NUMERICAL PROBLEMS

12.1 An object 10.0 cm in front of a convex mirror forms in image 5.0 cm behind the mirror. What is the focal length of the mirror?

Answer

Distance of object, P = 10m

Distance of image = 5 cm

focal length, f =?

As we know that,

$$\frac{1}{f} = \frac{1}{p} + \left(-\frac{1}{q}\right)$$

Negative sign indicates that the image formed by convex mirror is virtual,

$$\Rightarrow \quad \frac{1}{f} = \frac{1}{10} + \left(-\frac{1}{5}\right)$$

$$= \frac{f-2}{10}$$

$$\frac{1}{f} = -\frac{1}{10}$$

f = -10 cm

12.2 An object 30.0 cm tall is located 10.5 cm from a concave mirror with focal length 16.0 cm. (a) where is the image located? (b) How high is it?

Answer

Distance of object, P - 10.5 cm

Focal length, f =16 cm

Height of object, h_o = 30 cm

Height of image, h_I=?

Distance of image, q =?

we know the formula

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}, \text{ or } \frac{1}{q} = \frac{1}{f} - \frac{1}{p}$$

Putting the values, we get.

$$\frac{1}{q} = \frac{1}{16} - \frac{1}{10.5}$$

or $$= \frac{1}{16} - \frac{10}{105}$$

$$= \frac{1}{16} - \frac{2}{21}$$

$$= \frac{21-32}{336}$$

$$\frac{1}{q} = -\frac{11}{336}$$

or $$q = -\frac{336}{11}$$

or $$\boxed{q = 30.54 \text{ cm}}$$

As only the magnitude is required, so

$$q = 30.54 \text{ cm}$$

Also, we know that,

$$\frac{\text{height of image}}{\text{height of object}} = \frac{\text{Distance of image}}{\text{Distance of object}}$$

⇒ $$\frac{h_1}{h_0} = \frac{p}{q}$$

or $$h_I = \frac{p}{q} \times h_0$$

$$= \frac{30.54}{10.5} \times 30$$

$$h_I = 87.26 \text{ cm}$$

⇒ Height of image, h_I = 87.26 cm

12.3 An object and its image in a concave mirror are of the same height, yet inverted, when the object is 20.0 cm from the mirror. What is the focal length of the mirror?

Answer

As it is given that

Height of image = height of object

=> $h_I = h_p$

Distance of object, p = 20 cm

Focal length, f =?

Let us find the distance of image 'q' first.

We know that,

$$\frac{h_1}{h_0} = \frac{q}{p}$$

or $q = \frac{h_1}{h_0} \times p$

As $h_I = h_0$

So, $q = p$

or $\boxed{q = 20cm}$ (p =20cm)

Applying spherical mirror formula

i.e. $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$

⇒ $\frac{1}{f} = \frac{1}{20} + \frac{1}{20}$

$= \frac{1+1}{20}$

$\frac{1}{f} = \frac{2}{20}$

f = 10cm

12.4 Find the focal length of a mirror that forms an image 5.66 cm behind a mirror of an object placed at 34.4 cm in front of the mirror.

Answer

Focal length f =?

Distance of image, q = 5.66 cm

Distance of object, p = 34.4 cm

Applying spherical mirror formula;

i.e. $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$

$= \frac{1}{34.4} - \frac{1}{5.66}$ (-ve sign is due to virtual image)

$= \frac{1}{344} - \frac{1}{566}$

$= \frac{5}{172} - \frac{50}{283}$

$= \frac{1415 - 8600}{48676}$

$\frac{1}{f} = -\frac{7185}{48676}$

$f = \frac{1876}{7185}$

f = 677cm

12.5 An image of a statue appears to be 11.5 cm behind a convex mirror with focal length 13.5 cm. Find the distance from the statue to the mirror

Answer

Distance of image, q = 11.5 cm

Focal length f =13.5 cm

Distance of object, p =?

