Chapter 10

Trigonometric Identities

Exercise 10.1

1. Without using tables, find the value of:

i. $\sin(-780^0)$ **ii.** $\cot(-855^0)$ **iii.** $\csc(2040^0)$

iv. $\sec(-960^0)$ **v.** $\tan(1110^0)$ **vi.** $\sin(-300^0)$

Solutions:

i. $\sin(-780^0)$

$$\sin(-780^0) = -\sin(780^0)$$

$$= -\sin(2\times 360 + 60^0)$$

$$= -\sin 60^0$$

$$= -\frac{\sqrt{3}}{2}$$

$$= -\frac{\sqrt{3}}{2}$$

Hence, $\sin(-780^0) = -\frac{\sqrt{3}}{2}$

ii. $\cot(-855^0)$

$$\cot(-855^0) = -\cot(855^0)$$

$$= -\cot(2\times 360^0 + 135^0)$$

$$= -\cot(135^0)$$

$$= -\cot(90 + 45^0)$$

$$= -\tan 45^0$$

$$= -1$$

Hence, $\cot(-855^0) = -1$

iii. $\csc(2040^0)$

$$\csc(2040^0) = \cos ec(1800 + 780^0)$$

$$= \cos ec(5 \times 360^0 + 240^0)$$

$$= \cos ec(240^0)$$

$$= \frac{1}{\sin(240^0)}$$

$$= \frac{1}{\sin(180^0 + 60^0)}$$

$$= \frac{1}{-\sin 60^0}$$

$$= \frac{1}{-\frac{\sqrt{3}}{2}}$$

$$\Rightarrow \cos ec(2040^0) = -\frac{2}{\sqrt{3}}$$

Hence, $\cos ec(2040^0) = -\frac{2}{\sqrt{3}}$

iv. $\sec(-960^0)$

$$\sec(-960^0) = -\sec(960^0)$$

$$= \sec(720^0 + 240^0)$$

$$= \sec(2 \times 360^0 + 240^0)$$

$$= \sec(240^0)$$

$$= \sec(180^0 + 60^0)$$

$$= \sec 60^0$$

$$= -\frac{1}{\cos 60^0} = \frac{1}{1/2} = -2$$

Hence, $\sec(-960^0) = -2$

v. $\tan(1110^0)$

$$\tan(1110^0) = \tan(3(360^0) + 30^0)$$

$$\tan 30^0 = \tan \frac{1}{\sqrt{3}}$$

Hence, $\tan(1110^0) = \frac{1}{\sqrt{3}}$

vi. $\sin(-300^0)$

$$\sin(-300^0) = -\sin(300^0)$$

$$= -\sin(360^0 - 60^0)$$

$$= -\left[-\sin 60^0\right]$$

$$= \sin 60^0$$

$$= \frac{\sqrt{3}}{2}$$

Hence, $\sin(-300^0) = \frac{\sqrt{3}}{2}$

2. Express each of the following as a trigonometric function of an angle of positive degree measure of less than 45°.

i. $\sin 196^0$ **ii.** $\cos 147^0$ **iii.** $\sin 319^0$ **iv.** $\cos 254^0$

v. $\tan 294^0$ **vi.** $\cos 728^0$ **vii.** $\sin(-625^0)$ **viii.** $\cos(-435^0)$

xi. $\sin 150^0$

Solutions:

i. $\sin 196^0$

$$\sin(196^0) = \sin(180^0 + 16^0)$$

$$= -\sin 16^0$$

Hence,

$$\sin 196^0 = -\sin 16^0$$

ii. $\cos 147^0$

$$\cos 147^0 = \cos(180^0 - 33^0)$$

$$= \cos 33^0$$

Hence,

$$\cos(147^0) = -\cos 33^0$$

iii. $\sin 319^0$

$$\sin 319^0 = \sin(360^0 - 41^0)$$

$$= -\sin(-41^0)$$

$$= -\sin 41^0$$

Hence,

$$\sin 319^0 = -\sin 41^0$$

iv. $\cos 254^0$

$$\cos 254^0 = \cos(270^0 - 16^0)$$

$$= -\sin 16^0$$

Hence,

$$\cos 254^0 = -\sin 16^0$$

v. $\tan 294^0$

$$\tan 294^0 = \tan(270^0 + 24^0)$$

$$= -\cot 24^0$$

Hence,

$$\tan 294^0 = -\cot 24^0$$

vi. $\cos 728^0$

$$\cos 728^0 = \cos(720^0 + 8^0)$$

$$= \cos(2 \times 360^0 + 8^0)$$

$$= \cos 8^0$$

Hence,

$$\cos 728^0 = \cos 8^0$$

vii. $\sin(-625^0)$

$$\sin(-625^0) = -\sin(625^0)$$

$$= -\sin(360^0 + 265^0)$$

$$= -\sin(265^0)$$

$$= -\sin(270^0 - 5^0)$$

$$= \cos 5^0$$

Hence,

$$\sin(-625^0) = \cos 5^0$$

viii. $\cos(-435^0)$

$$\cos(-435^0) = \cos(435^0)$$

$$= \cos(360^0 + 75^0)$$

$$= \cos 75^0$$

$$= \cos(90^0 - 15^0)$$

$$= \sin(15^0)$$

Hence,

$$\cos(-435^0) = \sin(15^0)$$

ix. $\sin 150^0$

$$\sin 150^0 = \sin(180^0 - 30^0)$$

$$= \cos 30^0$$

Hence,

$$\sin 150^0 = \sin 30^0$$

3. Prove the following.

i. $\sin(180^0 + \alpha)\sin(90 - \alpha) = -\sin\alpha\cos\alpha$

ii. $\sin 780^0 \sin 480^0 + \cos 120^0 \sin 30^0 = \frac{1}{2}$

iii. $\cos 306^0 + \cos 234^0 + \cos 162^0 + \cos 18^0 = 0$

iv. $\cos 330^0 \sin 600^0 + \cos 120^0 \sin 150^0 = -1$

i. $\sin(180^0 + \alpha)\sin(90 - \alpha) = -\sin\alpha\cos\alpha$

L.H.S $= \sin(180^0 + \alpha)\sin(90 - \alpha)$

$$= \cos\alpha\left[-\sin\alpha\right]$$

$$= -\sin\alpha.\cos\alpha$$

Hence proved L.H.S = R.H.S

$$\sin(180^0 + \alpha)\sin(90 - \alpha) = -\sin\alpha\cos\alpha$$

ii. $\sin 780^0 \sin 480^0 + \cos 120^0 \sin 30^0 = \frac{1}{2}$

L.H.S $= \sin 780^0 \sin 480^0 + \cos 120^0 \sin 30^0$

$$= \sin(720^0 + 60^0).\sin(360^0 + 120^0) + \cos 120^0.\sin 30^0$$

$$= \sin 60^0.\sin 120^0 + \cos 120^0.\sin 30^0$$

$$= \frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2} + \left(-\frac{1}{2}\right)\frac{1}{2}$$

$$= \frac{3}{4} - \frac{1}{4} = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2} = R.H.S$$

Hence proved L.H.S = R.H.S

$$\sin 780^0 \sin 480^0 + \cos 120^0 \sin 30^0 = \frac{1}{2}$$

iii. $\cos 306^0 + \cos 234^0 + \cos 162^0 + \cos 18^0 = 0$

L.H.S = $\cos 306^0 + \cos 234^0 + \cos 162^0 + \cos 18^0$

$$= \cos(270^0 + 36^0) + \cos(270^0 - 36^0) + \cos(180^0 - 18^0) + \cos 18^0$$

$$= \sin 36^0 - \sin 36^0 - \cos 18^0 + \cos 18^0$$

= 0 + 0 = 0 = R.H.S

Hence proved L.H.S = R.H.S

$$\cos 306^0 + \cos 234^0 + \cos 162^0 + \cos 18^0 = 0$$

vi. $\cos 330^0 \sin 600^0 + \cos 120^0 \sin 150^0 = -1$

L.H.S = $\cos 330^0 \sin 600^0 + \cos 120^0 \sin 150^0$

$$= \cos(360^0 + 30^0).\sin(540^0 + 60^0) + \left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$= \cos(30^0).\sin(360^0 + 180^0 + 60^0) - \frac{1}{4}$$

$$=\left(\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right)-\frac{1}{4}$$

$$=-\frac{3}{4}-\frac{1}{4}=\frac{-3-1}{4}=\frac{-4}{4}=-1=R.H.S$$

Hence proved, L.H.S = R.H.S

$$\cos 330^0 \sin 600^0+\cos 120^0 \sin 150^0=-1$$

4. Prove that

i. $$\frac{\sin^2(\pi+\theta)\tan\left(\frac{3\pi}{2}+\theta\right)}{\cot^2\left(\frac{3\pi}{2}-\theta\right)\cos^2(\pi-\theta)\cos ec(2\pi-\theta)}=\cos\theta$$

ii. $$\frac{\cos(90^0+\theta)\sec(-\theta)\tan(180^0-\theta)}{\sec(360^0-\theta)\sin(180^0+\theta)\cot(90^0-\theta)}=-1$$

i. $$\frac{\sin^2(\pi+\theta)\tan\left(\frac{3\pi}{2}+\theta\right)}{\cot^2\left(\frac{3\pi}{2}-\theta\right)\cos^2(\pi-\theta)\cos ec(2\pi-\theta)}=\cos\theta$$

