

Numerical Problems

11.1 A normal conversation involves sound intensities of about $3.0 \times 10^{-6} \text{ Wm}^{-2}$. What is the decibel level for this intensity? What is the intensity of the sound for 100dB?

Answer

$$I = 3.0 \times 10^{-6} \text{ Wm}^{-2}$$

$$\text{Sound level} = ?$$

$$\text{Then (Sound level)} = 100\text{dB.}$$

$$I' = ?$$

For the 1st part, we have,

$$\begin{aligned} \text{Sound level} &= 10 \log \left(\frac{I}{I_0} \right) \text{ dB} \\ &= 10 \log \left(\frac{3 \times 10^{-6}}{10^{-12}} \right) \end{aligned}$$

because, the faintest intensity of sound is; $I_0 = 10^{-12} \text{ Wm}^{-2}$

$$\Rightarrow \text{Sound level} = 10 \log (3 \times 10^6) \text{ dB}$$

$$\text{or } \boxed{\text{Sound level} = 64.8 \text{ dB}}$$

For the second part, we have,

$$\begin{aligned} \text{(Sound level)} &= 10 \log \\ &= 10 \log \left(\frac{I'}{I_0} \right) \text{ dB.} \end{aligned}$$

$$100 = 10 \log \left(\frac{I'}{10^{-12}} \right) \text{ dB}$$

$$\text{or } 10 = \log \left(\frac{I'}{10^{-12}} \right)$$

As given in mathematics; $\log\left(\frac{a}{b}\right) = \log a - \log b$

So, $10 = \log I' - \log (10^{-12})$

Also, according to logarithm rules, $\log (10)^a = a \log (10)$

So, $10 = \log I' - (-12) \log (10)$

As, $\log 10 = 1$

So, $10 = \log I' + 12$

$$\log I' = -2$$

Taking antilog;

$$I' = \text{Antilog} (-2)$$

$$I' = 0.01 \text{ Wm}^{-2}$$

11.2 If at Anarkali bazaar Lahore, the sound level is 80 dB, what will be the intensity level of sound there?

Answer

$$\text{Sound level} = 80 \text{ dB}$$

$$\text{Intensity level } I = ?$$

$$\text{Intensity of faintest sound, } I_0 \text{ We have, } = 10^{-12} \text{ Wm}^{-2}$$

$$\text{Sound level} = 10 \log \left(\frac{I}{I_0}\right) \text{ dB}$$

or $80 = 10 \log \left(\frac{I}{10^{-12}}\right) \text{ dB}$

$$8 = \log I - \log (10^{-12})$$

$$8 = \log I - (-12) \log (10)$$

$$8 = \log I + 12$$

$$\log I = -4$$

Taking antilog we have,

$$I = \text{Antilog}(-4)$$

$$I = 10^{-4} \text{ Wm}^{-2}$$

11.3 At a particular temperature, the speed of sound in air is 330 ms^{-1} . If the wavelength of a note is 5 cm , calculate the frequency of the sound wave. Does its frequency lie in the audible range of the human ear?

Answer

$$v = 330 \text{ ms}^{-1}$$

$$\lambda = 5 \text{ cm}$$

$$= 0.05 \text{ m}$$

$$f = ?$$

$$v = f \lambda$$

or $f = \frac{v}{\lambda}$

Putting the values,

$$f = \frac{330}{0.05}$$

$$f = 6.6 \times 10^3 \text{ Hz}$$

11.4 A doctor counts 72 heartbeats in 1 min. Calculate the frequency and period of the heartbeats.

Answer

Number of heart beats, $n = 72$

Time, $t = 1 \text{ min}$

$= 60 \text{ Seconds}$

Time period, $T = ?$

frequency, $f = ?$

We have that, the time period is equal to number of vibrations per total time, and frequency is its inverse.

So, $f = \frac{n}{t}$

Putting the values:

$$f = \frac{72}{60}$$

$$f = 1.2 \text{ Hz}$$

$$f = \frac{1}{T}, \text{ or } T = \frac{1}{f}$$

(As, frequency and time period are reciprocal to each other.)

$$T = 0.83 \text{ Seconds}$$

11.5 A marine survey ship sends a sound wave straight to the sea bed. It receives an echo 1.5 s later. The speed of sound in sea water is 1500 ms^{-1} . Find the depth of the sea at this position.

