Exercise 10.2

Q1. Prove that:

i.	$\sin(180^0+\theta)=-\sin\theta$	**ii.**	$\cos(180^0+\theta)=-\cos\theta$
iii.	$\tan(270^0-\theta)=\cot\theta$	**iv.**	$\cos(\theta-180^0)=-\cos\theta$
v.	$\cos(270^0+\theta)=\sin\theta$	**vi.**	$\sin(\theta+270^0)=-\cos\theta$
vii.	$\tan(180^0+\theta)=\tan\theta$	**viii.**	$\cos(360^0-\theta)=\cos\theta$

i. $\sin(180^0+\theta)=-\sin\theta$

Solution:

L.H.S $\sin(180^0+\theta)=\sin 180^0\cos\theta+\cos 180^0\sin\theta$

$=(0)\cos\theta+(-1)\sin\theta$

$=0-\sin\theta$

$=-\sin\theta$

=R.H.S

Hence proved

L.H.S = R.H.S

$\sin(180^0+\theta)=-\sin\theta$

ii. $\cos(180^0+\theta)=-\cos\theta$

Solution:

L.H.S $\cos(180^0+\theta)=\cos 180^0.\cos\theta-\sin 180^0\sin\theta$

$$= (-1)\cos\theta - (0)\sin\theta$$

$$= -\cos\theta - 0$$

$$= -\cos\theta$$

= R.H.S

Hence proved L.H.S = R.H.S

$$\cos(180^0 + \theta) = -\cos\theta$$

iii. $\tan(270^0 - \theta) = \cot\theta$

Solution:

L.H.S $\tan(270^0 - \theta) = \left(\dfrac{\tan 270^0 - \tan\theta}{1 + \tan 270^0 \tan\theta}\right)$

$$= \left(\frac{\frac{1}{0} - \tan\theta}{1 + \frac{1}{0}.\tan\theta}\right)$$

$$= \left(\frac{\frac{1-0}{0}}{\frac{0+\tan\theta}{0}}\right)$$

$$= \frac{1-0}{0+\tan\theta}$$

$$= \frac{1}{\tan\theta} = \cot\theta = R.H.S$$

Hence proved L.H.S = R.H.S

$$\tan(270^0 - \theta) = \cot\theta$$

iv. $\cos(\theta - 180^0) = -\cos\theta$

Solution:

L.H.S $\cos(\theta - 180^0) = \cos\theta.\cos 180^0 + \sin\theta.\sin 180^0$

$= \cos\theta(-1) + \sin\theta(0)$

$= -\cos\theta$

$= R.H.S$

Hence proved L.H.S = R.H.S

$\cos(\theta - 180^0) = -\cos\theta$

v. $\cos(270^0 + \theta) = \sin\theta$

Solution:

L.H.S $\cos(270^0 + \theta) = \cos 270^0.\cos\theta - \sin 270^0.\sin\theta$

$= (0)\cos\theta - (-1)\sin\theta$

$= 0 + \sin\theta$

$= R.H.S$

Hence proved L.H.S = R.H.S

$\cos(270^0 + \theta) = \sin\theta$

vi. $\sin(\theta + 270^0) = -\cos\theta$

Solution:

L.H.S $\cos(\theta + 270^0) = \sin\theta.\cos 270^0 - \sin 270^0.\cos\theta$

$= \sin\theta.(0) + (-1)\cos\theta$

$$= 0 - \cos\theta$$

$$= -\cos\theta$$

= R.H.S

Hence proved L.H.S=R.H.S

$$\cos(\theta + 270^0) = \cos\theta$$

vii. $\tan(180^0 + \theta) = \tan\theta$

Solution:

L.H.S $\tan(180^0 + \theta) = \left(\dfrac{\tan 180^0 + \tan\theta}{1 - \tan 180^0 . \tan\theta}\right)$

$$= \left(\frac{0 + \tan\theta}{1 - (0)\tan\theta}\right)$$

$$= \frac{\tan\theta}{1 - 0} = \tan\theta$$

$$= \tan\theta$$

$$= R.H.S$$

Hence proved L.H.S = R.H.S

$$\tan(180^0 + \theta) = \tan\theta$$

viii. $\cos(360^0 - \theta) = \cos\theta$

Solution:

L.H.S $\cos(360^0 - \theta) = \cos 360^0 . \cos\theta + \sin 360^0 . \sin\theta$

$$= (1)\cos\theta + 0\sin\theta$$

$= \cos\theta + 0$

$= \cos\theta$

$= R.H.S$

Hence proved L.H.S = R.H.S

$\cos(360^0 - \theta) = \cos\theta$

Q2. Find the values of the following:

i. $\sin 15^0$ **ii.** $\cos 15^0$ **iii.** $\tan 15^0$ **iv.** $\sec 15^0$

v. $\sin 105^0$ **vi.** $\cos 105^0$ **vii.** $\tan 105^0$ **viii.** $\sec 105^0$

$\left(H\text{int}: 15^0 = (45^0 - 30^0) and 105^0 = (60^0 + 45^0)\right)$

i. $\sin 15^0$

Solution:

$\Rightarrow \sin(60^0 - 45^0) = \sin 60^0.\cos 45^0 - \sin 45^0.\cos 60^0$

$= \frac{\sqrt{3}}{2}.\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}.\frac{1}{2}$

$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$

$\Rightarrow \sin 15^0 = \frac{\sqrt{3}-1}{2\sqrt{2}}$ is required value

ii. $\cos 15^0$

Solution:

$\Rightarrow \cos(60^0 - 45^0) = \cos 60^0.\cos 45^0 + \sin 60^0.\sin 45^0$

$$= \frac{1}{2} \cdot \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}}$$

$$= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$$

$$\Rightarrow \cos 15^0 = \frac{1+\sqrt{3}}{2\sqrt{2}}$$ is required value

iii. $\tan 15^0$

Solution:

$$\Rightarrow \tan 15^0 = \frac{\sin 15^0}{\cos 15^0}$$

$$= \frac{\frac{\sqrt{3}-t}{2\sqrt{2}}}{\frac{1+\sqrt{3}}{2\sqrt{2}}}$$

$$\Rightarrow \tan 15^0 = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$\cos 15^0 = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$ is required value

iv. $\sec 15^0$

Solution:

$$\sec 15^0 = \frac{1}{\cos 15^0} = \frac{1}{\frac{1+\sqrt{3}}{2\sqrt{2}}} = \frac{2\sqrt{2}}{1+\sqrt{3}}$$

