \beta = 180^\circ - 44^\circ 29'$$

$$\alpha + \beta = 135^\circ 31'$$

By law of tangent

$$\frac{tan(\frac{\alpha \ \beta}{2})}{tan(\frac{\alpha+\beta}{2})} = \frac{a-b}{a+b}$$

$$\frac{tan(\frac{\alpha \ \beta}{2})}{tan(\frac{135^\circ 31'}{2})} = \frac{36.21-42.09}{36.21+42.09}$$

$$tan(\frac{\alpha-\beta}{2}) = tan\,(67^\circ 45').\frac{-5.83}{78.3}$$

$$=(2.445](\frac{-5.83}{78.3})$$

$$tan(\frac{\alpha-\beta}{2}) = -0.182$$

$$\frac{\alpha-\beta}{2} = tan^{-1}(-0.182) = -10^\circ 18'$$

$$\alpha - \beta = 2(-10^\circ 18') = -20^\circ 37'$$

$$\alpha + \beta = 135^\circ 31' \ and \ \alpha - \beta = -20^\circ 37'$$

$$\frac{\alpha-\beta= -20^\circ 37'}{2\alpha = 114^\circ 54'}$$

$$\beta = \frac{114^\circ 54'}{2} = 57^\circ 27'$$

$$\alpha + \beta = 135^\circ 31'$$

$$\frac{\alpha-\beta= -20^\circ 37'}{2\beta =156^\circ 8'}$$ (by subtracting )

$$\beta = \frac{156^\circ 8'}{2} = 78^\circ 4'$$

By law of sine

$$\frac{c}{\sin\gamma} = \frac{a}{\sin\alpha}$$

$$= 30.13.\frac{\sin(44^\circ 29')}{\sin(57^\circ 26')}$$

$$= \frac{30.13\times 0.7007}{0.8427} = 21.38$$

b= 21.38

hence

| | |
|---|---|
| a = 36.21 | $\alpha = 57^\circ 26'$ |
| b = 21.38 | $\beta = 78^\circ 34'$ |
| c= 21.38 | $\gamma = 44^\circ 29'$ |

**7. a = 93 , b = 101, and $\beta = 80^\circ$**

**Solution**

We know that

$$\alpha+\beta+\gamma = 180^\circ$$

$$\alpha+\gamma = 180^\circ - 80^\circ$$

$$\alpha+\gamma = 100^\circ$$

By law of tangent

$$\frac{\tan(\frac{\alpha\ \gamma}{2})}{\tan(\frac{\alpha+\gamma}{2})} = \frac{a-c}{a+c}$$

$$\frac{\tan(\frac{\alpha\ \gamma}{2})}{\tan(\frac{100^\circ}{2})} = \frac{93-101}{93+101}$$

$$\tan(\frac{\alpha-\gamma}{2}) = \tan(50^\circ).\frac{-8}{194}$$

$$=(-1.1917](0.04123)$$

$$\tan(\frac{\alpha-\gamma}{2}) = -0.04914$$

$$\frac{\alpha-\gamma}{2} = \tan^{-1}(-0.04914)$$

$$\alpha - \gamma = 2(2^{\circ}48') = 5^{\circ}36'$$

$$\alpha + \gamma = 100^{\circ} \; and \; \alpha - \gamma = 5^{\circ}36'$$

$$\frac{\alpha - \gamma = 5^{\circ}36'}{2\gamma = 105^{\circ}36'}$$

$$\beta = \frac{105^{\circ}36'}{2} = 52^{\circ}48'$$

$$\alpha + \gamma = 100^{\circ}$$

$$\frac{\alpha - \gamma = 5^{\circ}36'}{2\gamma = 94^{\circ}54'}$$ (by subtracting )

$$\beta = \frac{94^{\circ}54'}{2} = 47^{\circ}27'$$

By law of sine

$$\frac{b}{\sin\beta} = \frac{a}{\sin\alpha}$$

$$= 93.\frac{\sin(80^{\circ})}{\sin(52^{\circ}48')}$$

$$= \frac{93 \times 0.9848}{0.7965} = 125$$

b= 125

hence

| | |
|---|---|
| a = 93 | $\alpha = 52^{\circ}48'$ |
| b = 124 | $\beta = 80^{\circ}$ |
| c= 101 | $\gamma = 47^{\circ}27'$ |

