Exercise 10.3

1. Find the values of $\sin 2\alpha, \cos 2\alpha,$ **and** $\tan 2\alpha,$ **when:**

i. $\sin\alpha = \frac{12}{13}$ **ii.** $\cos\alpha = \frac{3}{5}$,

where $0 < \alpha < \frac{\pi}{2}$

1. $\sin \propto = \frac{12}{13}$

Solution:

$$\cos\alpha = \pm\sqrt{1-\sin^2\alpha}$$

$$= \pm\sqrt{1-\left(\frac{12}{13}\right)}$$

$$= \pm\sqrt{1-\frac{144}{169}}$$

$$= \pm\sqrt{\frac{169-144}{169}}$$

$$= \pm\sqrt{\frac{25}{169}}$$

$$= \pm\frac{5}{13} = \frac{5}{13}$$ [As Angle of terminal rays lies in I quad]

a) sin 2α

$$\cos 2\alpha = 2\sin\alpha\cos\alpha$$

$$= 2\times\frac{12}{13}\times\frac{5}{13} = \frac{120}{169}$$

Hence $= \sin 2\alpha = \frac{120}{169}$

b) **cos** 2α

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$= \left(\frac{5}{12}\right)^2 - \left(\frac{12}{13}\right)^2$$

$$= \frac{25}{169} - \frac{144}{169}$$

$$= \frac{25-144}{169} = \frac{-199}{169}$$

Hence; $\cos 2\alpha = -\frac{199}{169}$

and $\tan 2\alpha = \frac{\sin 2\alpha}{\cos 2\alpha} = \frac{\frac{120}{169}}{\frac{-199}{169}} = -\frac{120}{119}$

Hence, $\tan 2\alpha = \frac{-120}{119}$

ii. **cos** $\alpha = \frac{3}{5}$

$\Rightarrow$ $\sin \alpha = \pm\sqrt{1-\cos^2 \alpha}$

$$= \pm\sqrt{1-\frac{9}{25}}$$

$$= \pm\sqrt{\frac{25-9}{25}}$$

$$= \pm\sqrt{\frac{16}{25}}$$

$$= \pm\frac{4}{5}$$

$\Rightarrow \quad \sin\alpha = \pm\frac{4}{5} = \frac{4}{5}$ [The angle of terminal arm lies in I quard]

a). $\sin 2\alpha \; = 2\sin\alpha \;\; \cos\alpha$

$$= 2\times\frac{4}{5}\times\frac{3}{5} = \frac{24}{25}$$

Hence $\sin 2\alpha = \frac{24}{25}$

b). $\cos 2\alpha \; = \cos^2\alpha - \sin^2\alpha$

$$= \left(\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2$$

$$= \frac{9}{25} - \frac{16}{25}$$

$$= \frac{9-16}{25} = \frac{7}{25}$$

Henc $\cos 2\alpha = \frac{7}{25}$

c). $\tan 2\alpha$

$$\tan 2\alpha = \frac{\sin 2\alpha}{\cos 2\alpha}$$

$$=\frac{\frac{24}{25}}{\frac{-7}{25}}$$

$$=-\frac{24}{7}$$

Hence, $\tan 2\alpha = -\frac{24}{7}$

2. Prove the following identities

2. $\cot\alpha - \tan\alpha = 2\cot 2\alpha$

3. $\frac{\sin 2\alpha}{1+\cos 2\alpha} = \tan\alpha$

4. $\frac{1-\cos\alpha}{\sin\alpha} = \tan\frac{\alpha}{2}$

5. $\frac{\cos\alpha - \sin\alpha}{\cos\alpha + \sin\alpha} = see2\alpha - \tan 2\alpha$

6. $\sqrt{\frac{1+\sin\alpha}{1-\sin\alpha}} = \frac{\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}-\cos\frac{\alpha}{2}}$