Hence $\sin 15^0 = \frac{2\sqrt{2}}{1+\sqrt{3}}$ is required value

v. $\sin 105^0$

Solution:

$$\sin(60^0 + 45^0) = \sin 60^0 . \cos 45^0 + \cos 60^0 . \sin 45^0$$

$$= \frac{1}{2} . \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} . \frac{1}{\sqrt{2}}$$

$$= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$$

$$\Rightarrow \sin 105^0 = \frac{1+\sqrt{3}}{2\sqrt{2}}$$

$\sin 105^0 = \frac{1+\sqrt{3}}{2\sqrt{2}}$ is required value

vi. $\cos 105^0$

Solution:

$$\cos(60^0 + 45^0) = \cos 60^0 . \cos 45^0 - \sin 60^0 . \sin 45^0$$

$$= \frac{1}{2} . \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} . \frac{1}{\sqrt{2}}$$

$$= \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}$$

$$\Rightarrow \cos 105^0 = \frac{1-\sqrt{3}}{2\sqrt{3}}$$

$\cos 105^0 = \frac{1-\sqrt{3}}{2\sqrt{2}}$ is required value

vii. $\tan 105^0$

Solution:

$$\tan 105^0 = \frac{\sin 105^0}{\cos 105^0}$$

$$= \frac{\frac{1+\sqrt{3}}{2\sqrt{2}}}{\frac{1-\sqrt{3}}{2\sqrt{2}}}$$

$$\Rightarrow \tan 105^0 = \frac{1+\sqrt{3}}{1-\sqrt{2}}$$

$$\tan 105^0 = \frac{1+\sqrt{3}}{1-\sqrt{3}}$$ is required value

viii. $\sec 105^0$

Solution:

$$\sec 105^0 = \frac{1}{\cos 105^0} = \frac{1}{\frac{1-\sqrt{3}}{2\sqrt{2}}} = \frac{2\sqrt{2}}{1-\sqrt{3}}$$

$$\sec 105^0 = \frac{2\sqrt{2}}{1-\sqrt{3}}$$ is required value

Q3. Prove that:

i. $\sin(45^0+\alpha) = \frac{1}{\sqrt{2}}(\sin\alpha + \cos\alpha)$ **ii.** $\cos(\alpha + 45^0) = \frac{1}{\sqrt{2}}(\cos\alpha - \sin\alpha)$

i. $\sin(45^0+\alpha)=\frac{1}{\sqrt{2}}(\sin\alpha+\cos\alpha)$

Solution:

L.H.S = $\sin(45^0+\alpha)=\sin 45^0.\cos\alpha+\cos 45^0.\sin\alpha$

$$=\frac{1}{\sqrt{2}}\cos\alpha+\frac{1}{\sqrt{2}}\sin\alpha$$

$$=\frac{1}{\sqrt{2}}(\cos\alpha+\sin\alpha)$$

$$=\frac{1}{\sqrt{2}}(\sin\alpha+\cos\alpha)$$

$$=R.H.S$$

Hence proved

$$\sin(45^0+\alpha)=\frac{1}{\sqrt{2}}(\sin\alpha+\cos\alpha)$$

ii. $\cos(\alpha+45^0)=\frac{1}{\sqrt{2}}(\cos\alpha-\sin\alpha)$

Solution:

L.H.S $\cos(\alpha+45^0)=(\cos\alpha.\cos 45^0-\sin\alpha.\sin 45^0)$

$$=\cos\alpha.\frac{1}{\sqrt{2}}-\sin\alpha.\frac{1}{\sqrt{2}}$$

$$=\frac{1}{\sqrt{2}}(\cos\alpha-\sin\alpha)$$

$$=R.H.S$$

Hence proved;

$$\cos\left(\alpha+45^0\right)=\left(\cos\alpha.\cos 45^0-\sin\alpha.\sin 45^0\right)$$

Q4. Prove that:

i. $\tan\left(45^0+A\right)\tan\left(45^0-A\right)=1$ **ii.** $\tan\left(\frac{\pi}{4}-\theta\right)+\tan\left(\frac{3\pi}{4}+\theta\right)=0$

iii. $\sin\left(\theta+\frac{\pi}{6}\right)+\cos\left(\theta+\frac{\pi}{3}\right)=\cos\theta$ **iv.** $\frac{\sin\theta-\cos\theta\tan\frac{\theta}{2}}{\cos\theta+\sin\theta\tan\frac{\theta}{2}}=\tan\frac{\theta}{2}$

v. $\frac{1-\tan\theta\tan\phi}{1+\tan\theta\tan\phi}=\frac{\cos(\theta+\phi)}{\cos(\theta-\phi)}$

i. $\tan\left(45^0+A\right)\tan\left(45^0-A\right)=1$

Solution:

L.H.S $=\tan\left(45^0+A\right).\tan\left(45^0-A\right)$

$=\frac{\tan 45^0+\tan A}{1-\tan 45^0.\tan A}.\frac{\tan 45^0-\tan A}{1+\tan 45^0.\tan A}$

$=\frac{1+\tan A}{1-\tan A}.\frac{1-\tan A}{1+\tan A}$ $\left[\because \tan 45^0=1\right]$

$=1$

= R.H.S

Hence proved; $\tan\left(45^0+A\right)\tan\left(45^0-A\right)=1$

ii. $\tan\left(\frac{\pi}{4}-\theta\right)+\tan\left(\frac{3\pi}{4}+\theta\right)=0$

Solution:

L.H.S $= \tan\left(\frac{\pi}{4}-\theta\right)+\tan\left(\frac{3\pi}{4}+\theta\right)$

$$= \frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}.\tan\theta}+\frac{\tan\frac{3\pi}{4}+\tan\theta}{1-\tan\frac{3\pi}{4}.\tan\theta}$$

$$= \frac{1-\tan\theta}{1+(1)\tan\theta}+\frac{(-1)+\tan\theta}{1-(-1)\tan\theta}$$

$$= \frac{1-\tan\theta}{1+\tan\theta}+\frac{-1-\tan\theta}{1+\tan\theta}$$

$$= \frac{1-\tan\theta-1+\tan\theta}{1+\tan\theta}$$

$$= \frac{0}{1+\tan\theta} = 0 = R.H.S$$

Hence proved; $\tan\left(\frac{\pi}{4}-\theta\right)+\tan\left(\frac{3\pi}{4}+\theta\right)=0$

iii. $\sin\left(\theta+\frac{\pi}{6}\right)+\cos\left(\theta+\frac{\pi}{3}\right)=\cos\theta$