**8. b = 14.8, c = 16, $\alpha = 42^{\circ}45'$**

**Solution**

We know that

$$\alpha + \beta + \gamma = 180^{\circ}$$

$$\alpha + \gamma = 180^{\circ} - 42^{\circ}45'$$

$$\alpha + \gamma = 137^{\circ}15'$$

By law of tangent

$$\frac{tan(\frac{\beta-\gamma}{2})}{tan(\frac{\beta+\gamma}{2})} = \frac{b-c}{b+c}$$

$$\frac{tan(\frac{\beta-\gamma}{2})}{tan(\frac{137^\circ 15'}{2})} = \frac{14.8-16.1}{14.8+16.1}$$

$$tan(\frac{\beta-\gamma}{2}) = tan\,(68^\circ 37').\frac{-13}{30.9}$$

$$=\frac{-13}{30.9}(2.554)$$

$$tan(\frac{\beta-\gamma}{2}) = -0.10749$$

$$\frac{\beta-\gamma}{2} = tan^{-1}(-0.10749)$$

$$\beta-\gamma = 2(6.135) = 12^\circ 16'$$

$$\beta+\gamma = 137^\circ 15' \; and \; \beta-\gamma = 12^\circ 16'$$

$$\frac{\beta-\gamma= 12^\circ 16'}{2\beta = 149.52}$$

$$\beta = \frac{149.52}{2} = 74^\circ 48'$$

$$\alpha+\beta = 137^\circ 15'$$

$$\frac{\alpha-\beta=12^\circ 16'}{2\beta =124^\circ 52'}$$ (by subtracting )

$$\beta = \frac{124^\circ 52'}{2} = 62^\circ 26'$$

By law of sine

$$\frac{a}{sin\,\alpha} = \frac{b}{sin\,\beta}$$

$$= 14.8.\frac{sin\,(42^\circ 45')}{sin\,(62^\circ 26')}$$

$$= \frac{14.8 \times 0.6788}{0.8868} = 11.33$$

a= 11.33

hence

a = 11.33 $\alpha = 42^\circ 45'$

b = 14.8 $\beta = 62^{\circ}26'$

c= 16.1 $\gamma = 74^{\circ}48'$

**9. a= 319, b = 168 and $\gamma = 110^{\circ}22'$**

**Solution**

We know that

$$\alpha + \beta + \gamma = 180^{\circ}$$

$$\alpha + \beta = 180^{\circ} - 110^{\circ}22'$$

$$\alpha + \beta = 69^{\circ}38'$$

By law of tangent

$$\frac{tan(\frac{\alpha\ \ \beta}{2})}{tan(\frac{\alpha+\beta}{2})} = \frac{b-c}{b+c}$$

$$\frac{tan(\frac{\alpha\ \ \beta}{2})}{tan(\frac{69^{\circ}38'}{2})} = \frac{319-168}{319+168}$$

$$tan(\frac{\alpha-\beta}{2}) = tan\ (34^{\circ}49').\frac{151}{487}$$

$$= \frac{151}{487}(0.6954)$$

$$tan(\frac{\alpha-\beta}{2}) = 0.2156$$

$$\frac{\alpha-\beta}{2} = tan^{-1}(0.2156)$$

$$\alpha - \beta = 2(12^{\circ}10') = 24^{\circ}20'$$

$$\alpha - \beta = 24^{\circ}20'\ and\ \alpha + \beta = 69^{\circ}38'$$

$$\alpha - \beta = 24^{\circ}20'$$

$$\frac{\alpha+\beta = 69^{\circ}38'}{2\alpha = 93^{\circ}58'}$$

$$\alpha = \frac{93^{\circ}58'}{2} = 46^{\circ}59' \cong 47^{\circ}$$

$$\alpha + \beta = 69^{\circ}38'$$

$$\frac{\alpha-\beta=24^\circ 20'}{2\beta=45^\circ 18'}$$ (by subtracting )