Applying spherical mirror formula;

i.e. $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$

$\frac{1}{p} = \frac{1}{f} - \frac{1}{q}$

$= \frac{1}{13.5} - \frac{1}{11.5}$ (-ve sign is due to virtual image)

$= \frac{10}{135} + \frac{10}{115}$

$= \frac{2}{27} + \frac{2}{23}$

$= \frac{46+54}{621}$

$\frac{1}{p} = \frac{100}{621}$

p = 621/100

p = 6.2cm

12.6 An image is produced by a concave mirror of focal length 8.70 cm. The object is 13.2 cm tall and at a distance 19.3 cm from the mirror, (a) Find the location and height of the image, (b) Find the height of the image produced by the mirror if the object is twice as far from the mirror.

Answer

Focal length, f = 8.7 cm

Distance of image, p = 19.3cm

Distance of object, q =?

Height of image, h_i =?

Height of object h_0 = 13.2 cm

New distance of object, p' = 38.6 cm (double of 19.3 cm)

New height of image h_i,' =?

Applying spherical mirror formula:

i.e. $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$$

$$= \frac{1}{8.70} - \frac{1}{19.3}$$

$$= \frac{10}{870} - \frac{10}{193}$$

$$= \frac{10}{87} - \frac{10}{193}$$

$$= \frac{1930-870}{16791}$$

$$\frac{1}{q} = \frac{1060}{16791}$$

$$q = \frac{16791}{1060}$$

$$\boxed{q = 15.84 \text{ cm}}$$

Also, we know that

$$\frac{h_1}{h_0} = \frac{q}{p}$$

or $h_i = \frac{q}{p} \times h_0$

Putting the values, $h_i = \frac{15.84}{19.3} \times 13.2$

$$\boxed{h_i = 10.83 \text{ cm}}$$

When distance of object is made double, then,

$$\frac{h'_1}{h_0} = \frac{q}{p'}$$

or $h'_i = \frac{q}{p'} \times h_0$

$= \frac{15.84}{38.6} \times 13.2$

$h'_1 = 5.42cm$

'q' will remain the same, and it is formed at the same distance.

12.7 Nabeela uses a concave mirror when applying makeup. The mirror has a radius of curvature of 38.0 cm. (a) What is the focal length of the mirror? (b) Nabeela is located SO cm from the mirror, where will her image appear? (c) Will the image be upright or inverted?

Answer

Radius of curvature, R = 38 cm

Focal length, f =?

Distance of object, p = 50 cm

Distance of image, q =?

As we know that. f = R/2

So, $f = \frac{38}{2}$

f = 19 cm

Applying spherical mirror formula.

i.e. $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$

$\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$

Putting the values, we get

$\frac{1}{q} = \frac{1}{19} - \frac{1}{50}$

$= \frac{50-19}{950}$

$$\frac{1}{q} = \frac{31}{950}$$

$$q = \frac{950}{31}$$

or $\boxed{q = 30.64\text{ cm}}$

(Image will be erect as we are viewing concave mirror)

12.8 An object 4 cm high is placed at a distance of 12 cm from a convex lens of focal length 8 cm. Calculate the position and size of the image. Also state the nature of φe image.

Answer

Height of object, h_0 = 4cm

Height of image, h_1 =?

Distance of object, p = 12 cm

Focal length, f = 8 cm

Distance of image, q =?

Applying spherical mirror formula.

i.e. $$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$$

Putting the values, we get

$$\frac{1}{q} = \frac{1}{8} - \frac{1}{12}$$

$$= \frac{3-2}{24}$$

$$\frac{1}{q} = \frac{1}{24}$$

$$\boxed{q = 24\text{ cm}}$$

Also, we know that

$$\frac{h_1}{h_o} = \frac{q}{p}$$

⇒ $$h_I = \frac{q}{p} \times h_0$$

or $$h_I = \frac{24}{12} \times 4$$

$$\boxed{h_I = 8\text{ cm}}$$

As, we are using convex lens and the object is placed away from the focal point, so the image will be.