Solution:

L.H.S $= \sin\left(\theta+\frac{\pi}{6}\right)+\cos\left(\theta+\frac{\pi}{3}\right)$

$$= \sin\theta.\cos\frac{\pi}{6}+\cos\theta.\sin\frac{\pi}{6}+\cos\theta.\cos\frac{\pi}{3}-\sin\theta.\sin\frac{\pi}{3}$$

$$= \sin\theta\left(\frac{\sqrt{3}}{2}\right)+\cos\theta\left(\frac{1}{2}\right)+\cos\theta\left(\frac{1}{2}\right).\sin\theta\left(\frac{\sqrt{3}}{2}\right)$$

$$= 2\cos\theta\left(\frac{1}{2}\right)$$

$$= \cos\theta = R.H.S$$

Hence proved; $\sin\left(\theta+\frac{\pi}{6}\right)+\cos\left(\theta+\frac{\pi}{3}\right)=\cos\theta$

iv. $$\frac{\sin\theta-\cos\theta\tan\frac{\theta}{2}}{\cos\theta+\sin\theta\tan\frac{\theta}{2}}=\tan\frac{\theta}{2}$$

Solution:

L.H.S $$= \frac{\sin\theta-\cos\theta\tan\frac{\theta}{2}}{\cos\theta+\sin\theta\tan\frac{\theta}{2}}$$

$$= \frac{\sin\theta-\cos\theta\frac{\sin\theta/2}{\cos\theta/2}}{\cos\theta+\sin\theta\frac{\sin\theta/2}{\cos\theta/2}}$$

$$= \frac{\frac{\sin\theta.\cos\theta/2-\cos\theta.\sin\theta/2}{\cos\theta/2}}{\frac{\cos\theta\cos\theta/2+\sin\theta.\sin\theta/2}{\cos\theta/2}}$$

$$= \frac{\sin(\theta-\theta/2)}{\cos(\theta-\theta/2)}$$

$$= \frac{\sin(\theta/2)}{\cos(\theta/2)}$$

Hence proved; $$\frac{\sin\theta-\cos\theta\tan\frac{\theta}{2}}{\cos\theta+\sin\theta\tan\frac{\theta}{2}}=\tan\frac{\theta}{2}$$

v. $\dfrac{1-\tan\theta\tan\phi}{1+\tan\theta\tan\phi}=\dfrac{\cos(\theta+\phi)}{\cos(\theta-\phi)}$

Solution:

$$\text{L.H.S} = \frac{1-\tan\theta\tan\phi}{1+\tan\theta\tan\phi}$$

$$= \frac{1-\dfrac{\sin\theta}{\cos\theta}.\dfrac{\sin\phi}{\cos\phi}}{1+\dfrac{\sin\theta}{\cos\theta}.\dfrac{\sin\phi}{\cos\phi}}$$

$$= \frac{\dfrac{\cos\theta.\cos\phi-\sin\theta.\sin\phi}{\cos\theta.\cos\phi}}{\dfrac{\cos\theta.\cos\phi+\sin\theta.\sin\phi}{\cos\theta.\cos\phi}}$$

$$= \frac{\cos\theta.\cos\phi-\sin\theta.\sin\phi}{\cos\theta.\cos\phi+\sin\theta.\sin\phi}$$

$$= \frac{\cos(\theta+\phi)}{\cos(\theta-\phi)}$$

$= \text{R.H.S}$

Hence proved; $\dfrac{1-\tan\theta\tan\phi}{1+\tan\theta\tan\phi}=\dfrac{\cos(\theta+\phi)}{\cos(\theta-\phi)}$

Q5. Show that: $\cos(\alpha+\beta)\cos(\alpha-\beta)=\cos^2\alpha-\sin^2\beta=\cos^2\beta-\sin^2\alpha$

Solution:

$\cos(\alpha+\beta)\cos(\alpha-\beta)=\cos^2\alpha-\sin^2\beta=\cos^2\beta-\sin^2\alpha$

$$\text{L.H.S} = \cos(\alpha+\beta).\cos(\alpha-\beta)$$

$=\left[\cos\alpha.\cos\beta-\sin\alpha.\sin\beta\right]\left[\cos\alpha.\cos\beta+\sin\alpha.\sin\beta\right]$

$=\cos^2\alpha.\cos^2\beta-\sin^2\alpha.\sin^2\beta$

$=\cos^2\alpha(1-\sin^2\beta)-\sin^2\beta\left(1-\cos^2\alpha\right)$

$=\cos^2\alpha-\cos^2\alpha.\sin^2\beta-\sin^2\beta+\sin^2\beta.\cos^2\alpha$

$=\cos^2\alpha-\sin^2\beta$

= R.H.S

Hence Proved $=\cos(\alpha+\beta)\cos(\alpha-\beta)=\cos^2\alpha-\sin^2\beta$

Now take L.H.S $=\cos(\alpha+\beta)\cos(\alpha-\beta)$

$=\left[\cos\alpha.\cos\beta-\sin\alpha.\sin\beta\right]\left[\cos\alpha.\cos\beta+\sin\alpha.\sin\beta\right]$

$=\cos^2\alpha.\cos^2\beta-\sin^2\alpha.\sin^2\beta$

$=\cos^2\alpha.\cos^2\beta-\sin^2\alpha.\sin^2\beta$

$=(1-\sin^2\alpha).\cos^2\beta-\sin^2\alpha\left(1-\cos^2\beta\right)$

$=\cos^2\beta-\sin^2\alpha.\cos^2\beta-\sin^2\alpha+\sin^2\alpha.\cos^2\beta$

$=\cos^2\beta-\sin^2\alpha$

= R.H.S

Hence proved $\cos(\alpha+\beta)\cos(\alpha-\beta)=\cos^2\beta-\sin^2\alpha=\cos^2\alpha-\sin^2\beta$

Q6. Show that: $\dfrac{(\sin\alpha+\beta)+\sin(\alpha+\beta)}{\cos(\alpha+\beta)+\cos(\alpha-\beta)}=\tan\alpha$