$$\beta = \frac{45^\circ 18'}{2} = 22^\circ 39'$$

By law of sine

$$\frac{c}{\sin\gamma} = \frac{a}{\sin\alpha}$$

$$= 319.\frac{\sin(110^\circ 22')}{\sin 47^\circ}$$

$$= \frac{319\times 0.9375}{0.73135} = 408.91$$

c= 408.91

hence

| | |
|---|---|
| a = 319 | $\alpha = 47^\circ$ |
| b = 168 | $\beta = 22^\circ 39'$ |
| c= 408.91 | $\gamma = 110^\circ 22'$ |

**10. b= 61, a = 32 and $\alpha = 59^\circ 30'$**

**Solution**

Since the statement of the question is not correct so it cannot be solved. It should be 'b' and 'c' instead of 'b' and 'a'

$$\alpha+\beta+\gamma = 180^\circ$$ by law of sine

$$\beta+\gamma = 180^\circ - 59^\circ 30'$$ $$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$$

$$\beta+\gamma = 120^\circ 30'$$ a = 61 $\frac{\sin(59^\circ 30')}{\sin(88^\circ 51')}$

By law of tangents a =61 × $\frac{0.861}{0.999}$ =

a= 53

$$\frac{\tan(\frac{\beta-\gamma}{2})}{\tan(\frac{\beta+\gamma}{2})} = \frac{b-c}{b+c}$$

$$\frac{tan(\frac{\beta\ \gamma}{2})}{tan(\frac{120^{\circ}30'}{2})} = \frac{61-32}{61+32}$$

$$tan(\frac{\beta-\gamma}{2}) = \frac{29}{93} \times tan(60^{\circ}15')$$

$$tan(\frac{\beta-\gamma}{2}) = 28.61 = 28^{\circ}36'$$

$$\beta - \gamma = 57^{\circ}12'$$

So

$$\beta + \gamma = 12^{\circ}30'$$

$$\beta - \gamma = 57^{\circ}12'$$

$$2\beta = 177^{\circ}42'$$

$$\beta = \frac{177^{\circ}42'}{2} = 88^{\circ}51'$$

$$\beta + \gamma = 120^{\circ}30'$$

$$\beta - \gamma = 57^{\circ}12'$$

$$2\gamma = 63^{\circ}18'$$

$$\gamma = \frac{63^{\circ}18'}{2} = 31^{\circ}39'$$

**11. Measure of two sides of a triangle are in the ratio 3 :2 and they include an angle of measure $57^{\circ}$. Find the remaining two angles.**

**Solution**

Let two sides of a triangle are a and b.

Then $\frac{a}{b} = \frac{3}{2}$; $\gamma = 57^{\circ}$

By componendo- division law

$$\frac{a+b}{a-b} = \frac{3+2}{3-2}$$

$$\frac{a+b}{a-b} = \frac{5}{1}$$

We know that

$$\alpha + \beta + \gamma = 180^\circ$$

$$\alpha + \beta = 180^\circ - 57^\circ$$

$$\alpha + \beta = 123^\circ$$

By law of tangent

$$\frac{tan(\frac{\alpha+\beta}{2})}{tan(\frac{\alpha-\beta}{2})} = \frac{a+b}{a-b}$$

