Exercise 12.4

1. Solve the triangle ABC, If

1) $\beta = 60$, $\gamma = 15$, $b = \sqrt{16}$

2) $\beta = 52°$, $\gamma = 89° 35'$, $a = 89.35$

3) $b = 125$, $\gamma = 53°$, $\alpha = 47°$

4) $c = 16.1$, $\alpha = 42°45'$, $\gamma = 74° 32'$

5) $a = 53$, $\beta = 88°36$, $\gamma = 31°54'$

1. $\beta = 60°$, $\gamma = 15°$, $b = \sqrt{6}$

Solution

$\beta = 60$ $\gamma = 15°$, $a = ?$

We know that

$$\alpha + \beta + \gamma = 180°$$

$$\Rightarrow \alpha = 150°$$

By law of Sine

$$\frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

$$c = b\frac{\sin\gamma}{\sin\beta}$$

$$= \sqrt{6}\,\frac{\sin(15^{\circ})}{\sin(60^{\circ})}$$

$$c = 0.732$$

By law of Sine

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$$

$$a = b\frac{\sin\alpha}{\sin\beta}$$

$$= \sqrt{6}\ \frac{\sin(15^\circ)}{\sin(60^\circ)}$$

a = 2.732

Hence

a= 2.732 $\alpha = 105°$

b = 2.45 $\beta = 60°$

c = 0.7320 $\gamma = 15°$

2. β = 52°, γ = 89° 35', a= 89.35

$\beta = 52°, \gamma = 89°\ 35', \alpha = ?$

we know that

$\alpha + \beta + \gamma = 180°$

$\alpha = 180° - \beta - \gamma$

$= 180° - 52° - 89°35'$

$\alpha = 38°25$

By law of Sine

$$\frac{b}{\sin\beta} = \frac{a}{\sin\alpha}$$

$$b = a\frac{\sin(\beta)}{\sin\alpha}$$

$$= 89.35 \times \frac{\sin(52^\circ)}{\sin(38^\circ 25')}$$

b =113.38

By law of Sine

$$\frac{c}{\sin\gamma} = \frac{a}{\sin\alpha}$$

$$c = a\frac{\sin\gamma}{\sin\alpha}$$

$$= 89.25 \times \frac{Sin(52^{\circ})}{\sin(38^{\circ}25')}$$

c=143.79

Hence,

b=89.35 α =38°25

b = 113.38 β = 52°

c= 143.79 γ =89°35'

3. b=125 , γ = 53° , α = 47°

α =47°, β **=?**, γ =53

we know that

$\alpha + \beta + \gamma = 180°$

$\beta = 180° - \alpha - \gamma$

=180°-47°-53°=80°

β =80°

By law of Sine

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$$

$$a = b\frac{\sin\alpha}{\sin\beta}$$

$$= 125\frac{\sin(47^{\circ})}{\sin(80^{\circ})}$$

a=92.83

By law of Sine.

$$\frac{c}{\sin\gamma} = \frac{b}{\sin\beta}$$

$$c = b\frac{\sin\gamma}{\sin\beta}$$

$$= 125\frac{\sin(53^\circ)}{\sin(80^\circ)}$$

$$c = 101.37$$

Hence.

a = 92.83 $\alpha = 47°$

b=125 $\beta = 80°$

c =101.37 $\gamma = 53°$

4. c=16.1, α =42°45' , γ =74°32'

we know that

$$\alpha + \beta + \gamma = 180°$$

$$\beta = 180° - \alpha - \gamma$$

$$= 180° - 42°45' - 74°32'$$

$$B = 62°43'$$

By law of Sine.

$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$$

$$a = c\frac{\sin\alpha}{\sin\gamma}$$

$$= 16.1\frac{\sin(42^\circ 45')}{\sin(74^\circ 32')}$$

$$a = 11.339$$

By law of Sine.

$$\frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

$$b = c\frac{\sin\beta}{\sin\gamma}$$

$$= 16.1\frac{\sin(62^{\circ}43')}{\sin(74^{\circ}32')}$$

b = 14.846

Hence,

a = 11.339 ; $\alpha = 42°45'$

b = 14.846 ; $\beta = 62°43'$

c = 1.16 ; $\gamma = 74°32$

5. β =88°36' , γ=31°54'

α =?; β = 88°36'; γ = 31°54'

we know that

$\alpha + \beta + \gamma = 180°$

$\alpha = 180° - \beta - \gamma$

= 180°-88°36'-31°54'

$\alpha = 59°30'$

By law of Sine

$$\frac{b}{\sin\beta} = \frac{a}{\sin\alpha}$$

$$b = a\frac{\sin(\beta)}{\sin\alpha}$$

$$= 53\frac{\sin(88^{\circ}36')}{\sin(59^{\circ}30')}$$

b =61.4929

By law of Sine

$$\frac{c}{\sin\gamma} = \frac{a}{\sin\alpha}$$

$$c = a\frac{\sin\gamma}{\sin\alpha}$$

$$= 53\frac{Sin(31^{\circ}54')}{\sin(59^{\circ}30')}$$

c=32.51

Hence

a = 53 α =59.30

b= 61.50 β = 88°36'

c = 32.51 γ = 31°54'