7. $\frac{\cos ec\theta + 2\cos ec2}{\sec\theta} = \cos\frac{\theta}{2}$

8. $1+\tan\alpha\tan 2\alpha = \sec 2\alpha$

9. $\frac{2\sin\theta\sin 2\theta}{\cos\theta + \cos 3\theta} = \tan 2\theta\tan\theta$

10. $\frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta} = 2$

11. $\frac{\cos 3\theta}{\cos\theta} + \frac{\sin 3\theta}{\sin\theta} = 4\cos 2\theta$

12. $\frac{\tan\frac{\theta}{2}+\cot\frac{\theta}{2}}{\cot\frac{\theta}{2}\tan\frac{\theta}{\theta}} = \sec\theta$

13. $\frac{\sin 3\theta}{\cos\theta} + \frac{\cos 3\theta}{\sin\theta} = 2\cot 2\theta$

2. $\cot\alpha - \tan\alpha = 2\cot 2\alpha$

Solution:

L.H.S $= \cot\alpha - \tan\alpha$

$$= \frac{\cos\alpha}{\sin\alpha} - \frac{\sin\alpha}{\cos\alpha}$$

$$= \frac{\cos^2\alpha - \sin^2\alpha}{\sin\alpha\cos\alpha}$$

$$= \frac{2\left[\cos^2\alpha - \sin^2\alpha\right]}{2\sin\alpha\cos\alpha}$$

$$= \frac{2\cos 2\alpha}{\sin 2\alpha}$$

$$= 2\cot 2\alpha$$

$= \text{R.L.S}$

Hence Proved

$$\cot\alpha - \tan\alpha = 2\cot 2\alpha$$

3. $\dfrac{\sin 2\alpha}{1+\cos 2\alpha} = \tan\alpha$

Solution:

$$\text{L.H.S} = \frac{\sin 2\alpha}{1+\cos 2\alpha}$$

$$= \frac{2\sin\alpha\cos\alpha}{1+2\cos^2\alpha - 1}$$

$$= \frac{2\sin\alpha\cos\alpha}{2\cos^2\alpha}$$

$$= \frac{\sin\alpha}{\cos\alpha}$$

$$= \tan\alpha$$

$= \text{R.H.S}$

Hence Proved $\frac{\sin 2\alpha}{1+\cos 2\alpha} = \tan\alpha$

4. $\frac{1-\cos\alpha}{\sin\alpha} = \tan\frac{\alpha}{2}$

Solution:

$$\text{L.H.S} = \frac{1-\cos\alpha}{\sin\alpha}$$

$$= \frac{1-\left[1-2\sin^2\frac{\alpha}{2}\right]}{2\sin\frac{\alpha}{2}.\cos\frac{\alpha}{2}}$$

$$= \frac{1-1+2\sin^2\frac{\alpha}{2}}{2\sin\frac{\alpha}{2}.\cos\frac{\alpha}{2}}$$

$$= \frac{2\sin^2\frac{\alpha}{2}}{2\sin\frac{\alpha}{2}.\cos\frac{\alpha}{2}}$$

$$= \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}$$

$$= \tan\frac{\alpha}{2}$$

= R.H.S

Hence proved

$$\frac{1-\cos\alpha}{\sin\alpha} = \tan\frac{\alpha}{2}$$

5. $\dfrac{\cos\alpha-\sin\alpha}{\cos\alpha+\sin\alpha}=\sec 2\alpha-\tan 2\alpha$

Solution:

$$\text{L.H.S} = \frac{\cos\alpha-\sin\alpha}{\cos\alpha+\sin\alpha}$$

$$= \frac{\cos\alpha-\sin\alpha}{\cos\alpha+\sin\alpha}\times\frac{\cos\alpha-\sin\alpha}{\cos\alpha-\sin\alpha}$$

$$= \frac{(\cos\alpha-\sin\alpha)^2}{\cos^2\alpha-\sin^2\alpha}$$

$$= \frac{\cos^2\alpha+\sin^2\alpha-2\sin\alpha\cos\alpha}{\cos^2\alpha-\sin^2\alpha}$$

$$= \frac{1-\sin 2\alpha}{\cos 2\alpha}$$

$$= \frac{1}{\cos 2\alpha}-\frac{\sin 2\alpha}{\cos 2\alpha}$$

$$= \sec 2\alpha-\tan 2\alpha$$

$$= \text{R.H.S}$$

Hence proved

$$\frac{\cos\alpha+\sin\alpha}{\cos\alpha-\sin\alpha}=\sec 2\alpha-\tan 2\alpha$$