Solution:

$$\frac{(\sin\alpha+\beta)+\sin(\alpha+\beta)}{\cos(\alpha+\beta)+\cos(\alpha-\beta)}=\tan\alpha$$

L.H.S = $\frac{(\sin\alpha+\beta)+\sin(\alpha-\beta)}{\cos(\alpha+\beta)+\cos(\alpha-\beta)}$

$$=\frac{\sin\alpha.\cos\beta+\cos\alpha.\sin\beta+\sin.\cos\beta-\cos\alpha\sin\beta}{\cos\alpha\cos\beta-\sin\alpha.\sin\beta+\cos\alpha.\cos\beta+\sin\alpha.\sin\beta}$$

$$=\frac{2\sin\alpha\cos\beta}{2\cos\alpha\cos\beta}$$

$$=\frac{\sin\alpha}{\cos\alpha}$$

$$=\tan\alpha$$

=R.H.S

Hence; $\frac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{\cos(\alpha+\beta)+\sin(\alpha-\beta)}=\tan\alpha$

Q7. Show that:

i. $\cot(\alpha+\beta)=\frac{\cot\alpha\cot\beta-1}{\cot\alpha+\cot\beta}$ **ii.** $\cot(\alpha-\beta)=\frac{\cot\alpha\cot\beta+1}{\cot\beta-\cot\alpha}$

iii. $\frac{\tan\alpha+\tan\beta}{\tan\alpha-\tan\beta}=\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}$

i. $\cot(\alpha+\beta)=\frac{\cot\alpha\cot\beta-1}{\cot\alpha+\cot\beta}$

Solution

L.H.S $=\cot(\alpha+\beta)$

$$= \frac{\cos(\alpha+\beta)}{\sin(\alpha+\beta)}$$

$$= \frac{\cos\alpha.\cos\beta - \sin\alpha.\sin\beta}{\sin\alpha.\cos\beta + \cos\alpha.\sin\beta A}$$

$$= \frac{(\sin\alpha\sin\beta)\left[\frac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta} - 1\right]}{(\sin\alpha\sin\beta)\left[\frac{\cos\alpha}{\cos\alpha} + \frac{\cos\beta}{\cos\alpha}\right]}$$

$$= \frac{\cot\alpha\cos\beta - 1}{\cot\alpha + \cot\beta}$$

= R.H.S.

Hence proved

$$\cot(\alpha+\beta) = \frac{\cot\alpha\cot\beta}{\cot\alpha + \cot\beta}$$

ii. $\cot(\alpha-\beta) = \frac{\cot\alpha\cot\beta + 1}{\cot\alpha - \cot\beta}$

Solution

L.H.S. = $\cot(\alpha-\beta)$

$$= \frac{\cos(\alpha-\beta)}{\sin(\alpha-\beta)}$$

$$= \frac{\cos\alpha.\cos\beta + \sin\alpha\sin\beta}{\sin\alpha.\cos\beta - \cos\alpha\sin\beta}$$

$$= \frac{\sin\alpha\sin\beta\left(\frac{\cos\alpha\cos\beta}{\sin\alpha\cos\alpha} + 1\right)}{\sin\alpha\sin\beta\left(\frac{\cos\beta}{\sin\beta} - \frac{\cos\alpha}{\sin\beta}\right)}$$

$$= \frac{\cot\alpha\cot\beta + 1}{\cot\beta - \cot\alpha}$$

= R.H.S.

Hence proved

$$\cot(\alpha - \beta) = \frac{\cot\alpha\cot\beta + 1}{\cot\beta - \cot\alpha}$$

iii. $= \dfrac{\tan\alpha + \tan\beta}{\tan\alpha - \tan\beta} = \dfrac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}$

Solution.

L.H.S. $= \dfrac{\tan\alpha + \tan\beta}{\tan\alpha - \tan\beta}$

$$= \frac{\frac{\sin\alpha}{\cos\alpha} + \frac{\sin\beta}{\cos\beta}}{\frac{\sin\alpha}{\cos\alpha} - \frac{\sin\beta}{\cos\beta}}$$

$$= \frac{\frac{\sin\alpha.\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\frac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\cos\alpha\cos\beta}}$$

$$= \frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}$$

= R.H.S.

Hence prove $\dfrac{\tan\alpha + \tan\beta}{\tan\alpha - \tan\beta} = \dfrac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}$

Q8. **If sin** $a = \frac{4}{5}$ **and cos** $\beta \frac{40}{41}$**, where 0 <a<** $\frac{\pi}{2}$

Show that sin $(\alpha - \beta) = \frac{133}{205}$

Solution

$$\sin\alpha = \frac{4}{5}$$

$$\Rightarrow \cos\alpha = \pm\sqrt{1-\sin^2\alpha}$$

$$\therefore \cos^2\alpha + \sin^2\theta = 1$$
$$\cos^2\alpha = 1-\sin^2\alpha$$
$$\cos\alpha = \sqrt{1-\sin^2\alpha}$$

$$= \pm\sqrt{1-\left(\frac{4}{5}\right)^2}$$
$$= \pm\sqrt{1-\frac{16}{25}}$$
$$= \pm\sqrt{\frac{25-16}{25}}$$
$$= \pm\sqrt{\frac{9}{25}}$$
$$= \pm\frac{3}{5}$$

Neglect the'-ve' sign.

$$\Rightarrow \cos\alpha = \frac{3}{5}$$

$$\cos\beta = \frac{40}{41}$$

$$\sin\beta = \pm\sqrt{1-\left(\frac{40}{41}\right)^2}$$

$$= \pm\sqrt{1 - \frac{1600}{1681}}$$

$$= \pm\sqrt{\frac{1681 - 1600}{1681}}$$

$$= \pm\sqrt{\frac{81}{1681}}$$

$$= \pm\frac{9}{41}$$

neglect the '-ve' sigh

$$\sin\beta = \frac{9}{41}$$

So

L.H.S $\sin(\alpha - \beta) \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$$= \frac{4}{5}.\frac{40}{41} - \frac{3}{5}.\frac{9}{41}$$

$$= \frac{160}{205} - \frac{27}{205}$$

$$= \frac{160 - 27}{205}$$

$$= \frac{133}{205}$$

= R.H.S

Hence proved

$$\sin(\alpha - \beta) = \frac{133}{205}$$

Q9. If sin $\alpha = \frac{4}{5}$ **and** $\sin\beta = \frac{12}{13}$ **where** $\frac{\pi}{2} < \alpha < \pi$ **and** $\frac{\pi}{2} < \beta < \pi.$ **find**

i. $\sin(\alpha+\beta)$ **ii.** $\cos(\alpha+\beta)$

iii. $\tan(\alpha+\beta)$ **iv.** $\sin(\alpha-\beta)$

v. $\cos(\alpha-\beta)$ **vi.** $\tan(\alpha-\beta)$

In which quadrants do the terminal sides of the angles of measures $(\alpha+\beta)$ **and** $(\alpha-\beta)$ **lie?**

