Exercise 1.3

Q1. Graph the following number on the complex plane.

i) 2+3i ii) 2-3i iii) -2-3i iv) -2+3i

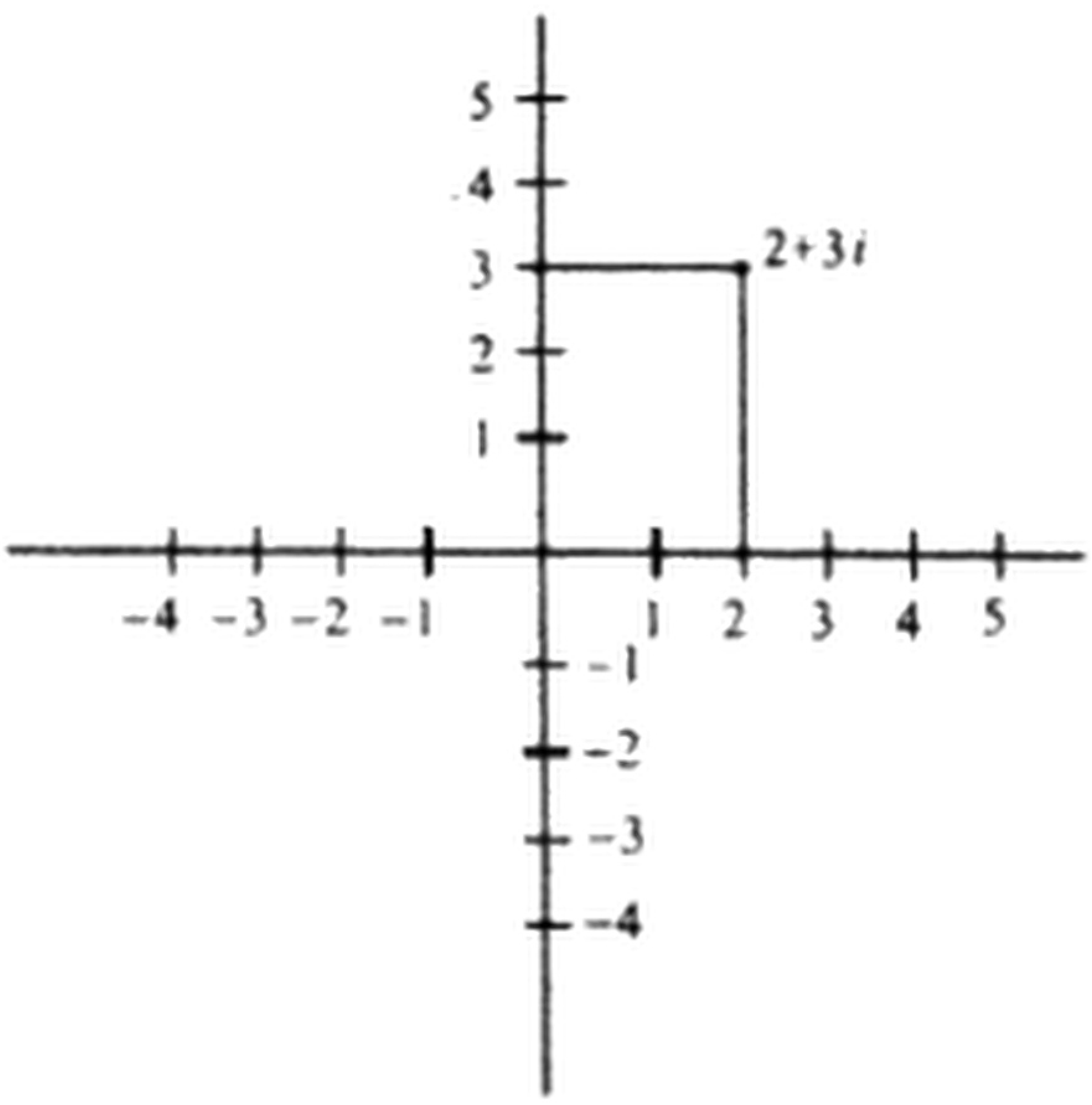
v) -6 vi) i vii) $\frac{3}{5}-\frac{4}{5}i$ viii) -5-6i

Solution:

i) 2+3i

Point on x-axis = 2 units

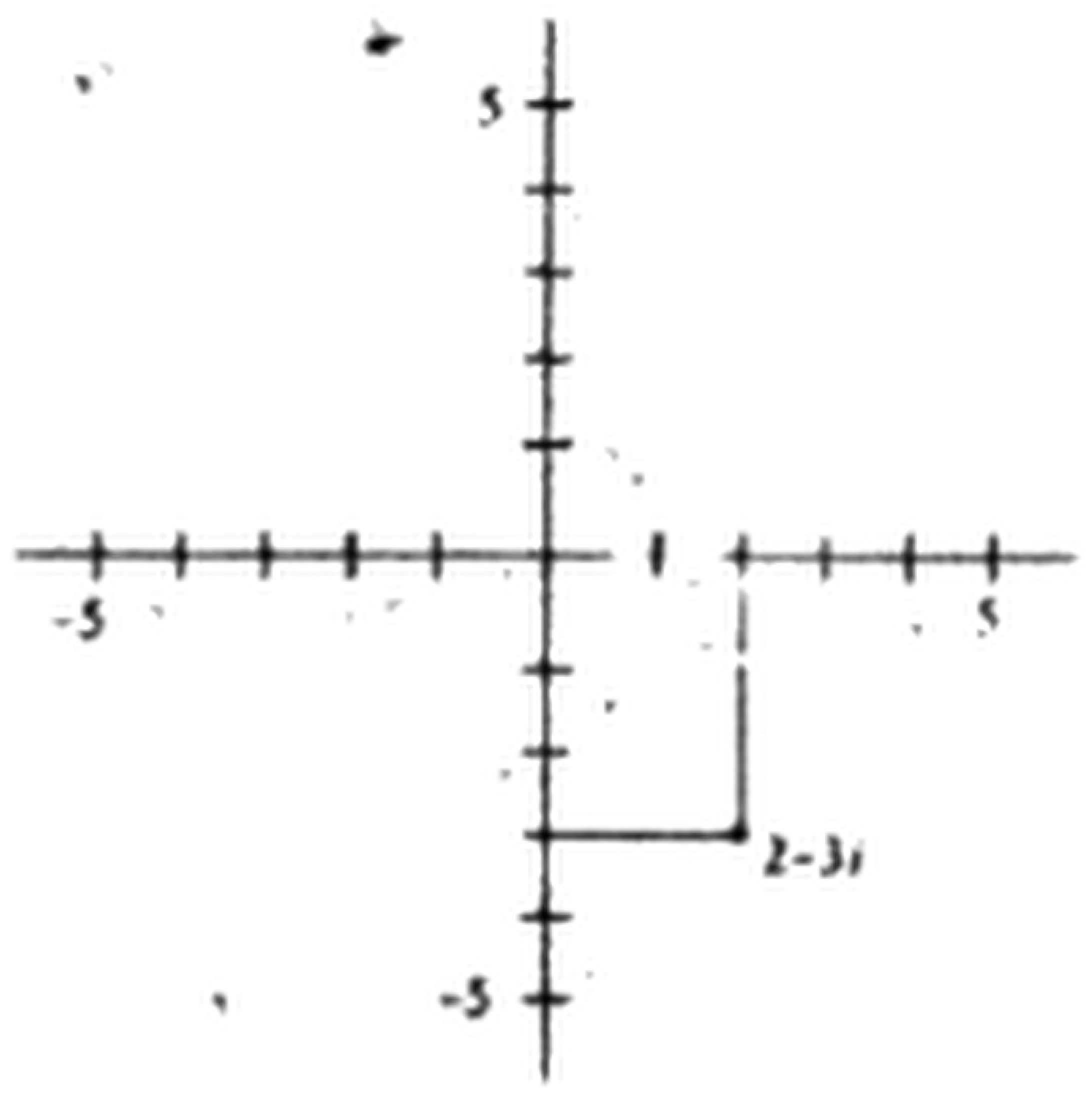
Point along y-axis = 3 units



ii) 2-3i

Point on x-axis = 2 units

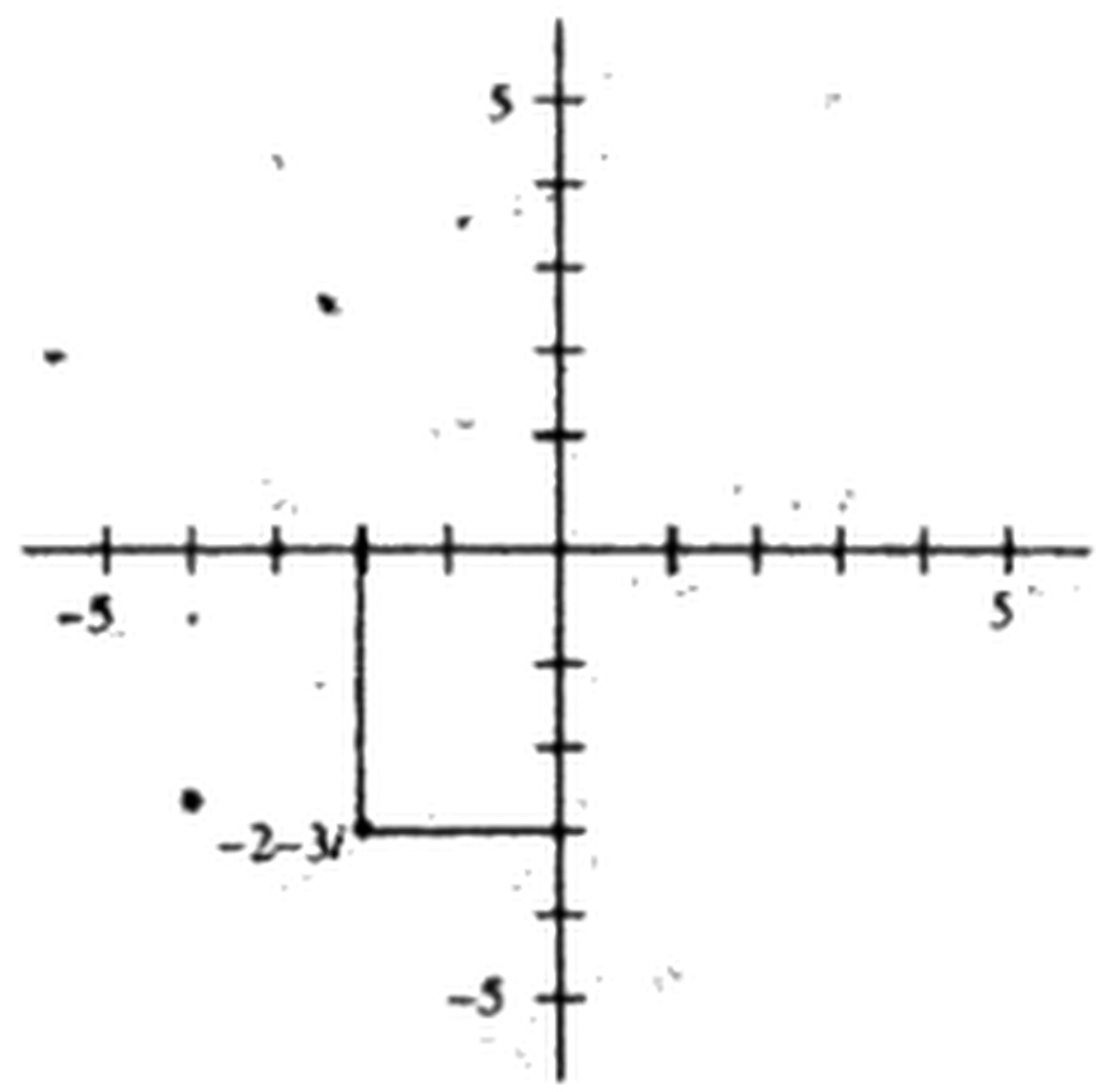
Point along y-axis = -3 units



iii) -2-3i

Point on x-axis = -2 units

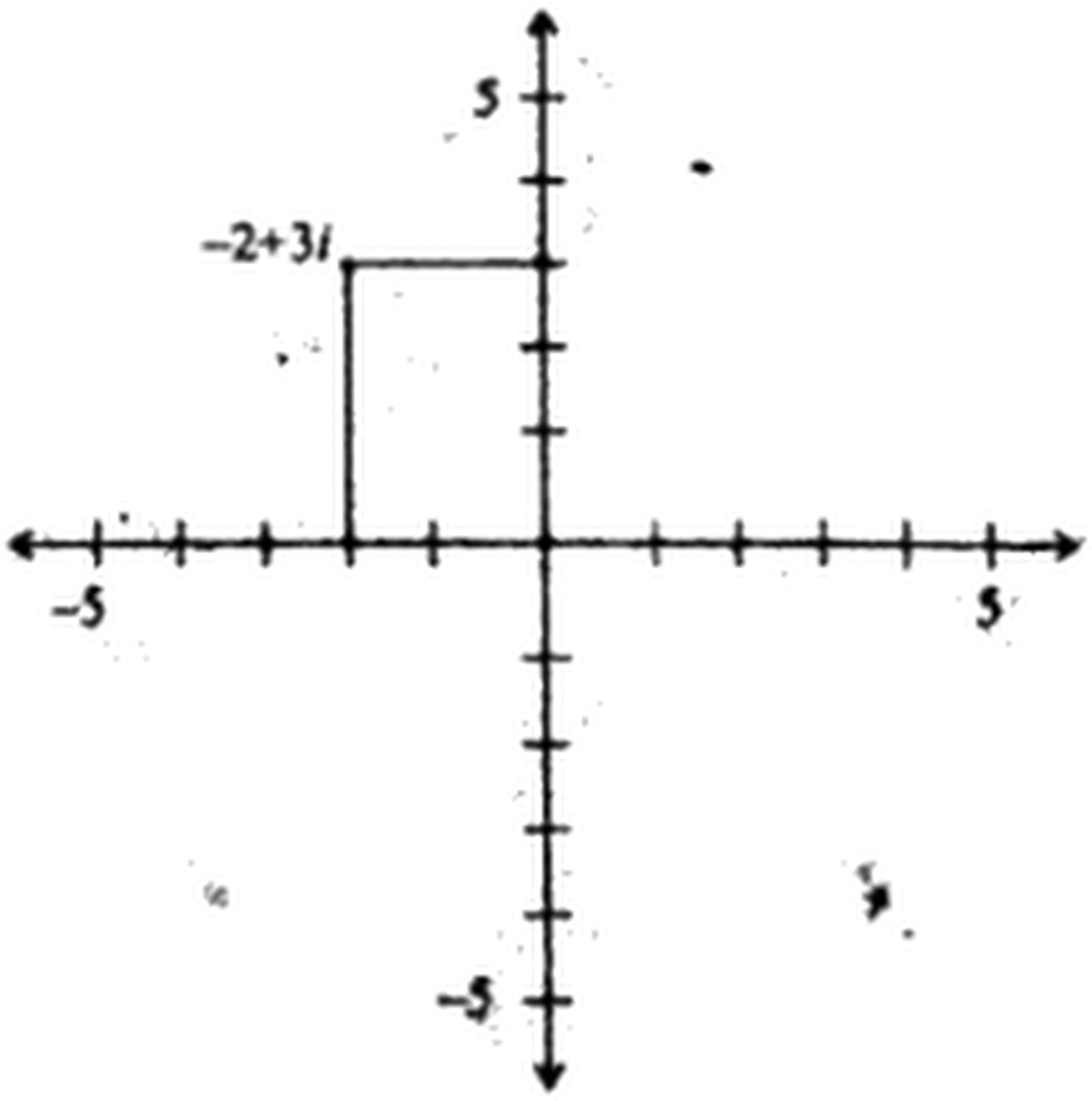
Point along y-axis = -3 units



iv) -2+3i

Point on x-axis = -2 units

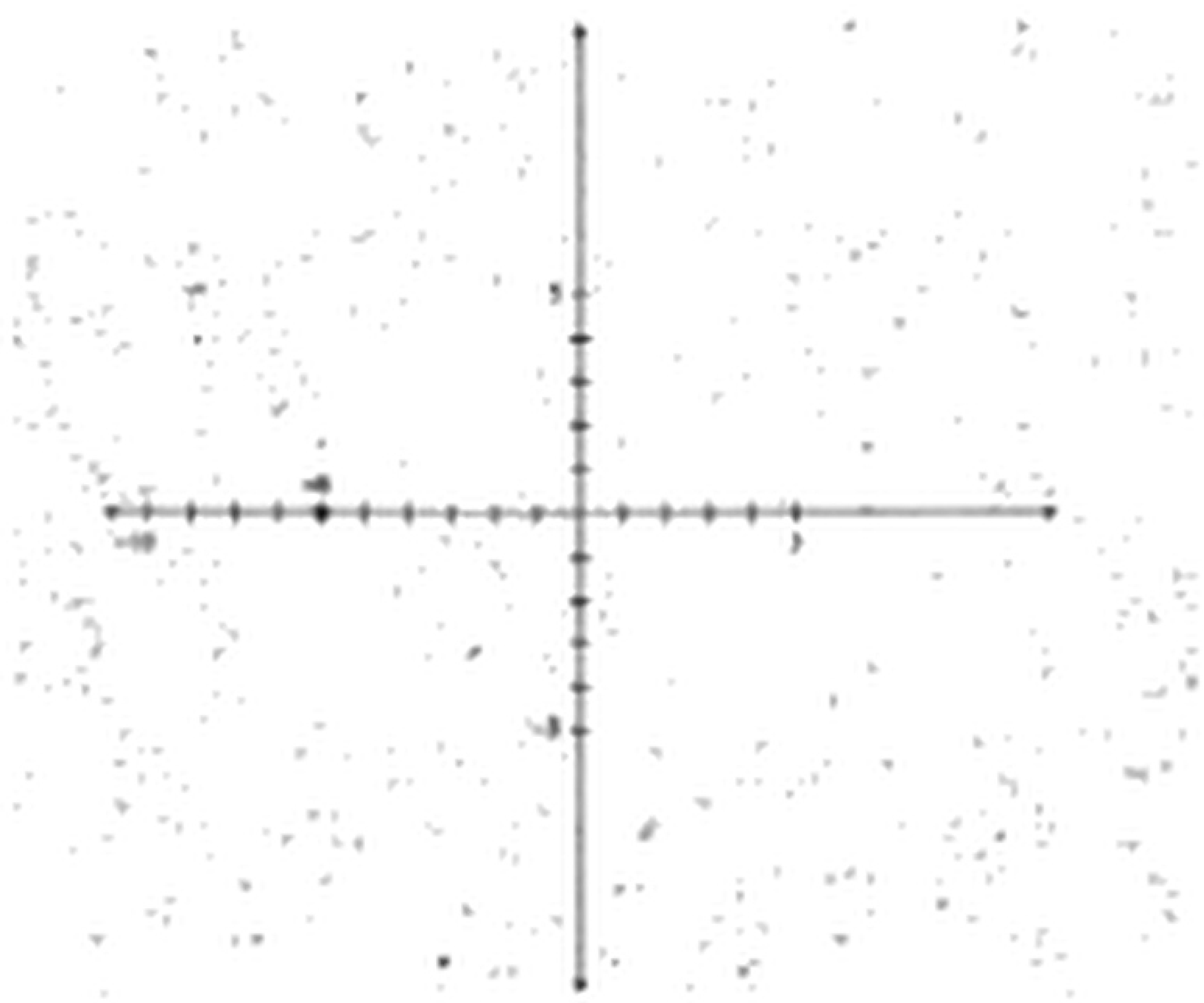
Point along y-axis = 3 units



v) -6

Point on x-axis = 6 units