[real, Inverted and magnified]

12.9 An object 10 cm high is placed at a distance of 20 cm from a concave lens of focal length 15 cm. Calculate the position and size of the image. Also state the nature of the image

Answer

Height of object, h_o = 10cm

Height of image, h_I =?

Distance of object, p =20cm

Distance of image, q =?

Applying the lens formula.

i.e. $$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$$

Putting the values, we get

$$\frac{1}{q} = -\frac{1}{15} - \frac{1}{20}$$ (-ve f means formation of virtual image)

$= \frac{-3-4}{60}$

$\frac{1}{q} = \frac{-7}{60}$

q = -60/7

$\boxed{q = -8.57 \text{ cm}}$

Also, we know that

$\frac{h_1}{h_0} = \frac{q}{p}$

⇒ $h_1 = \frac{q}{p} \times h_0$

or $h_1 = \frac{8.57}{20} \times 10$

$\boxed{h_1 = 4.28 \text{ cm}}$

As, we are using convex lens and the object is placed away from the focal point, so the image will be.

[virtual, erect and diminished]

12.10 A convex lens of focal length 6 cm is to be used to form a virtual image three times the size of the object. Where must the lens be placed?

Answer

Focal length, f = 6 cm

As given: Height of image h_1 = 3 (height of object)

⇒ $h_1 = 3h_0$

Distance of object, p=?

As we know that

$\frac{h_1}{h_0} = \frac{q}{p}$

or $\frac{3h_1}{h_o} = \frac{q}{p}$

or $\frac{q}{p} = 3$

or $\boxed{q = 3p}$ (i)

Applying the lens formula.

i.e. $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$

$\frac{1}{f} = \frac{1}{p} - \frac{1}{3p}$ (-ve sign is due to virtual image)

$\frac{1}{6} = \frac{3-1}{3p}$

$\frac{1}{6} = \frac{2}{3p}$

$3p = 12$

$\boxed{p = 4cm}$

12.11 A ray of light from air is incident on a liquid surface at an angle of incidence 35°. Calculate the angle of refraction if the refractive index of the liquid is 1.25. Also calculate the critical angle between the liquid air inter-face.

Answer

Incident angle, $\theta_i = 35°$

Refracted angle, θ_r =?

Refractive index, n = 1.25

Critical angle, θ_C=?

As we know that

$$n = \frac{\sin\theta_i}{\sin\theta_r}$$

$\sin \theta_r = \sin^{-1}\left[\frac{\sin 35^\circ}{1.25}\right]$

$\theta_r = 27.31°$

For critical angle,

$\sin \theta_c = \frac{1}{n}$, $\theta_C = \sin^{-1}\left[\frac{1}{n}\right]$

$\theta_C = \sin^{-1}\left[\frac{1}{1.25}\right]$

$\boxed{\theta_C = 53.13°}$

12.12 The power of a convex lens is 5D. At what distance the object should be placed from the lens so that its real and 2 times larger image is formed.

Answer

Power of lens, p = 5D

Distance of object, p =?

It is given that, $h_i = 2h_0$

(focal length T is also not given)

We know that,

$p = \frac{1}{f}$ (meters)	Also, $\frac{h_1}{h_0} = \frac{q}{p}$
$f(\text{meters}) = \frac{1}{p}$	$\frac{2h_1}{h_0} = \frac{q}{p}$
$= \frac{1}{5}$	$\frac{q}{p} = 2$
$= 0.2$ m	

or $\boxed{f = 20cm}$ q =2p

Applying the lens formula.

i.e. $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{2p}$$

$$\frac{1}{20} = \frac{2+1}{2p}$$

$$\frac{1}{20} = \frac{3}{2p}$$

$$2p = 60$$

$$\boxed{p = 30cm}$$