Solution

$$\sin\alpha = \frac{4}{5}$$

$$\therefore \cos^2\alpha + \sin^2\theta = 1$$
$$\cos^2\alpha = 1 - \sin^2\alpha$$
$$\cos\alpha = \sqrt{1-\sin^2\alpha}$$

$$\Rightarrow \cos\alpha = \pm\sqrt{1-\sin^2\alpha}$$

$$= \pm\sqrt{1-\left(\frac{4}{5}\right)^2}$$

$$= \pm\sqrt{1-\frac{16}{25}}$$

$$= \pm\sqrt{\frac{25-16}{25}}$$

$$= \pm\sqrt{\frac{9}{25}}$$

$$\Rightarrow \cos\alpha = \pm\frac{3}{5}$$

The angle of terminal arm lies in $\prod$ **quad cos** $\alpha = \frac{-3}{5}$

$$\sin\beta = \frac{12}{13}$$

$$\Rightarrow \cos\beta = \sqrt{1-(\sin\beta)^2}$$

$$= \sqrt{1-\left(\frac{12}{13}\right)^2}$$

$$= \sqrt{1-\frac{144}{169}}$$

$$= \pm\sqrt{\frac{169-144}{169}}$$

$$= \pm\sqrt{\frac{25}{169}}$$

$$\Rightarrow \cos\beta = \pm\frac{5}{13}$$

The angle of terminal arm lie in Π quad $\cos\beta = \frac{5}{13}$

i. $\sin(\alpha+\beta)$

Solution.

$$\sin(\alpha+\beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$$

$$= \frac{4}{5}\left(\frac{-5}{13}\right) + \left(\frac{-3}{5}\right)\left(\frac{12}{13}\right)$$

$$= \frac{-20}{65} - \frac{36}{65}$$

$$\Rightarrow \sin(\alpha+\beta) = \frac{-20-36}{65} = \frac{-56}{65}$$

Hence sin $(\alpha+\beta) = \frac{-56}{65}$

ii. $\cos(\alpha+\beta)$

Solution.

$$\cos(\alpha+\beta)=\cos\alpha.\cos\beta-\sin\alpha.\sin\beta$$

$$=\left(\frac{-3}{5}\right)\left(\frac{-5}{13}\right)-\left(\frac{4}{5}\right)\left(\frac{12}{13}\right)$$

$$=\frac{15}{65}-\frac{48}{65}$$

$$=\frac{15-48}{65}$$

$$\Rightarrow \cos(\alpha+\beta)=\frac{-33}{65}$$

Hence

$$\cos(\alpha+\beta)=\frac{-33}{65}$$

iii. $\tan(\alpha+\beta)$

Solution.

$$\tan(\alpha+\beta)=\frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}$$

$$\Rightarrow \tan(\alpha+\beta)=\frac{\frac{-56}{65}}{\frac{-33}{65}}=\frac{56}{33}$$

Hence, $\tan(\alpha+\beta)=\frac{56}{33}$

iv. $\sin(\alpha-\beta)$

Solution:

$$\sin(\alpha-\beta)=\sin\alpha.\cos\alpha-\cos\alpha.\sin\beta$$

$$=\left(\frac{4}{5}\right)\left(\frac{-5}{13}\right)-\left(\frac{-3}{5}\right)\left(\frac{12}{13}\right)$$

$$=\frac{-20}{65}+\frac{36}{65}$$

$$\Rightarrow \sin(\alpha-\beta)=\frac{16}{65}$$

Hence

$$\sin(\alpha-\beta)=\frac{16}{65}$$

v. $\cos(\alpha-\beta)$

Solution:

$$\cos(\alpha-\beta)=\cos\alpha.\cos\beta+\sin\alpha.\sin\beta$$

$$=\left(\frac{-3}{5}\right)\left(\frac{-5}{13}\right)+\left(\frac{4}{5}\right)\left(\frac{12}{13}\right)$$

$$=\frac{15}{65}+\frac{48}{65}$$

$$=\frac{15+48}{65}$$

$$\Rightarrow \cos(\alpha-\beta)=\frac{63}{65}$$

Hence;

$$\cos(\alpha-\beta)=\frac{63}{65}$$

vi. $\tan(\alpha-\beta)$

Solution:

$$\tan(\alpha-\beta)=\frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)}$$

$$=\frac{\frac{16}{65}}{\frac{63}{65}}$$

$$\Rightarrow \tan(\alpha-\beta)=\frac{16}{63}$$

Hence; $\tan(\alpha-\beta)=\frac{16}{63}$

b. $\sin(\alpha+\beta)=\frac{-56}{65}<0$

$$\cos(\alpha+\beta)=\frac{-33}{65}<0$$

$$\tan(\alpha+\beta)=\frac{56}{33}>0$$

Thus, the terminal side of $(\alpha+\beta)$ is in III quad.

$$\sin(\alpha-\beta)=\frac{16}{65}>0$$

$$\cos(\alpha-\beta)=\frac{63}{65}>0$$

$$\tan(\alpha-\beta)=\frac{16}{32}>0$$

Thus, the terminal side of $(\alpha-\beta)$ is in I quad.