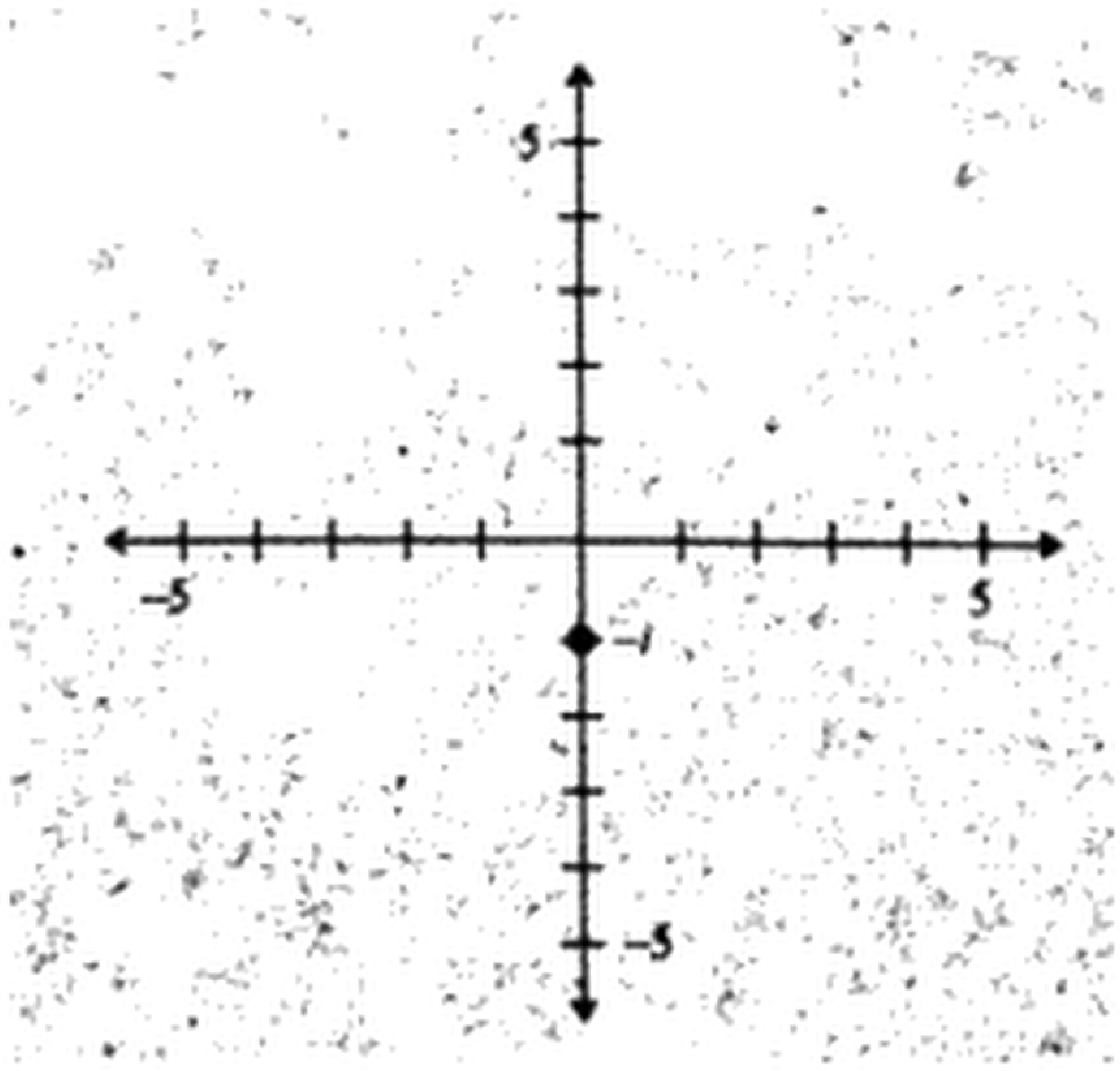
Point along y-axis = 0 units



vi) **i**

Point on x-axis = 0 units

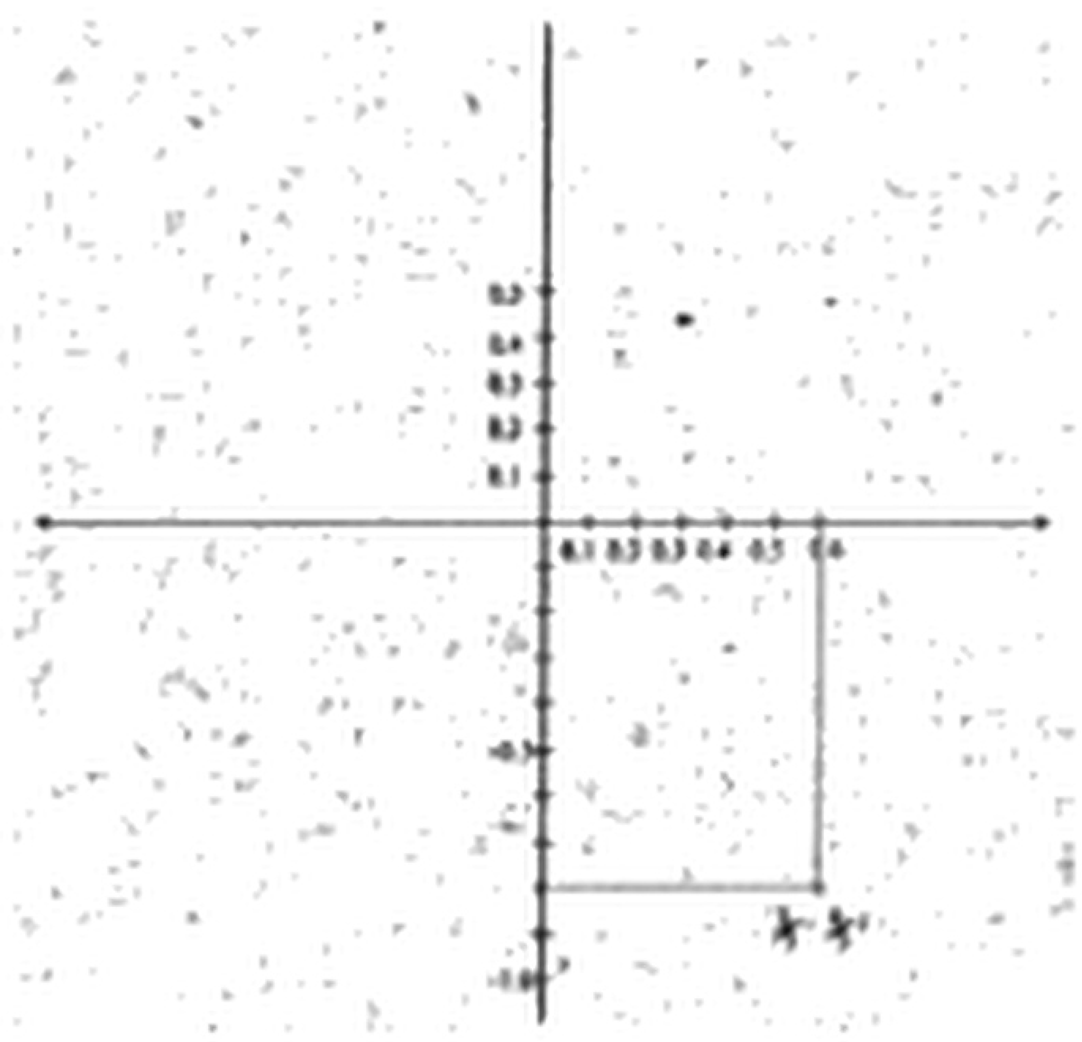
Point along y-axis = 1 units



vii) $\frac{3}{5} - \frac{4}{5}i$

Point on x-axis = $\frac{3}{5}$ = 0.6 units

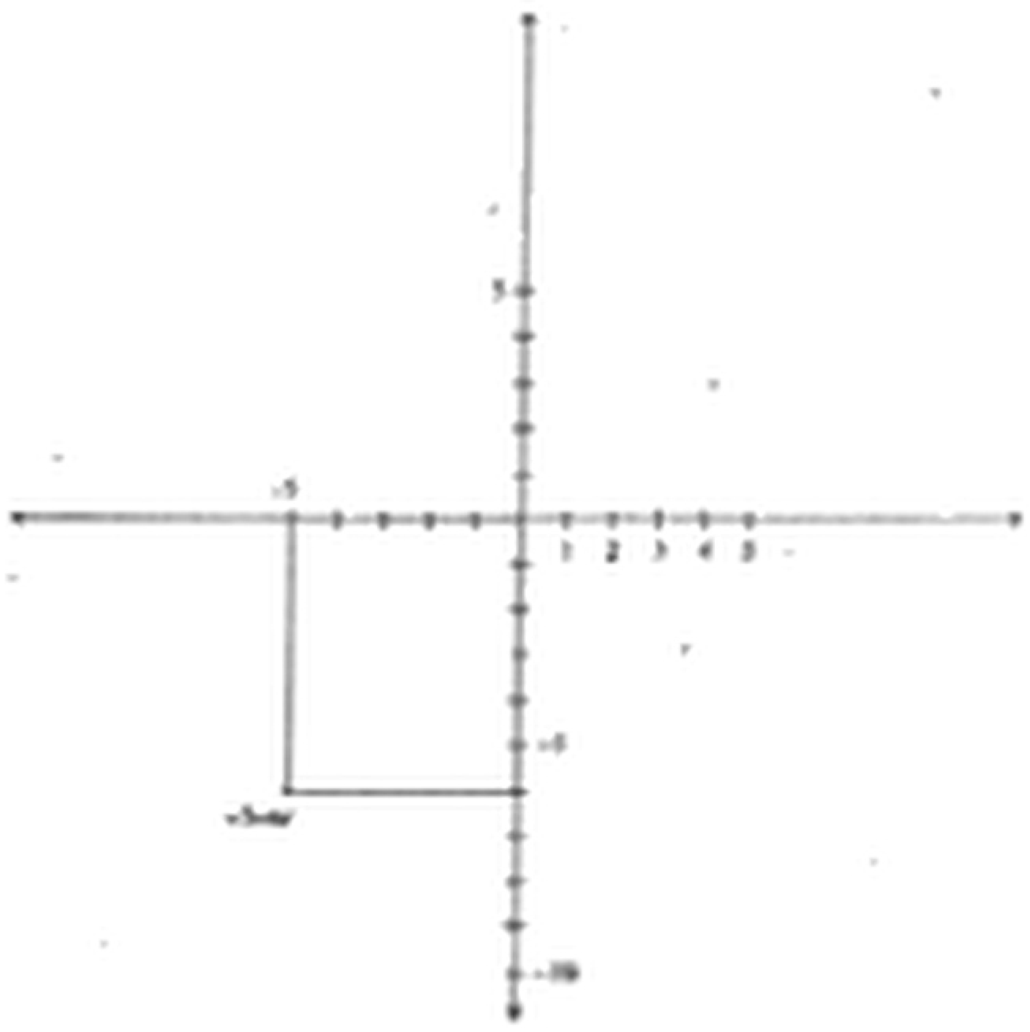
Point on y-axis = $-\frac{4}{5}$ = -0.8 units



viii) -5-6i

Point on x-axis = -5 units

Point on y-axis = -6 units



Q2. Find the multiplicative inverse of each of the following numbers.

