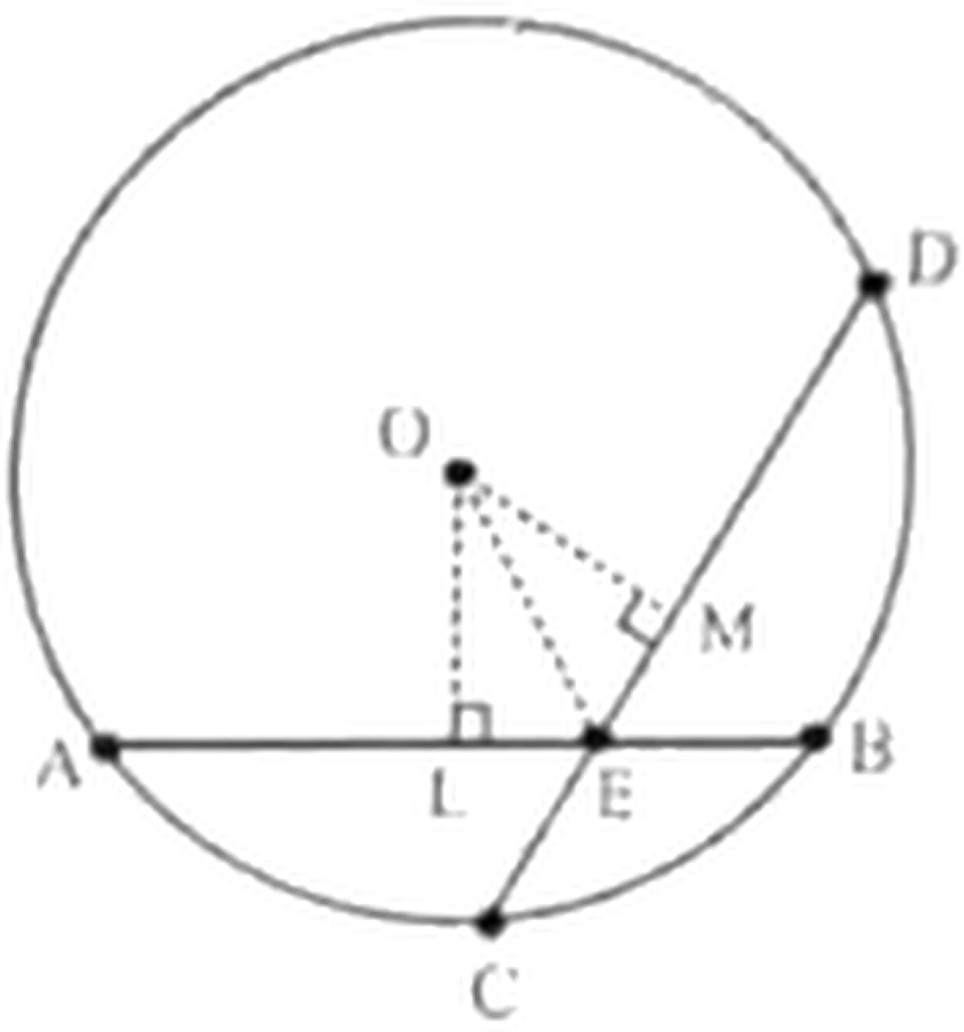
Exercise 9.2

1. Two equal chords of a circle intersect, show that the segments of the one are equal corresponding to the segments of the other.

Given:

A circle with center "O". Two equal chords $\overline{AB}$ and $\overline{CD}$ (i.e. $m\overline{AB} = m\overline{CD}$) intersect each other at point E.