Q10. Find $\sin(\alpha+\beta)$ **and** $\cos(\alpha+\beta)$ **given that**

i. $\tan\alpha = \frac{3}{4}, \cos\beta = \frac{5}{13}$ **and neither the terminal side of the angle of measure** α **nor that of** β **is in the** I **quadrant.**

ii. $\tan\alpha = -\frac{15}{18}, \sin\beta = \frac{7}{25}$ **and neither the terminal side of the angle of measure** α **nor that of** β **is in the** IV **quadrant.**

i. $\tan\alpha = \frac{3}{5}; \cos\beta = \frac{5}{13}$

Solution:

We know that

$$\sec^2\alpha = 1 + \tan^2\alpha$$

$$= 1 + \left(\frac{3}{5}\right)^2$$

$$= 1 + \frac{9}{25}$$

$$= \frac{25+9}{25}$$

$$= \frac{34}{25}$$

$$\Rightarrow \qquad \sec\alpha = \pm\frac{\sqrt{34}}{5}$$

The angle of terminal are lies in III quad.

$$\Rightarrow \qquad \cos\alpha = \frac{1}{\sec\alpha} = -\frac{5}{\sqrt{34}}$$

So, $$\sin\alpha = \tan\alpha\cos\alpha$$

$$\sin = \frac{\cdots}{5}\left(\frac{-5}{\sqrt{34}}\right) = \frac{-3}{\sqrt{34}}$$

$$\cos\beta = \frac{5}{13}$$

$\Rightarrow$ $$\sin\beta = \pm\sqrt{1-\cos^2\beta}$$

$$= \pm\sqrt{1-\left(\frac{5}{13}\right)^2}$$

$$= \pm\sqrt{1-\frac{25}{169}}$$

$$= \pm\sqrt{1-\frac{25}{169}}$$

$$= \pm\sqrt{\frac{144}{169}}$$

$\Rightarrow$ $$\sin\beta = \pm\frac{12}{13}$$

The angle of terminal arm lies in IV quad

$$\sin\beta = -\frac{12}{13}$$

Therefore

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$= \left(\frac{-3}{\sqrt{34}}\right)\left(\frac{5}{13}\right) + \left(\frac{-5}{\sqrt{34}}\right)\left(\frac{-12}{13}\right)$$

$$= \frac{-15}{13\sqrt{34}} + \frac{60}{13\sqrt{34}}$$

$$= \frac{60-15}{13\sqrt{34}} = \frac{45}{13\sqrt{34}}$$

Hence $\sin(\alpha+\beta)=\dfrac{45}{13\sqrt{34}}$

ii. $\tan\alpha=-\dfrac{15}{8};\sin\beta=-\dfrac{7}{25}$

Solution:

$$\therefore \sec^2\alpha=1+\tan^2\alpha$$

$$=1+\left(\frac{-15}{8}\right)^2$$

$$=1+\frac{225}{64}$$

$$=\frac{64+225}{64}$$

$$\sec\alpha=\frac{289}{64}$$

$$\Rightarrow \sec\alpha=\pm\frac{\sqrt{298}}{64}$$

$$\Rightarrow \sec\alpha=\pm\frac{17}{8}$$

The angle of terminal arm lies in II quad.

$$\sec\alpha=\pm\frac{17}{8}$$

$$\Rightarrow \cos\alpha=\frac{1}{\sec\alpha}=\frac{1}{-17/8}=-\frac{8}{17}$$

So, $\sin\alpha=\cos\alpha\tan\alpha$

$$=\frac{-8}{17}\times\frac{-15}{8}=\frac{15}{17}$$

$$\Rightarrow \sin\alpha = \frac{15}{17}$$

$$\Rightarrow \sin\beta = \frac{-7}{25}$$

$$\cos\beta = \pm\sqrt{1-\sin^2\beta}$$

$$= \pm\sqrt{1-\left(\frac{-7}{25}\right)^2}$$

$$= \pm\sqrt{1-\frac{49}{625}}$$

$$= \pm\sqrt{\frac{625-49}{625}}$$

$$\Rightarrow \sin\beta = \pm\frac{24}{25}$$

$$\Rightarrow \cos\beta = \pm\sqrt{\frac{576}{625}}$$

The angles of terminal arm lie in III quad

$$\cos\beta = -\frac{24}{25}$$

$$\Rightarrow \sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$= \frac{15}{17}\left(\frac{-24}{25}\right) + \left(\frac{-8}{17}\right)\cdot\left(\frac{-7}{25}\right)$$

$$= \frac{-360}{425} + \frac{56}{425}$$

$$= \frac{-360+56}{425}$$

$$= -\frac{304}{425}$$

$$\sin(\alpha+\beta)=\frac{-304}{425}$$

And

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

$$=\frac{-8}{17}\times\frac{-24}{25}-\frac{15}{17}\times\left(\frac{-7}{25}\right)$$

$$=\frac{192}{425}+\frac{105}{425}$$

$$=\frac{192+105}{425}$$

$$=\frac{297}{425}$$

$$\Rightarrow\cos(\alpha+\beta)=\frac{297}{425}$$

Hence

$$\cos(\alpha+\beta)=\frac{297}{425}$$

Q11. Prove that: $\frac{\cos 8^0-\sin 8^0}{\cos 8^0+\sin 8^0}=\tan 37^0$

Solution:

$$\text{L.H.S}\quad=\frac{\cos 8^0-\sin 8^0}{\cos 8^0+\sin 8^0}$$

$$=\frac{\cos 8^0\left(\frac{1-\sin 8^0}{\cos 8^0}\right)}{\cos 8^0+\left(\frac{1+\sin 8^0}{\cos 8^0}\right)}$$

$$= \frac{1 - \tan 8^0}{1 + \tan 8^0}$$

$$= \frac{\tan 45^0 - \tan 8^0}{\tan 45^0 + \tan 8^0} = \left[\therefore \tan 45^0 = 1\right]$$

$$= \tan\left(45^0 - 8\right)^0$$

$$= \tan 37^0$$

= R.H.S

Hence proved

$$\frac{\cos 8^0 - \sin 8^0}{\cos 8^0 + \sin 8^0} = \tan 37^0$$

Q12. If α, β, γ are the angles of a triangle ABC, show that

$$\cot \alpha/2 + \cot \beta/2 + \cot \frac{\gamma}{2} = \cot \frac{\alpha}{2} \cot \frac{\beta}{2} \cot \frac{\gamma}{2}$$