i) -3i ii) 1-2i iii) -3-5i iv) (1,2)

Solution:

i) -3i

Let z = -3i

Then multiplication inverse of z is:

$z^{-1} = (-3i)^{-1} = -\frac{1}{3i} \times \frac{i}{i}$

$= -\frac{i}{3i^2}$

$= -\frac{i}{3(-1)}$

$= \frac{i}{3}$

$z^{-1} = \frac{i}{3}$

ii) 1-2i

Let z = 1-2i

Then multiplication inverse of z is:

$z^{-1} = (1-2i)^{-1} = \frac{1}{1-2i} \times \frac{1+2i}{1+2i}$

$= \frac{1+2i}{(1)^2-(2i)^2}$

$= \frac{1+2i}{1-4i^2}$

$= \frac{1+2i}{1+4}$

$z^{-1} = \frac{1+2i}{5} = \frac{1}{5} + \frac{2i}{5}$

$z^{-1} = \frac{1}{5} + \frac{2i}{5}$

iv) (1,2)

Let z = 1+2i

Then multiplication inverse of z is:

$z^{-1} = (1+2i)^{-1} = \frac{1}{1+2i} \times \frac{1-2i}{1-2i}$

$= \frac{1-2i}{(1)^2-(2i)^2}$

$= \frac{1-2i}{1-4i^2}$

$= \frac{1-2i}{1+4}$

$z^{-1} = \frac{1-2i}{5} = \frac{1}{5} - \frac{2i}{5}$

$z^{-1} = \frac{1}{5} - \frac{2i}{5}$

Q3. Simplify

i) i^{101} **ii)** $(-ai)^4$ **iii)** i^{-3} **iv)** i^{-19}

Solution:

i) $i^{101} = i^{100+1}$

$= i^{100}.i$

$= (i^2)^{50}\, i$

$= (-1)^{50}\, i \qquad [\because i^2 = -1]$

$= 1.i$

$= i$

Hence $i^9 = i$

ii) $(-ai)^4$

$= (-a)^4\, (i)^4$

$= a^4\, i^4$

$= a^4\, (-1)^4$

$= a^4\, (1)$

$= a^4$

Thus $(-ai)^4 = a^4$

iii) i^{-3}

$= \frac{1}{i^3}$

$= \frac{1}{i^2.i}$

$= \frac{1}{(-1)i}$

$= \frac{-i}{i^2}$

$= \frac{-i}{-1} = i$

Thus $i^3 = i$

iv) i^{-10}

$= (i^{10})^{-1}$

$= [(i^2)^5]^{-1}$

$= [(-1)^5]^{-1}$

$= (-1)^{-1}$

$= -1$

Thus $i^{-10} = -1$

Q4. Prove that $\bar{z} = z$, iff z is real.