6. $\sqrt{\dfrac{1+\sin\alpha}{1-\sin\alpha}}=\dfrac{\sin\dfrac{\alpha}{2}+\cos\dfrac{\alpha}{2}}{\sin\dfrac{\alpha}{2}-\cos\dfrac{\alpha}{2}}$

Solution:

L.H.S $= \sqrt{\dfrac{1+\sin\alpha}{1-\sin\alpha}}$

$$= \sqrt{\frac{\sin^2\frac{\alpha}{2}+\cos^2\frac{\alpha}{2}+2\sin\frac{\cdots}{2}\cos\frac{\alpha}{2}}{\sin^2\frac{\alpha}{2}+\cos^2\frac{\alpha}{2}-2\sin\frac{\alpha}{2}.\cos\frac{\alpha}{2}}}$$

$$= \sqrt{\frac{\left(\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}\right)^2}{\left(\sin\frac{\alpha}{2}-\cos\frac{\alpha}{2}\right)^2}}$$

$$= \frac{\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}-\cos\frac{\alpha}{2}}$$

= R.H.S

Hence prove

$$= \sqrt{\frac{1+\sin\alpha}{1-\sin\alpha}} = \frac{\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}-\cos\frac{\alpha}{2}}$$

7. $\dfrac{\cos ec\theta+2\cos ec2\theta}{\sec\theta} = \cot\dfrac{\theta}{2}$

Solution

L.H.S $= \dfrac{\cos ec\theta+2\cos ec2\theta}{\sec\theta}$

$$= \frac{\frac{1}{\sin\theta}+\frac{2}{\sin 2\theta}}{\frac{1}{\cos\theta}}$$

$$= \cos\theta\left[\frac{1}{\sin\theta} + \frac{2}{\sin 2\theta}\right]$$

$$= \cos\theta\left[\frac{\sin 2\theta + 2\sin\theta}{\sin\theta.\sin 2\theta}\right]$$

$$= \cos\theta\left[\frac{2\sin\theta\cos\theta + 2\sin\theta}{\sin\theta.2\sin\theta\cos\theta}\right]$$

$$= \frac{2\sin\theta\left[\cos\theta + 1\right]}{2\sin^2\theta}$$

$$= \frac{\cos\theta + 1}{\sin\theta}$$

$$= \frac{\cos^2\frac{\theta}{2} - 1 + 1}{2\sin\frac{\theta}{2}.\cos\frac{\theta}{2}}$$

$$= \frac{2\cos^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}.\cos\frac{\theta}{2}}$$

$$= \frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}$$

$$= \cot\frac{\theta}{2}$$

$= \text{R.H.S}$

Hence proved

$$= \frac{\cos ec\theta + 2\cos ec\theta}{\sec\theta} = \cot\frac{\theta}{2}$$

8. 1 + tan α tan 2α = sec 2α

Solution:

$$-1+\tan\alpha\tan 2\alpha=\sec 2\alpha$$

L.H.S $=1+\tan\alpha\tan 2\alpha$

$$=1+\frac{\sin\alpha}{\cos\alpha}.\frac{\sin 2\alpha}{\cos 2\alpha}$$

$$=\frac{\cos\alpha\cos 2\alpha+\sin\alpha\sin 2\alpha}{\cos\alpha.\cos 2\alpha}$$

$$=\frac{\cos\alpha\left(\cos^2\alpha-\sin^2\alpha\right)+\sin\alpha\left(2\sin\alpha\cos\alpha\right)}{\cos\alpha\left[\cos^2\alpha-\sin^2\alpha\right]}$$