Solution:

$$\Rightarrow \alpha + \beta + \gamma = 180^0$$

$$\alpha + \beta = 180^0 - \gamma$$

$$\Rightarrow \frac{\alpha + \beta}{2} = \frac{180^0 - \gamma}{2} = 90^0 - \frac{\gamma}{2}$$

$$= \frac{\alpha}{2} + \frac{\beta}{2} = 90^0 - \frac{\gamma}{2}$$

So, $\cot\left(\frac{\alpha}{2} + \frac{\beta}{2}\right) = \cot\left(90^0 - \frac{\gamma}{2}\right)$

$$\frac{\cot\frac{\alpha}{2}\cot\frac{\beta}{2}-1}{\cot\frac{\alpha}{2}+\cot\frac{\beta}{2}}=\frac{\cot\left(90^{0}\right).\cot\frac{\gamma}{2}+1}{\cot\left(90^{0}\right)-\cot\left(\frac{\gamma}{2}\right)}$$

$$\frac{\cot\frac{\alpha}{2}\cot\frac{\beta}{2}-1}{\cot\frac{\alpha}{2}+\cot\frac{\beta}{2}}=\frac{(0).\cot\frac{\gamma}{2}+1}{(0)-\cot\frac{\gamma}{2}}$$

$$\frac{\cot\frac{\alpha}{2}\cot\frac{\beta}{2}-1}{\cot\frac{\alpha}{2}+\cot\frac{\beta}{2}}=\frac{1}{\cot\frac{\gamma}{2}}$$

$$\Rightarrow \cot\frac{\gamma}{2}\left(\cot\frac{\alpha}{2}\cot\frac{\beta}{2}-1\right)=\left(\cot\frac{\alpha}{2}+\cot\frac{\beta}{2}\right)$$

$$\cot\frac{\alpha}{2}\cot\frac{\beta}{2}\cot\frac{\gamma}{2}-\cot\frac{\gamma}{2}=\cot+\frac{\alpha}{2}\cot\frac{\beta}{2}$$

Hence proved

$$\cot\frac{\alpha}{2}.\cot\frac{\beta}{2}\cot\frac{\gamma}{2}=\cot\frac{\alpha}{2}.\cot\frac{\beta}{2}\cot\frac{\gamma}{2}$$

$$\cot\frac{\alpha}{2}+\cot\frac{\beta}{2}+\cot\frac{\gamma}{2}=\cot\frac{\alpha}{2}\cot\frac{\beta}{2}\cot\frac{\gamma}{2}$$

Q13. If $\alpha+\beta+\gamma=180^{0}$, show that

$$\cot\alpha\cot\beta+\cot\beta\cot\gamma+\cot\gamma\cot\alpha=1$$

Solution:

$\alpha+\beta+\gamma$ are the angles of triangle

Then

$\alpha+\beta+\gamma=180^{0}$

$$\Rightarrow \alpha + \beta = 180^0 - \gamma$$

$$\Rightarrow \cot(\alpha + \beta) = \cot(180^0 - \gamma)$$

$$\frac{\cot\alpha\cot\beta - 1}{\cot\alpha + \cot\beta} = \frac{\cot 180^0 \cot\gamma - 1}{\cot 180^0 + \cot\gamma}$$

$$\frac{\cot\alpha\cot\beta - 1}{\cot\alpha + \cot\beta} = \frac{\frac{\cot\gamma - 1}{0}}{\frac{1+0}{0}}$$

$$\frac{\cot\alpha\cot\beta - 1}{\cot\alpha + \cot\beta} = -\cot\gamma$$

$$\cot\alpha\cot\beta - 1 = -\cot\gamma\left(\cot\alpha + \cot\beta\right)$$

$$\cot\alpha\cot\beta - 1 = -\cot\alpha\cot\gamma - \cot\beta\cot\gamma$$

$$\Rightarrow \cot\alpha\cot\beta + \cot\beta\cot\gamma + \cot\gamma\cot\alpha = 1$$

Hence proved,

$$\cot\alpha.\cot\beta + \cot\beta.\cot\gamma + \cot\gamma.\cot\alpha = 1$$

Q14. Express the following in the form r $\sin(\theta+\phi)$, where terminal sides of the angles of measures θ and ϕ are in the first quadrant:

i. $12\sin\theta + 5\cos\theta$ **ii.** $3\sin\theta - 4\cos\theta$ **iii.** $\sin\theta - \cos\theta$

iv. $5\sin\theta - 4\cos\theta$ **v.** $\sin\theta + \cos\theta$ **vi.** $3\sin\theta - 5\cos\theta$

i. $12\sin\theta + 5\cos\theta$

Solution:

Let $\gamma\sin\phi = 12$ and r $\cos\phi = 5$

$$r = \sqrt{(12)^2 + (5)^2}$$

$$= \sqrt{144 + 25}$$

$$= \sqrt{169}$$

$$r = 13$$

And $\dfrac{r\sin\phi}{r\cos\phi} = \dfrac{12}{5}$

$$\tan\phi = \frac{12}{5}$$

$\Rightarrow$ $\phi = \tan^{-1}\left(\dfrac{12}{5}\right)$

$$\Rightarrow 12\sin\theta + 5\cos\theta = r\sin\phi\sin\theta + r\cos\theta\cos\phi$$

$$= r\left[\cos\theta\cos\phi + \sin\theta\sin\phi\right]$$

$$= r\left[\cos(\theta - \phi)\right]$$

$$\Rightarrow 12\sin\theta + 5\cos\theta = 13\cos(\theta - \phi); \phi = \tan^{-1}\left(\frac{12}{5}\right)$$

ii. $3\sin\theta - 4\cos\theta$

Solution:

Let $3 = r\cos\phi$; $-4 = r\sin\phi$

$$r = (r\cos\phi)^2 + (r.\sin\phi)^2 = (3)^2 + (-4)^2$$

$$= r^2\cos^2\phi + r^2\sin^2\phi = 9 + 16$$

$$r^2\left(\sin + \cos^2\phi\right) = 25$$

$$r^2 = 25$$

$\Rightarrow \qquad r = \pm\sqrt{25} = \pm 5$

Neglect '-ve' sign

$$r = 5$$

And

$$\frac{r\sin\phi}{r\cos\phi} = \frac{-4}{3}$$

$$\tan\phi = \frac{-4}{3}$$

$\Rightarrow \qquad \phi = \tan^{-1}\left(-4/3\right)$

So, $3\sin\theta - 4\cos\theta = r\cos\phi\sin\theta + r\sin\phi\cos\theta$

$$= r\left[\cos\phi\sin\theta + \sin\phi\cos\theta\right]$$

$$= r\left[\sin\left(\phi+\theta\right)\right]$$

$$= 5\sin\left(\phi+\theta\right); \phi = \tan^{-1}\left(-4/3\right)$$

Hence,

$$3\sin\theta - 4\cos\theta = 5\sin\left(\theta+\phi\right); \phi = \tan^{-1}\left(-4/3\right)$$

iii. $\sin\theta - \cos\theta$

Solution:

Let $\qquad 1 = r\cos\phi \qquad ; -1 = r\sin\phi$

$$\left(r\cos\phi\right)^2 + \left(r.\sin\phi\right)^2 = \left(1\right)^2 + \left(-1\right)^2$$

$$= r^2\cos^2\phi + r^2\sin^2\phi = 1+1$$

$$r^2\left(\cos^2\phi+\sin^2\phi\right)=2$$

$\Rightarrow$ $$r^2=2$$

$$r=\pm\sqrt{2}$$

Neglect '-ve' sign

$$r=\sqrt{2}$$

$$\frac{r\sin\phi}{r\cos\phi}=\frac{1}{-1}$$

$\Rightarrow$ $$\tan\phi=-1$$

$$\phi=\tan^{-1}(-1)=3\pi/4$$

So, $$\sin\theta-\cos\theta=r\cos\phi\sin\theta+r\sin\phi\cos\theta$$

$$=r\left[\sin\phi\cos\theta+\sin\phi\cos\theta\right]$$

$$=\sqrt{2}\left[\sin(\theta+\phi)\right]$$

$$=\sqrt{2}\sin(\theta+\phi);\qquad \phi=3\pi/4$$

Hence,

$$3\sin\theta-\cos\theta=\sqrt{2}\sin(\theta+\phi);\phi=3\pi/4$$

iv. $5\sin\theta-4\cos\theta$

Solution:

Let $5=r\cos\theta$; $-4=r\sin\phi$

$$(r\cos\phi)^2+(r\sin\phi)^2=(5)^2+(-4)^2$$

$$=r^2\cos^2\phi+r^2\sin^2\phi=25+16$$

$$r^2\left(\cos^2\phi+\sin^2\phi\right)=41$$

$\Rightarrow$ $$r^2=41$$

$$r=\pm\sqrt{41}$$

Neglect '-ve' sign

$$\gamma=\sqrt{41}$$

$$\frac{r\sin\phi}{r\cos\phi}=\frac{-4}{5}$$

$\Rightarrow$ $$\tan\phi=-4/5$$

$$\phi=\tan^{-1}\left(-4/5\right)$$

So, $5\sin\theta-4\cos\theta=r\cos\phi\sin\theta+r\sin\phi\cos\theta$

$$=r\left[\sin\theta\cos\phi+\cos\theta\sin\theta\right]$$

$$=r\left[\sin\left(\theta+\phi\right)\right]$$

$$=\sqrt{41}\left[\sin\left(\theta+\phi\right)\right]$$

$\Rightarrow 5\sin\theta-4\cos\theta\sqrt{41}\sin\left(\theta+\phi\right);\phi=\tan^{-1}\left(-4/5\right)$

Hence,

$$5\sin\theta-4\cos\theta=\sqrt{41}\left(\theta+\phi\right);\phi=\tan^{-1}\left(-4/5\right)$$

v. $\sin\theta+\cos\theta$

Solution:

Let $1=r\cos\phi$; $1=r\sin\phi$

$$(r\cos\phi)^2+(r\sin\phi)^2=(1)^2+(1)^2$$

$$=r^2\cos^2\phi+r^2\sin^2\phi=1+1$$

$$=r^2\left(\cos^2\phi+\sin^2\phi\right)=2$$

$$\Rightarrow \qquad r^2=2$$

$$r=\pm\sqrt{2}$$

Neglect '-ve' sign

$$r=\sqrt{2}$$

$$\frac{r\sin\phi}{r\cos\phi}=\frac{1}{1}$$

$$\tan\phi=1$$

$$\Rightarrow \qquad \phi=\tan^{-1}(1)=\pi/4$$

So, $\quad r\sin\theta+r\cos\theta=r\cos\phi\sin\theta+r\sin\phi\cos\theta$

$$=r[\sin\theta\cos\phi+\cos\theta\sin\theta]$$

$$=r[\sin(\theta+\phi)]$$

$$=\sqrt{2}\sin(\phi+\theta)$$

$$\Rightarrow \qquad \sin\theta+4\cos\theta=\sqrt{2}\sin(\phi+\theta); \qquad \phi=-\pi/4$$

Hence, $\quad \sin\theta+\cos\theta=\sqrt{2}\sin(\phi+\theta); \qquad \phi=-\pi/4$

Domain of θ

$$\theta\in R\wedge\theta\neq(2n+1)\frac{\pi}{2};n\in Z$$

vi. $3\sin\theta - 5\cos\theta$

Solution:

Let $3 = r\cos\phi$; $-5 = r\sin\phi$

$$(r\cos\phi)^2 + (r\sin\phi)^2 = (3)^2 + (-5)^2$$

$$r^2\cos^2\phi + r^2\sin^2\phi = 9 + 25$$

$$r^2\cos^2\phi + r^2\sin\phi = 9 + 25$$

$\Rightarrow$ $$r^2\left(\cos^2 + \sin^2\phi\right) = 34$$

$$r^2 = 34$$

Neglect '-ve' sign

$\Rightarrow$ $$r = \pm\sqrt{34}$$

$$\frac{r\sin\phi}{r\cos\phi} = \frac{-5}{3}$$

$$\tan\phi = -5/3$$

$\Rightarrow$ $$\phi = \tan^{-1}\left(-5/3\right)$$

So, $$3\sin\theta - 5\cos\theta = r\cos\phi\sin\theta + r\sin\phi\cos\theta$$

$$= r\left[\sin\phi\sin\theta + \sin\phi\cos\theta\right]$$

$$= r\left[\sin(\theta + \phi)\right]$$

$$= \sqrt{34}\sin(\theta + \phi)$$

$\Rightarrow$ $$3\sin\theta - 5\cos\theta = \sqrt{34}\sin(\theta + \varphi); \varphi = \tan^{-1}\left(-5/3\right)$$

Hence,

$$3\sin\theta - 5\cos\theta = \sqrt{34}\sin(\varphi + \theta); \varphi = \tan^{-1}\left(-5/3\right)$$