Solution:

Case I:

Let $Z = a+bi$

and $\bar{z} = a-bi$

Given $\bar{z} = z$

$a-bi = a+ib$

$-ib = ib$

$-2ib = 0$

$b = 0$ or $2i = 0$

only $b = 0$ $2i \neq 0$

put in equation $z = a+i(0)$

$= a+0i \in$ real number

Thus $\bar{z} = z$ iff z is real.

Case II:

Let $z \in R$

Then z = a+bi = a

So $\bar{z}$ = a-bi = a

Thus z = $\bar{z}$

Hence proved

Q5. Simplify by expressing in the form a+bi

i) $5+2\sqrt{-4}$ **ii)** $(2+\sqrt{-3})(3+\sqrt{-3})$ **iii)** $\frac{2}{\sqrt{5}+\sqrt{-8}}$

iv) $\frac{3}{\sqrt{6}-\sqrt{-12}}$

Solution:

i) $5+2\sqrt{-4}$

$= 5+2\sqrt{-1\times 4}$

$= 5+2\sqrt{4}i$

$= 5+2(2i)$

$=5+4i$

Thus $5+2\sqrt{-4} = 5+4i$

ii) $(2+\sqrt{-3})(3+\sqrt{-3})$

$= (2+\sqrt{-1\times 3})(3+\sqrt{-1\times 3})$

$= (2+i\sqrt{3})(3+i\sqrt{3})$

$= 6+2i\sqrt{3}+3i\sqrt{3})+(i\sqrt{3})^2$

$= 6+i^2(3)+5i\sqrt{3}$

$= 6-3+5i\sqrt{3})$

$= 3+5i\sqrt{3}$

Thus $(2+\sqrt{-3})(3+\sqrt{-3}) = (3+5i\sqrt{3})$

iii) $\frac{2}{\sqrt{5}+\sqrt{-8}}$

$= \frac{2}{\sqrt{5}+\sqrt{-1\times 8}}$

$= \frac{2}{\sqrt{5}+\sqrt{8}i}$

$= \frac{2}{\sqrt{5}+2\sqrt{2}i} \times \frac{\sqrt{5}-2\sqrt{2}i}{\sqrt{5}-2\sqrt{2}i}$

$= \frac{2(\sqrt{5}-2\sqrt{2}i)}{(\sqrt{5})^2-(2\sqrt{2}i)^2}$

$= \frac{2(\sqrt{5}-2\sqrt{2}i)}{5-8i^2}$

$= \frac{2(\sqrt{5}-2\sqrt{2}i)}{5+8}$

$= \frac{2}{13}(\sqrt{5} \quad 2\sqrt{2}i)$

Thus $\frac{2}{\sqrt{5}+\sqrt{-8}} = \frac{2}{13}(\sqrt{5} - 2\sqrt{2}i)$

iv) $\frac{3}{\sqrt{6}-\sqrt{-12}}$

$= \frac{3}{\sqrt{6}-\sqrt{-1\times 12}}$

$= \frac{3}{\sqrt{6}-\sqrt{12}i}$

$= \frac{3}{\sqrt{6}-\sqrt{12}i} \times \frac{\sqrt{6}+\sqrt{12}i}{\sqrt{6}+\sqrt{12}i}$

$= \frac{3(\sqrt{6}+\sqrt{12}i)}{(\sqrt{6})^2-(\sqrt{12}i)^2}$

$= \frac{3(\sqrt{6}+\sqrt{12}i)}{6-12i^2}$

$= \frac{3(\sqrt{6}+\sqrt{12}i)}{6+12}$

$=\frac{3}{18}(\sqrt{6}+\sqrt{12}i)$

$=\frac{1}{6}(\sqrt{6}+\sqrt{12}i)$

$=\frac{1}{6}[\sqrt{6}(1+\sqrt{2}i)]$

Thus $\frac{3}{\sqrt{6}+\sqrt{-12}}=\frac{1}{\sqrt{6}}(1+\sqrt{2}i)$

Q6. Show that $\forall z \in C$

i) $Z^2+\bar{Z}^2$ is real number ii) $(Z-\bar{Z})^2$ is real number

Solution:

Let Z = a+bi

& $\bar{Z}$ = a- bi

$Z^2=(a+bi)^2=a^2+(ib)^2+2(a)(bi)$

$=(a^2-b^2)+2abi$

$\bar{Z}^2=(a-bi)^2=a^2+(ib)^2-2(a)(bi)$

$=(a^2-b^2)-2abi$

i) $Z^2+\bar{Z}^2$

$=(a^2-b^2)+2abi+(a^2-b^2)-2abi$

$=2(a^2-b^2)\in R$

Hence proved

ii) $(Z-\bar{Z})^2$

$Z-\bar{Z}=(a+ib)+(a-bi)=2a$

$(Z-\bar{Z})^2=(2a)^2=4a^2\in R$

Q7. Simplify the following.

i) $[-\frac{1}{2}+\frac{\sqrt{3}}{2}i]^3$ ii) $[-\frac{1}{2}-\frac{\sqrt{3}}{2}i]^3$

iii) $[-\frac{1}{2}-\frac{\sqrt{3}}{2}i]^{-2}\ [-\frac{1}{2}-\frac{\sqrt{3}}{2}i]^1$ iv) $(a+bi)^2$

v) $(a+bi)^{-2}$ vi) $(a+bi)^3$

vii) $(a-bi)^3$ viii) $(3-\sqrt{-4})^{-3}$

Solution:

i) $[-\frac{1}{2}+\frac{\sqrt{3}}{2}i]^3$

$= \left(-\frac{1}{2}\right)^3 + \left(\frac{\sqrt{3}}{2}i\right)^3 + 3\left(-\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}i\right)\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)$