Answer

{ Time taken by sec and wave from
Ship to the sea bed and then back, $t = 1.5\text{sec.}$
to ship

Time for one side (from ship to sea bed), $t' = \frac{1.5}{2}$

$$= 0.75 \text{ Sec}$$

Speed of sound, $V = 1500 \text{ ms}^{-1}$

Depth of Sea, $d = ?$

We know that,

$$V = \frac{d}{t'}$$

or $d = v \times t'$

Putting the values, $d = 1500 \times 0.75$

$$\boxed{d = 1125 \text{ m}}$$

11.6 A student clapped his hands near a cliff and heard the echo after 5 s.

What is the distance of the cliff from the student if the speed "of the sound, v is taken as 346 ms^{-1} ?

Answer

{ Time taken Time taken by sound from
the student to cliff and then, $t = 5\text{sec.}$
back to student

Time for one side (from cliff to student) $t' = \frac{5}{2}$

$$t' = 2.5 \text{ Sec}$$

$$\text{Velocity, } V = 346 \text{ ms}^{-1}$$

Distance from cliff to student, $d = ?$

We know that,

$$V = \frac{d}{t'}$$

or $d = v \times t'$

Putting the values, $d = 346 \times 2.5$

$$\boxed{d = 865 \text{ m}}$$

11.7 A ship sends out ultrasound that returns from the seabed and is detected after 3.42s. If the speed of ultrasound through sea water is 1531 ms^{-1} , what is the distance of the seabed from the ship?

Answer

{ Time taken by ultrasound from
Ship to the sea bed and then back, $t = 3.42 \text{ sec}$.
to ship

Time for one side (from ship to sea bed), $t' = \frac{3.42}{2}$

$$= 1.71 \text{ Sec}$$

$$\text{Speed of ultrasound, } V = 1531 \text{ ms}^{-1}$$

Distance of sea bed from the ship, $d = ?$

We know that,

$$V = \frac{d}{t'}$$

or $d = v \times t'$

Putting the values, $d = 1531 \times 1.71$

$$\boxed{d = 2618 \text{ m}}$$

11.8 The highest frequency sound humans can hear is about 20,000 Hz. What is the wavelength of sound in air at this frequency at a temperature of 20°C? What is the wavelength of the lowest sounds we can hear of about 20Hz? Assume the speed of sound in air at 20°C is 343 ms⁻¹.

Answer

Highest sound frequency, $f = 20,000 \text{ Hz}$

Wave length sound, $\lambda = ?$

Lowest sound frequency, $f' = 20 \text{ Hz}$

Wave length of sound, $\lambda' = ?$

For the first part:

$$v = f\lambda$$

\Rightarrow while V in both the cases will be same;

i.e. $v = 343 \text{ ms}^{-1}$

So, $\lambda = \frac{v}{f}$

$$= 34320000$$

$$\boxed{\lambda = 1.7 \times 10^{-2} \text{ m}}$$

For the second part;

$$v = f' \lambda'$$

or $\lambda' = \frac{v}{f'}$

$$= \frac{343}{20}$$

$$\boxed{\lambda' = 17.2 \text{ m}}$$

11.9 A sound wave has a frequency of 2 kHz and wavelength 35 cm. How long will it take to travel 1.5 km?

Answer

frequency of sound waves, $f = 2 \text{ kHz}$

or $\quad \quad \quad = 20000 \text{ Hz}$

Wave length, $\lambda = 35 \text{ cm}$

$$= 0.35 \text{ m}$$

Time taken, $t = ?$

Distance covered, $d = 1.5 \text{ km}$

$$= 1500 \text{ m}$$

We know that $v = \frac{d}{t}$

$$t = \frac{d}{v} \quad (i)$$

but velocity is wave is not known,

So, $v = f \lambda$

$$= 20000 \times 0.35$$

$$v = 700 \text{ ms}^{-1}$$

putting the values of velocity and distance in equation. (1), we get,

$$t = \frac{d}{v}$$

$$= \frac{1500}{700} \Rightarrow t = 2.1 \text{ seconds}$$