L.H.S = $$\frac{\sin^2(\pi+\theta)\tan\left(\frac{3\pi}{2}+\theta\right)}{\cot^2\left(\frac{3\pi}{2}-\theta\right)\cos^2(\pi-\theta)\cos ec(2\pi-\theta)}$$

$$=\frac{\sin^2\theta.(-\cot\theta)}{(-\tan\theta)^2.\cos^2\theta(-\cos ec\theta)}$$

$$=\frac{-\cot\theta.\sin^2\theta(-\sin\theta)}{\tan^2\theta.\cos^2\theta}$$

$$= \frac{\cot\theta(\tan^2\theta)(\sin\theta)}{(\tan^2\theta)}$$

$$= \frac{\cos\theta}{\sin\theta}.\sin\theta = \cos\theta = R.H.S$$

Hence proved;

$$\frac{\sin^2(\pi+\theta)\tan\left(\frac{3\pi}{2}+\theta\right)}{\cot^2\left(\frac{3\pi}{2}-\theta\right)\cos^2(\pi-\theta)\cos ec(2\pi-\theta)} = \cos\theta$$

ii. $$\frac{\cos(90^0+\theta)\sec(-\theta)\tan(180^0-\theta)}{\sec(360^0-\theta)\sin(180^0+\theta)\cot(90^0-\theta)} = -1$$

L.H.S = $$\frac{\cos(90^0+\theta)\sec(-\theta)\tan(180^0-\theta)}{\sec(360^0-\theta)\sin(180^0+\theta)\cot(90^0-\theta)}$$

$$= \frac{[-\sin\theta].\sec\theta.[-\tan\theta]}{\sec\theta.[-\sin\theta].\tan\theta}$$

$$= -1 = R.H.S$$

Hence proved

$$\frac{\cos(90^0+\theta)\sec(-\theta)\tan(180^0-\theta)}{\sec(360^0-\theta)\sin(180^0+\theta)\cot(90^0-\theta)} = -1$$

5. **If α, β, γ are the angles of a triangle ABC, then prove that**

i. $\sin(\alpha+\beta) = \sin\gamma$ **ii.** $\cos\left(\frac{\alpha+\beta}{2}\right) = \sin\frac{\gamma}{2}$

iii. $\cos(\alpha+\beta) = -\sin\gamma$ **iv.** $\tan(\alpha+\beta) + \tan\gamma = 0$

i. $\sin(\alpha+\beta)=\sin\gamma$

L.H.S $=$ $\sin(\alpha+\gamma)$

$\alpha+\beta+\gamma=$ 180^0 $\Rightarrow \alpha+\beta=180^0-\gamma$

$=\sin(180^0-\gamma)$

$=\sin\gamma=R.H.S$

Hence proved

$\sin(\alpha+\beta)=\sin\gamma$

ii. $\cos\left(\frac{\alpha+\beta}{2}\right)=\sin\frac{\gamma}{2}$

L.H.S $=$ $\cos\left(\frac{\alpha+\beta}{2}\right)$

$\alpha+\beta+\gamma=180^0$ $\Rightarrow \alpha+\beta=180^0-\gamma$

$\frac{\alpha+\beta}{2}=\frac{180-\gamma}{2}=\left(90-\frac{\gamma}{2}\right)$

$\cos\left(\frac{\alpha+\beta}{2}\right)=\cos\left(90-\frac{\gamma}{2}\right)$

$=\sin\left(\frac{\gamma}{2}\right)=R.H.S$

Hence proved

$\cos\left(\frac{\alpha+\beta}{2}\right)=\sin\frac{\gamma}{2}$

iii. $\cos(\alpha+\beta)=-\sin\gamma$

L.H.S $= \cos(\alpha+\beta)$

$\alpha+\beta+\gamma=180^0$

$\Rightarrow \alpha+\beta=180^0-\gamma$

$\cos(\alpha+\beta)=\cos(180^0-\gamma)$

$=-\sin\gamma = R.H.S$

Hence proved

$\cos(\alpha+\beta)=-\sin\gamma$

vi. $\tan(\alpha+\beta)+\tan\gamma=0$

L.H.S $= \tan(\alpha+\beta)+\tan\gamma$

$\alpha+\beta+\gamma=180^0$ $\Rightarrow \alpha+\beta=180^0-\gamma$

$\tan(\alpha+\beta)=\tan(180^0-\gamma)$

$=-\tan\gamma$

$\Rightarrow \tan(\alpha+\beta)+\tan(\gamma)=-\tan\gamma+\tan\gamma$

$=0=R.H.S$

Hence proved

$\tan(\alpha+\beta)+\tan\gamma=0$