$= (-\frac{1}{8}) + (\frac{3\sqrt{3}}{8}i^3 - \frac{3\sqrt{3}}{4}i(-\frac{1}{2}+\frac{\sqrt{3}}{2}i)$

$= -\frac{1}{8} - \frac{3\sqrt{3}}{8}i + \frac{3\sqrt{3}}{8}i - \frac{9}{8}i^2$

$= -\frac{1}{8} + \frac{9}{8}$

$= \frac{-1+9}{8} = \frac{8}{8} = 1$

Thus, $[-\frac{1}{2}+\frac{\sqrt{3}}{2}i]^3 =$ **1**

ii) $[-\frac{1}{2}-\frac{\sqrt{3}}{2}i]^3$

$= \left(-\frac{1}{2}\right)^3 - \left(\frac{\sqrt{3}}{2}i\right)^3 - 3\left(-\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}i\right)\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)$

$= (-\frac{1}{8}) - (\frac{3\sqrt{3}}{8}i^3) + \frac{3\sqrt{3}}{4}i(-\frac{1}{2}-\frac{\sqrt{3}}{2}i)$

$= -\frac{1}{8} + \frac{3\sqrt{3}}{8}i - \frac{3\sqrt{3}}{8}i - \frac{9}{8}i^2$

$= -\frac{1}{8} + \frac{9}{8}$

$= \frac{-1+9}{8} = \frac{8}{8} = 1$

Thus, $[-\frac{1}{2} - \frac{\sqrt{3}}{2}i]^3 = 1$

iii) $[-\frac{1}{2} - \frac{\sqrt{3}}{2}i]^{-2}\ [-\frac{1}{2} - \frac{\sqrt{3}}{2}i]^1$

$= [\frac{-1+\sqrt{3}i}{2}]^{-2}\ [\frac{-1-\sqrt{3}i}{2}]^1$

$= [\frac{2}{-1+\sqrt{3}i}]^2\ [\frac{-1-\sqrt{3}i}{2}]^1$

$= \frac{4}{(-1)^2+2(-1)(\sqrt{3}i)+(\sqrt{3}i)^2}\ (\frac{-1-\sqrt{3}i}{2})$

$= (\frac{4}{1-2(\sqrt{3}i)+3i^2})\ (\frac{-1-\sqrt{3}i}{2})$

$= (\frac{4}{-2-2\sqrt{3}i})\ (\frac{-1-\sqrt{3}i}{2})$

$= \frac{1}{2}\ (\frac{4}{-1-\sqrt{3}i})\ (\frac{-1-\sqrt{3}i}{2})$

$= \frac{4}{2\times 2}$

$= \frac{4}{4} = 1$

iv) $(a + bi)^2$

$= (a)^2+2(a)(bi) + (bi)^2$

$= a^2+2abi + b^2i^2$

Thus $(a + bi)^2 = (a^2-b^2) + 2abi$

v) $(a + bi)^{-2}$

$= \frac{1}{(a+bi)^2}$

$= \frac{1}{(a)^2+2(a)(bi)+(bi)^2}$

$= \frac{1}{(a^2-b^2)+2abi}$

$= \frac{1}{(a^2-b^2)+2abi} \times \frac{(a^2-b^2)-2abi}{(a^2-b^2)-2abi}$

$= \frac{(a^2-b^2)-2abi}{(a^2-b^2)^2-(2abi)^2}$

$= \frac{(a^2-b^2)-2abi}{a^2+b^4-2a^2b^2+4a^2b^2i^2}$

$= \frac{(a^2-b^2)-2abi}{a^2+b^4+2a^2b^2}$

$= \frac{(a^2-b^2)-2abi}{(a^2-b^2)^2}$

Thus $a + bi)^{-2} = \frac{(a^2-b^2)-2abi}{(a^2-b^2)^2}$

vi) $\boldsymbol{(a+bi)^3}$

$= (a)^3 + (bi)^3 + 3(a)(bi)(a+bi)$

$= a^3 + b^3i^3 + 3abi(a+bi)$

$= a^3 - b^3i + 3a^2bi + 3ab^2i^2$

$= (a^3 - 3ab^2) + (3a^2b - b^3)i$

Thus $(a+bi)^3 = (a^3-3ab^2) + (3a^2b-b^3)i$

vi) $\boldsymbol{(a-bi)^3}$

$= (a)^3 - (bi)^3 - 3(a)(bi)(a-bi)$

$= a^3 - b^3i^3 - 3abi(a-bi)$

$= a^3 + b^3i - 3a^2bi + 3ab^2i^2$

$= (a^3 - 3ab^2) + (-3a^2b + b^3)i$

Thus $(a-bi)^3 = (a^3 - 3ab^2) + (b^3 - 3a^2b)i$

viii) $(3-\sqrt{-4})^{-3}$

$= (3-\sqrt{-1\times 4})^{-3}$

$= (3-\sqrt{4}i)^{-3}$

$= (3-2i)^{-3}$

$= \frac{1}{(3-2i)^3}$

$= \frac{1}{(3)^3-(2i)^3-3(3)(2i)(3-2i)}$

$= \frac{1}{27-8i^3-18i(3-2i)}$

$= \frac{1}{(27-36)+(8-54)i}$

$= \frac{1}{-9-46i}$

$= \frac{1}{-9-46i} \times \frac{-9+46i}{-9+46i}$

$= \frac{-9+46i}{(-9)^2+(46i)^2}$

$= \frac{-9+46i}{81-2116i^2}$

$= \frac{-9+46i}{2197}$

$= \frac{-9}{2197} + \frac{46i}{2197}$

Thus $(3-\sqrt{-4})^{-3} = \frac{-9}{2197} + \frac{46i}{2197}$