To Prove:

$m\overline{AE} = m\overline{ED}$ and $m\overline{EB} = m\overline{EC}$

Construction:

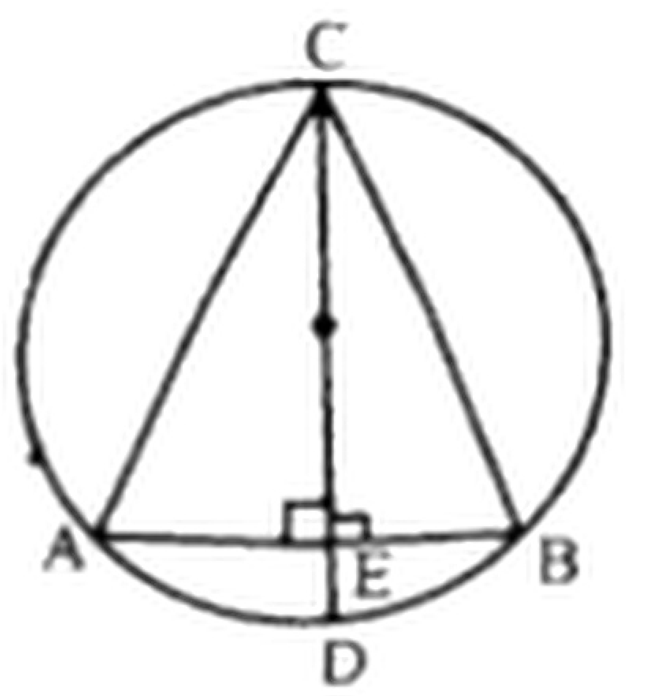
Draw perpendiculars $\overline{OL}$ and $\overline{OM}$ from the center "O" to the chords $\overline{AB}$ and $\overline{CD}$ respectively. L and M are the midpoints of $\overline{AB}$ and $\overline{CD}$ respectively.

Proof:

Statements	Reasons
In $\triangle OLE \leftrightarrow \triangle OME$	
$\overline{OL} \cong \overline{OM}$	Two equal chords of a circle are equidistant from the center.
$m\angle OLE = m\angle OME = 90^\circ$	$\overline{OL} \perp \overline{AB}$ and $\overline{OM} \perp \overline{CD}$
$m\overline{OE} \cong m\overline{OE}$	Common side
$\therefore$ $\triangle OLE \cong \triangle OME$	H.S ≅ H.S

$\overline{LE} \cong \overline{ME}$ ..... (i)	Corresponding sides of congruent triangles
$m\overline{AL} = \frac{1}{2} m\overline{AB}$	
$m\overline{DM} = \frac{1}{2} m\overline{CD}$	
$m\overline{AL} = \overline{DM}$ ..... (ii)	Both are half of equal chords.
$m\overline{AL} + m\overline{LE} = m\overline{DM} + m\overline{ME}$	Adding (i) and (ii).
$m\overline{AE} = m\overline{DE}$ ..... (iii)	
Now, $m\overline{AB} = m\overline{CD}$	Given
$m\overline{AE} + m\overline{EB} = m\overline{DE} + m\overline{EC}$	
$m\overline{AE} + m\overline{EB} = m\overline{AE} + m\overline{EC}$	From (iii)
$m\overline{EB} = m\overline{EC}$	By cancellation property.

2. AB is the chord of a circle and the diameter CD is perpendicular bisector of AB . Prove that $m\overline{AC}$ = $m\overline{BC}$.



Given:

In a circle.

AB ⊥ CD and AE ≅ EB

To Prove:

$m\overline{AC} = m\overline{BC}$

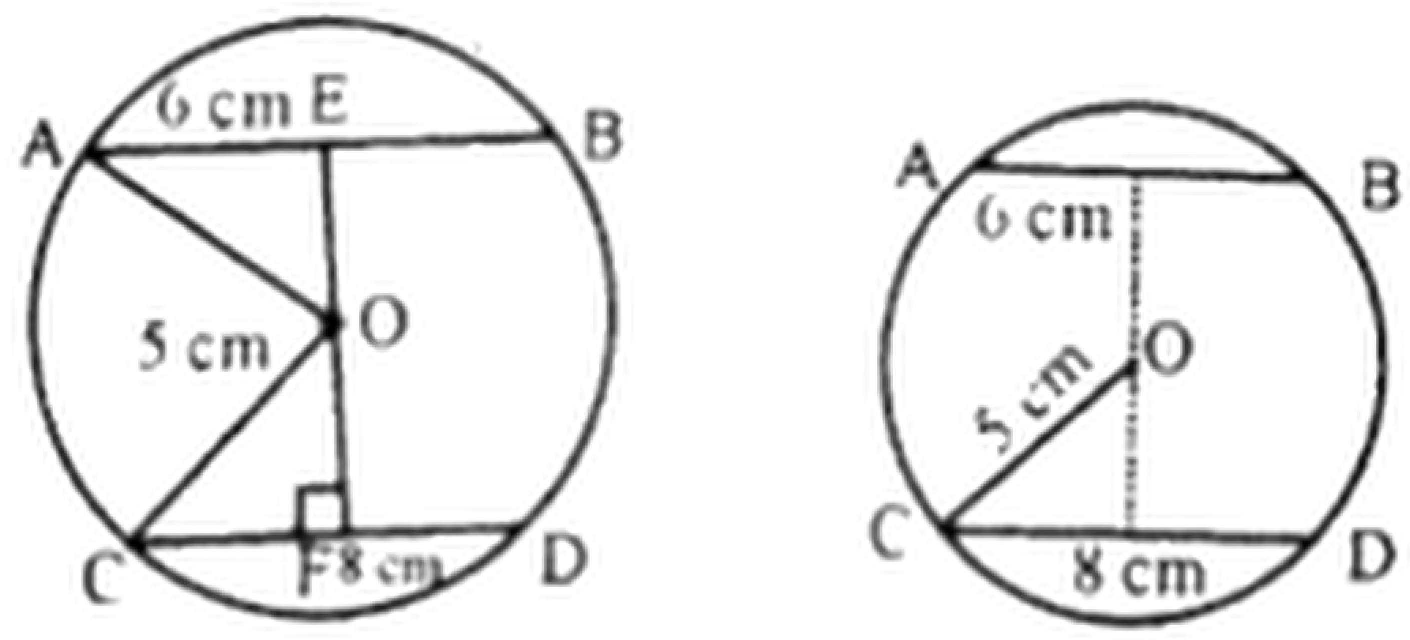
Construction:

Join C to A and B.

Proof:

Statements	Reasons
In $\triangle ACE \leftrightarrow \triangle EBC$	
$AE \cong EB$	A diameter $CD \perp$ on chord AB bisect it.
$m\angle AEC = m\angle CEB$	Given
$CE \cong CE$	Common side
$\therefore \triangle AEC \cong \triangle EBC$	S.A.S $\cong$ S.A.S
$\Rightarrow \overline{AC} \cong \overline{BC}$	Corresponding sides of congruent triangles.
$m\overline{AC} = m\overline{BC}$	

3. As shown in the figure, find the distance between two parallel chords $\overline{AB}$ and $\overline{CD}$.



Given:

$m\overline{AB} = 6cm$ and $m\overline{CD} = 8cm$

$m\overline{OC} = 5cm$

Required:

$m\overline{EF} = ?$

In $\triangle OCF$

$m\overline{OC}^2 = \overline{OF}^2 + \overline{FC}^2$

$5^2 = \overline{OF}^2 + 4^2$

$\Rightarrow \overline{OF}^2 = 25 - 16 = 9$

$\overline{OF} = \sqrt{9} = 3cm$

In □OAE

$\overline{OA}^2 = \overline{OE}^2 + \overline{EA}^2$

$5^2 = \overline{OE}^2 + 3^2$

$\Rightarrow \overline{OE}^2 = 25 - 9 = 16$

$\overline{OE} = \sqrt{16} = 4$

$\therefore \overline{EF} = \overline{OE} + \overline{OF} = 4 + 3 = 7cm$