$$=\frac{\cos\alpha\left[\cos^2\alpha-\sin^2\alpha+2\sin^2\alpha\right]}{\cos\alpha\left[\cos^2\alpha-\sin^2\alpha\right]}$$

$$=\frac{\cos^2\alpha+\sin^2\alpha}{\cos^2\alpha-\sin^2\alpha}$$

$$=\frac{1}{\cos 2\alpha}$$

$=$ R.H.S

Hence; $1+\tan\alpha.\tan 2\alpha=\sec 2\alpha$

9. $\dfrac{2\sin\theta.\sin 2\theta}{\cos\theta+\cos 3\theta}=\tan 2\theta.\tan\theta$

Solution:

L.H.S $=\dfrac{2\sin\theta.\sin 2\theta}{\cos\theta+\cos 3\theta}$

$$=\frac{2\sin\theta\left(2\sin\theta.\cos\theta\right)}{\cos\theta+4\cos^3\theta-3\cos\theta}$$

$$= \frac{4\sin^3\theta.\cos\theta}{4\cos^3\theta - 2\cos\theta}$$

$$= \frac{4\sin^3\theta\cos\theta}{2\cos\theta\left(2\cos^2\theta - 1\right)}$$

$$= \frac{2\sin^2\theta}{2\cos^2\theta - 1}$$

$$= \frac{2\sin^2\theta.\cos\theta}{\cos\theta\left[\cos 2\theta\right]}$$

$$= \frac{\sin\theta}{\cos\theta}.\frac{2\sin\theta.\cos\theta}{\cos 2\theta}$$

$$= \tan\theta.\frac{\sin 2\theta}{\cos 2\theta}$$

$$= \tan\theta.\tan 2\theta$$

$= \text{R.H.S}$

Hence proved

$$\frac{2\sin\theta.\sin 2\theta}{\cos\theta + \cos\theta} = \tan 2\theta.\tan\theta$$

10. $\frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta} = 2$

Solution:

$$\text{L.H.S} = \frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta}$$

$$= \frac{\sin 3\theta.\cos\theta - \cos 3\theta.\sin\theta}{\sin\theta.\cos\theta}$$

$$= \frac{\sin\left(3\theta - \theta\right)}{\sin\theta.\cos\theta}$$

$$= \frac{2\sin 2\theta}{2\sin\theta.\cos\theta}$$

$$= \frac{2\sin 2\theta}{\sin 2\theta}$$

$$= 2$$

$$= \text{R.H.S}$$

Hence proved

$$\frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta} = 2$$

11. $\frac{\cos 3\theta}{\cos\theta} + \frac{\sin 3\theta}{\sin\theta} = 4\cos 2\theta$

Solution:

$$\text{L.H.S} = \frac{\cos 3\theta}{\cos\theta} + \frac{\sin 3\theta}{\sin\theta}$$

$$= \frac{\sin\theta.\cos 3\theta + \sin 3\theta.\cos\theta}{\cos\theta.\sin\theta}$$

$$= \frac{\sin(\theta + 3\theta)}{\cos\theta.\sin\theta}$$

$$= \frac{2\sin 4\theta}{2\sin\theta.\cos\theta}$$

$$= \frac{2.(2\sin 2\theta.\cos 2\theta)}{(\sin 2\theta)}$$

$$= 4\cos 2\theta$$

Hence proved

$$\frac{\cos 3\theta}{\cos\theta} + \frac{\sin 3\theta}{\sin\theta} = 4\cos 2\theta$$

12. $\dfrac{\tan\frac{\theta}{2}+\cot\frac{\theta}{2}}{\cot\frac{\theta}{2}-\tan\frac{\theta}{2}}=\sec\theta$