$$\frac{tan(123^\circ)}{tan(\frac{\alpha-\beta}{2})} = \frac{5}{1}$$

$$tan(\frac{\alpha-\beta}{2}) = \frac{tan\,(61^\circ 30')}{5}$$

$$tan(\frac{\alpha-\beta}{2}) = \frac{1.8417}{5} = 0.3684$$

$$\frac{\alpha-\beta}{2} = tan^{-1}(0.3684) = 20.22$$

$$\alpha - \beta = 2(20.22) = 40.44$$

$$\alpha - \beta = 40.44 \text{ and } \alpha + \beta = 123^\circ$$

$$\frac{\alpha-\beta = 40.44}{2\alpha = 163.44} \quad \text{(by adding)}$$

$$\alpha = \frac{163.44}{2} = 81.72 = 81^\circ 44'$$

$$\alpha + \beta = 123$$

$$\frac{\alpha-\beta = 40.44}{2\beta = 82.56} \quad \text{(by subtracting )}$$

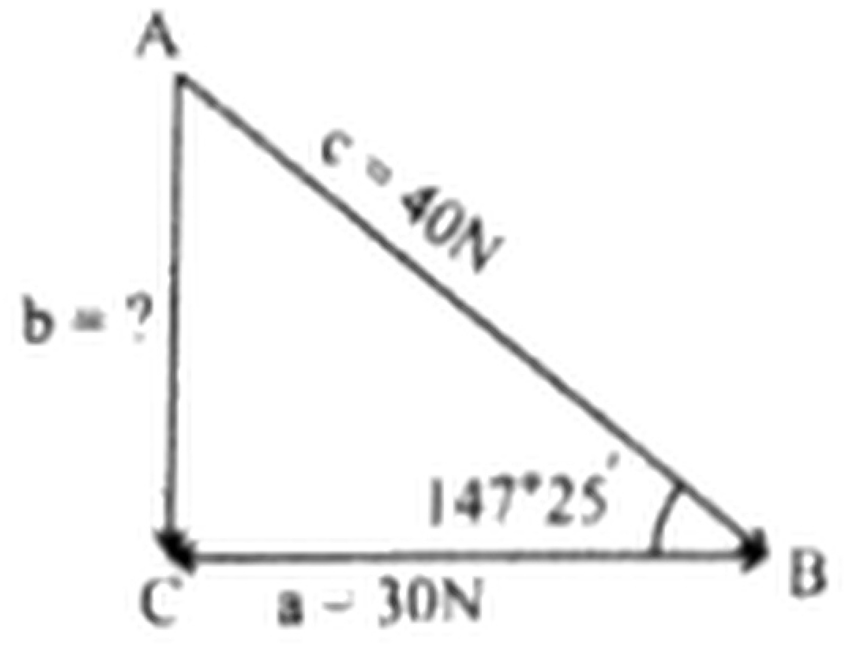
$$\beta = \frac{82.56}{2} = 41.28 = 41^\circ 16'$$

Hence the values of remaining

Angles $= 41^\circ 16'$ and $81^\circ 44'$

**12. Two force of 40 N and 30 N are represented by $\vec{A}\,\overline{B}$ and $\vec{B}\,\overline{C}$ which are inclined at an angle of $147^\circ 25'$ . find $\vec{A}\,\overline{C}$, the resultant of $\vec{A}\,\overline{B}$ and $\vec{B}\,\overline{C}$.**

**Solution**




Let $a = 30$ N

$c = 40$ N

$b = 147^{\circ}25'$

then $b = ?$

by law of cosine

$$b^2 = a^2 + c^2 - 2ac \cdot \cos\beta$$

$$= (30)^2 + (40)^2 - 2(30)(40) \cdot \cos(147^{\circ}25')$$

$$= 900+1600-2400(-0.8426)$$

$$= 2500 + 2022.26$$

$$= 4522.26$$

$$b = \sqrt{4522.26} = 67.25\ N$$

hence

$\vec{AC}$ the resultant of $\vec{AB}$ $\vec{BC}$ = 67.25 N