Exercise 9.1

1. Prove that, only the diameters of a circle are the intersecting chords which bisect each other.

Given: A circle having diameters $\overline{AC}$ and $\overline{BD}$ which passes through centre O.

To Prove: Diameters $\overline{AC}$ and $\overline{BD}$ bisect each other.

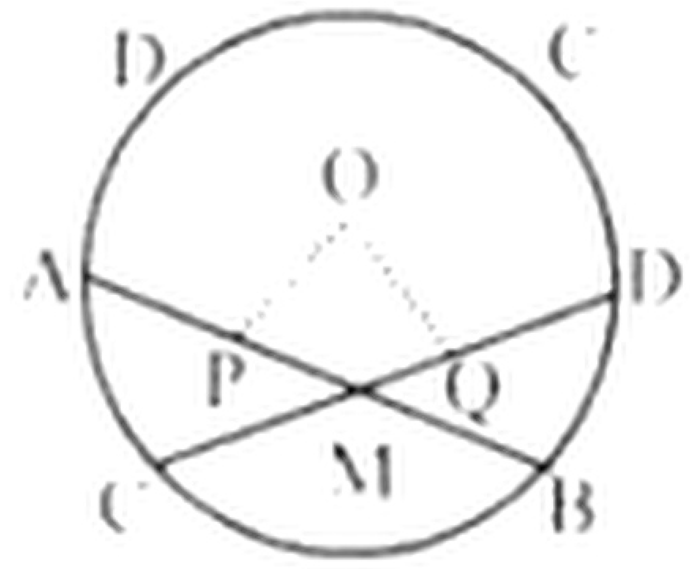
Proof:

Statements	Reasons
$\overline{OA} \cong \overline{OC}$ (i)	
	Common
Similarly, $m\overline{OC} \cong \overline{OD}$ (ii)	
$m\overline{OA} = m\overline{OD}$ (iii)	
From (i), (ii) and (iii), we have	Radii of the same circle
$m\overline{OA} = m\overline{OB} = m\overline{OC} = \overline{OD}$	

Hence AC and BD are intersecting chords which bisect each other.

2. Two chords of a circle do not pass through the center. Prove that they cannot bisect each other.

Given:



A circle with center O having two chords $\overline{AB}$ and $\overline{CD}$.

To Prove:

M is not the mid-point of chords $\overline{AB}$ and $\overline{CD}$.

Construction:

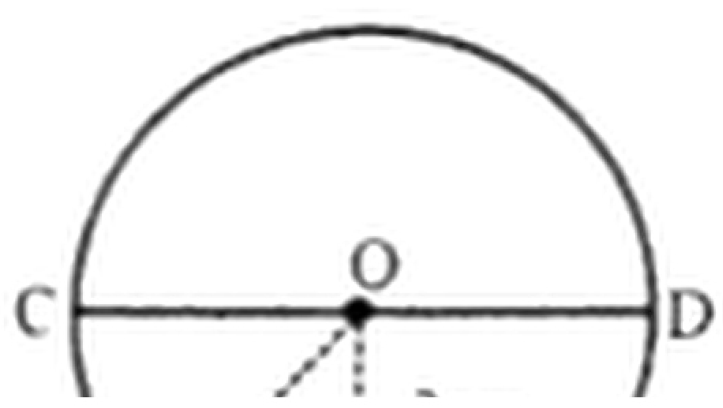
Join O to P and Q such that $\overline{OP} \perp \overline{AB}$ and $\overline{OQ} \perp \overline{CD}$.

Proof:

Statements	Reasons
O is the center of the circle with $\overline{OP} \perp \overline{AB}$ Thus $\overline{OP} \perp \overline{AB}$ Now point M lies between P and B. Therefore, M is not the midpoint of AB. Hence $\overline{AB}$ and $\overline{CD}$ cannot bisect each other.	Construction

3. If the length of the chord AB = 8 cm. Its distance from the center is 3 cm, then measure the diameter of such circle.

Given:



to find the length of diameter

i.e., mCD = ?

Construction:

Join O to A and E.

Proof:

Statements	Reasons
In △ AEO $(AO)^2 = \overline{AE}^2 + \overline{EO}^2$ $= \left[\frac{1}{2}(\overline{AB})\right]^2 + (3)^2$ $= \left[\frac{1}{2} \times 8\right]^2 + 9$ $= (4)^2 + 9$ $= 16 + 9 = 25cm$ $\Rightarrow \overline{AO} = \sqrt{25} = 5cm$	

$$m\overline{AO} = m\overline{OC} = m\overline{OD} = 5cm$$

$$\Rightarrow \overline{CD} = \overline{CO} + m\overline{OD}$$
$$= 5cm + 5cm$$
$$= 10cm$$

Hence

$\boxed{\text{Diameter} = 10cm}$

4. Calculate the length of a chord which stands at a distance 5cm from the center of a circle whose radius is 9cm.

Given:

$m\ \overline{OD} = 5cm$

Required:

$m\ \overline{AB} = ?$

Proof:

Statements	Reasons
In $\triangle$ OAD $m\overline{OA}^2 = m\overline{OD}^2 + m\overline{AD}^2$ $m\overline{OA}^2 - m\overline{OD}^2 = m\overline{AD}^2$ $9^2 - 5^2 = \left[\frac{1}{2}m\left(\overline{AB}\right)\right]^2$ $\left[\frac{1}{2}m\left(\overline{AB}\right)\right]^2 = 81 - 25$ $\frac{1}{4}m\left(\overline{AB}\right)^2 = 56$ $\Rightarrow m\overline{AB}^2 = 56 \times 4 = 224$ $AB = 14.97cm$	$\left[\because AD = \frac{1}{2}\overline{AB}\right]$