Solution:

$$\text{L.H.S} = \frac{\tan\frac{\theta}{2}+\cot\frac{\theta}{2}}{\cot\frac{\theta}{2}-\tan\frac{\theta}{2}}$$

$$= \frac{\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}+\frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}}{\frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}-\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}}$$

$$= \frac{\frac{\sin\frac{\theta}{2}.\sin\frac{\theta}{2}+\cos\frac{\theta}{2}.\cos\frac{\theta}{2}}{\cos\frac{\theta}{2}.\sin\frac{\theta}{2}}}{\frac{\cos\frac{\theta}{2}.\cos\frac{\theta}{2}-\sin\frac{\theta}{2}.\sin\frac{\theta}{2}}{\sin\frac{\theta}{2}.\cos\frac{\theta}{2}}}$$

$$= \frac{\sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}}{\cos^2\frac{\theta}{2}-\sin\frac{\theta}{2}}$$

$$= \frac{1}{\cos^2\theta}$$

$$= \sec^2\theta$$

$$= \text{R.H.S}$$

Hence proved $\dfrac{\tan\dfrac{\theta}{2}+\cot\dfrac{\theta}{2}}{\cot\dfrac{\theta}{2}-\tan\dfrac{\theta}{2}}=\sec^2\theta$

13. $\dfrac{\sin 3\theta}{\cos\theta}+\dfrac{\cos 3\theta}{\sin\theta}=2\cot 2\theta$

Solution:

L.H.S $=\dfrac{\sin 3\theta}{\cos\theta}+\dfrac{\cos 3\theta}{\sin\theta}$

$=\dfrac{\sin 3\theta.\sin\theta+\cos 3\theta.\cos\theta}{\cos\theta.\sin\theta}$

$=\dfrac{\cos(3\theta-\theta)}{\cos\theta.\sin\theta}$

$=\dfrac{2\cos 2\theta}{2\cos\theta.\sin\theta}$

$=\dfrac{2\cos 2\theta}{\sin 2\theta}$

$=2\cot 2\theta$

$=$R.H.S

Hence proved

$\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos 3\theta}{\sin\theta}=2\cot 2\theta$

14. Reduce $\sin^4\theta$ to an expression involving only function of multiplies of θ, raised to the first power.

Solution:

$$\sin^4 = \left(\sin^2\theta\right)^2$$

$$= \left(\frac{1-\cos 2\theta}{2}\right)^2$$

$$= \frac{1}{4}\left(1+\cos^2 2\theta - 2\cos 2\theta\right)$$

$$= \frac{1}{4} + \frac{1}{4}\cos^2 2\theta - \frac{1}{2}\cos 2\theta$$

$$= \frac{1}{4} + \frac{1}{4}\left[\frac{1+\cos 4\theta}{2}\right] - \frac{1}{2}\cos 2\theta$$

$$= \frac{1}{4} + \frac{1}{8} + \frac{1}{8}\cos 4\theta - \frac{1}{2}\cos 2\theta$$

$$= \frac{2+1}{8} + \frac{1}{8}\cos 4\theta - \frac{1}{2}\cos 4\theta$$

$$= \frac{3}{8} - \frac{1}{2}\cos 2\theta + \frac{1}{8}\cos 4\theta$$

$$\sin^4\theta = \frac{3-4\cos 2\theta+\cos 4\theta}{8}$$

Hence $\sin^4\theta = \dfrac{3-4\cos 2\theta+\cos 4\theta}{8}$

15. Find values of $\sin\theta$ and $\cos\theta$ without using table or calculator , when θ is

i. $18°$ **ii.** $36°$ **iii.** $54°$ **iv.** $72°$

Hence proved that: $\cos 36° \ \cos 72° \ \cos 144° = \frac{1}{16}$

i. $18°$

Solution:

Let θ 18°

We know that

$$5\theta = 90°$$

$$\Rightarrow \quad 2\theta + 3\theta = 90°$$

$$2\theta = 90° - 3\theta$$

$$\sin(2\theta) = \sin(90° - 3\theta)$$

$$2\sin\theta\cos\theta = \cos 3\theta$$

$$2\sin\theta\cos\theta = 4\cos^3\theta - 3\cos\theta$$

$$2\sin\theta.\cos\theta = \cos\theta\left(4\cos^2\theta - 3\right)$$

$$2\sin\theta = 4\cos^2\theta - 3$$

$$2\sin\theta = 4\left(1 - \sin^2\theta\right) - 3$$

$$2\sin\theta = 4 - 4\sin^2\theta - 3$$

$$4\sin^2\theta + 2\sin\theta - 1 = 0$$

Let $\sin\theta = y$

$$4y^2 + 4y - 1 = 0$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \quad y = \frac{-2 \pm \sqrt{4 + 16}}{8}$$

$$= \frac{-2 \pm \sqrt{20}}{8}$$

$$= \frac{-2 \pm \sqrt{20}}{8}$$

$$= \frac{-2 \pm 2\sqrt{5}}{8}$$

$$\Rightarrow \qquad y = \frac{-1 \pm \sqrt{5}}{4}$$

So, $$\sin\theta = \frac{-1 \pm \sqrt{5}}{4}$$

$$\sin 18° = \frac{-1 \pm \sqrt{5}}{4}$$

Neglect the –ve sign

Hence: $$\sin 18° = \frac{-1 \pm \sqrt{5}}{4}$$

and

$$\cos^2 18° = 1 - \sin^2 18°$$

$$= 1 - \left[\frac{-1 + \sqrt{5}}{4}\right]^2$$

$$\cos^2 18° = 1 - \left[\frac{1 + 5 - 2\sqrt{5}}{16}\right]$$

$$= \frac{16 - 6 + 2\sqrt{5}}{16}$$

$$= \frac{10 + 2\sqrt{5}}{16}$$

$$\cos^2 18° = \frac{5 + \sqrt{5}}{8}$$

$$\Rightarrow \qquad \cos^2 18° = \pm\sqrt{\frac{5 + \sqrt{5}}{8}}$$

neglect the negative sign

$$\cos 18° = \sqrt{\frac{5+\sqrt{5}}{8}}$$

ii. 36⁰

Let $\theta = 36^0$

$$5\theta = 180^0$$

$$2\theta + 3\theta = 180^0$$

$$\Rightarrow 2\theta = 180^0 - 3\theta$$

$$\sin(2\theta) = \sin(180 - 3\theta)$$

$$2\sin\theta.\cos\theta = \sin 3\theta$$

$$2\sin.\cos\theta = 3\sin\theta - 4\sin^2\theta$$

$$2\cos\theta = 3 - 4\sin^2\theta$$

$$2\cos\theta = 3 - 4\left(1 - \cos^2\theta\right)$$

$$2\cos\theta = 3 - 4 + 4\cos^2\theta$$

$$4\cos^2\theta - 1 - 2\cos\theta = 0$$

$$4\cos^2\theta - 2\cos\theta - 1 = 0$$

Let $\cos\theta = y$

$$4y^2 - 2y - 1 = 0$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(4)(-1)}}{2(4)}$$

$$= \frac{2 \pm \sqrt{4+16}}{8}$$

$$= \frac{2 \pm \sqrt{20}}{8}$$

$$= \frac{2 \pm 2\sqrt{5}}{8}$$

$$y = \frac{1 \pm \sqrt{5}}{4}$$

$$\Rightarrow \cos 36^0 = \frac{1 \pm \sqrt{5}}{4}$$

Neglect the '-ve' sign

Hence; $\cos 36^0 = \frac{1+\sqrt{5}}{4}$

$$\sin^2 36 + \cos^2 36^0 = 1$$

$$\sin^2 36^0 = 1 - \cos^2 .36^0$$

$$= 1 - \left[\frac{1+\sqrt{5}}{4}\right]^2$$

$$= 1 - \left[\frac{1+5+2\sqrt{5}}{16}\right]$$

$$= \frac{16 - \left[6 + 2\sqrt{5}\right]}{16}$$

$$= \frac{16 - 6 - 2\sqrt{5}}{16}$$

$$= \frac{10 - 2 - \sqrt{5}}{16}$$

$\Rightarrow \quad \sin 36^0 = \pm\sqrt{\frac{10-2\sqrt{5}}{16}}$

Neglect the '-ve' sign

$$\sin 36^0 = \frac{\sqrt{10-2\sqrt{5}}}{4}$$

Hence $\sin 36^0 = \frac{\sqrt{10-2\sqrt{5}}}{4}$

iii. 54^0

Let $\theta = 54^0$

$$54^0 = 90^0 - 36^0$$

$$\sin 54^0 = \sin(90^0 - 36^0)$$

$$= \cos 36^0$$

$$\Rightarrow \sin 54^0 = \frac{1+\sqrt{5}}{4}$$

Hence $\sin 54^0 = \frac{1+\sqrt{5}}{4}$

$$\cos\left(54^0\right) = \cos\left(90^0 - 36^0\right)$$

$$= \sin 36^0$$

$$= \frac{\sqrt{10-2\sqrt{5}}}{4}$$

$$\Rightarrow \cos 54^0 = \frac{\sqrt{10-2\sqrt{5}}}{4}$$

Hence; $\cos 54^0 = \frac{\sqrt{10-2\sqrt{5}}}{4}$

iv. 72^0

Let $\theta = 72^0$

$\Rightarrow$ $72^0 = 90^0 - 18^0$

$$\sin 72^0 = \sin\left(90^0 - 18^0\right)$$

$$\sin 72^0 = \cos 18^0$$

$$= \sqrt{\frac{5+\sqrt{5}}{8}}$$

Or

$$= \frac{\sqrt{10+2\sqrt{5}}}{4}$$

Hence $\sin 72^0 = \frac{\sqrt{10+2\sqrt{5}}}{4}$

And

$$72^0 = 90^0 - 18^0$$

$$\cos 72^0 = \cos\left(90^0 - 18^0\right)$$

$$\cos 72 = \sin 18^0$$

$$= \frac{-1+\sqrt{5}}{4}$$

$\Rightarrow$ $\cos 72^0 = \frac{-1+\sqrt{5}}{4}$

Hence $\cos 72^0 = \frac{-1+\sqrt{5}}{4}$

Therefore

$$\sin 18^0 = \frac{-1+\sqrt{5}}{4}; \cos 18^0 = \sqrt{\frac{5+\sqrt{5}}{8}}$$

$$\sin 36^0 = \frac{\sqrt{10-2\sqrt{5}}}{4}; \cos 36^0 = \frac{1+\sqrt{5}}{4}$$

$$\sin 54^0 = \frac{1+\sqrt{5}}{4}; \cos 54^0 = \frac{\sqrt{10-2\sqrt{5}}}{4}$$

$$\sin 72^0 = \frac{\sqrt{10+2\sqrt{5}}}{4}; \cos 72^0 = \frac{-1+\sqrt{5}}{4}$$

b. Prove that

$$\cos 36^0.\cos 72^0.\cos 108^0.\cos 144^0 = \frac{1}{16}$$

L.H.S $= \cos 36^0.\cos 72^0.\cos 108^0.\cos 144^0$

$$= \cos 36^0.\cos 72^0.\cos\left(180^0 - 72^0\right).\cos\left(180^0 - 36^0\right)$$

$$= \cos 36^0.\cos 72^0.\left(-\cos 72^0\right)\left(-\cos 36^0\right)$$

$$= \cos^2 36^0.\cos 72^0$$

$$= \left(\cos 36^0 - \cos 72\right)^2$$

$$= \left[\left[\frac{\sqrt{5}+1}{4}\right]\left[\frac{\sqrt{5}-1}{4}\right]\right]^2$$

$$= \left[\frac{5-1}{16}\right]^2$$

$$= \left[\frac{4}{16}\right]^2$$

$$= \left[\frac{1}{4}\right]^2$$

$$= \frac{1}{16} = R.H.S$$

Hence proved

$$\cos 36^0 . \cos 72^0 . \cos 108^0 . \cos 144^0 = \frac{1}{16}$$