EXERCISE 8.2

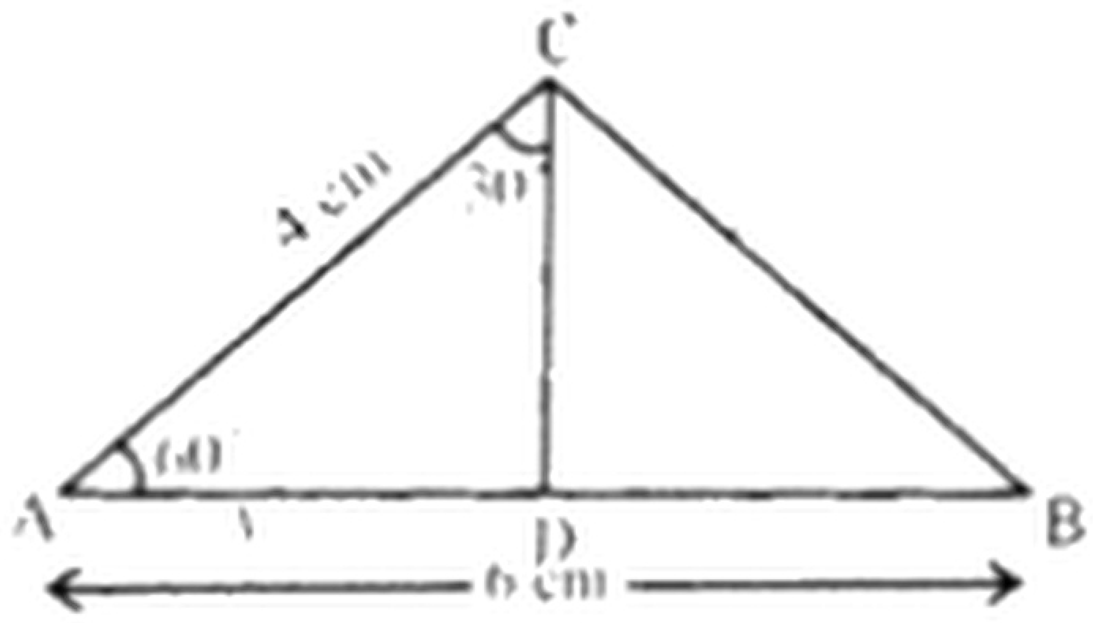
Q1. In a $\triangle ABC$ calculate m $\overline{BC}$ when m $\overline{AB}$ = 6cm, m$\overline{AC}$ = 4cm and m$\angle A$ = 60°.

Solution:

Given: mAB = 6cm; mAC = 4cm;

m$\angle$A =60°.

Required: m$\overline{CB}$ = ?



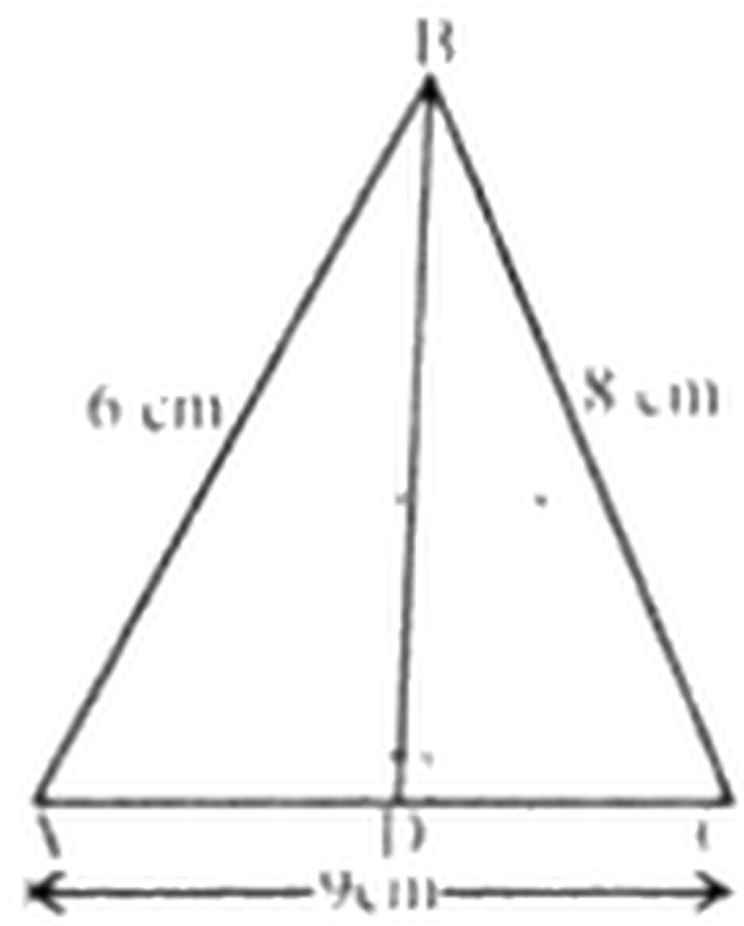
In $\triangle$ABC, we have

$$\left(\overline{BC}\right)^2 = \left(\overline{AB}\right)^2 + \left(\overline{AC}\right)^2 - 2\left(\overline{AB}\right).\left(\overline{AD}\right)$$
$$= (6)^2 + (4)^2 \times 2(6)(x)$$
$$= 36 + 16 - 2(6)(2)$$
$$= 52 - 24$$
$$= 28$$
$$m\overline{BC} = \sqrt{28}$$
$$= 2\sqrt{7}\text{ cm} \Rightarrow = 5.29\text{cm}$$

$$\because \cos 60^\circ = \frac{x}{4}$$
$$\frac{1}{2} = \frac{x}{4}$$
$$2x = 4$$
$$\Rightarrow \quad x = 2$$

Q2. In a $\triangle ABC$, m$\overline{AB}$ = 6cm, m$\overline{BC}$ = 8cm ,m$\overline{AC}$ = 9cm and D is the midpoint of side $\overline{AC}$. Find length of the median $\overline{BD}$

Solution:



According to the figure, we have

$$m\overline{AD} = \overline{DC}$$
$$and \quad \overline{mAC} = m\overline{AD} + m\overline{DC}$$
$$\overline{mAC} = m\overline{AD} + m\overline{AD}$$
$$9 = 2m\overline{AD}$$
$$Or \quad 2m\overline{AD} = 9$$

$$m\overline{AD} = \frac{9}{2} = 4.5 \text{ cm}$$

We know that

$$\left(\overline{AC}\right)^2 + \left(\overline{BC}\right)^2 = 2\left[\left(\overline{AD}\right)^2 + \left(\overline{BD}\right)^2\right]$$

$$(6)^2 + (8)^2 = 2\left[(4.5)^2 + \left(\overline{BD}\right)^2\right]$$

$$36 + 64 = 2(4.5)^2 + 2\left(\overline{BD}\right)^2$$

$$100 = 40.5 + 2\left(\overline{BD}\right)^2$$

Or $2\left(\overline{BD}\right)^2 = 100 - 40.5$

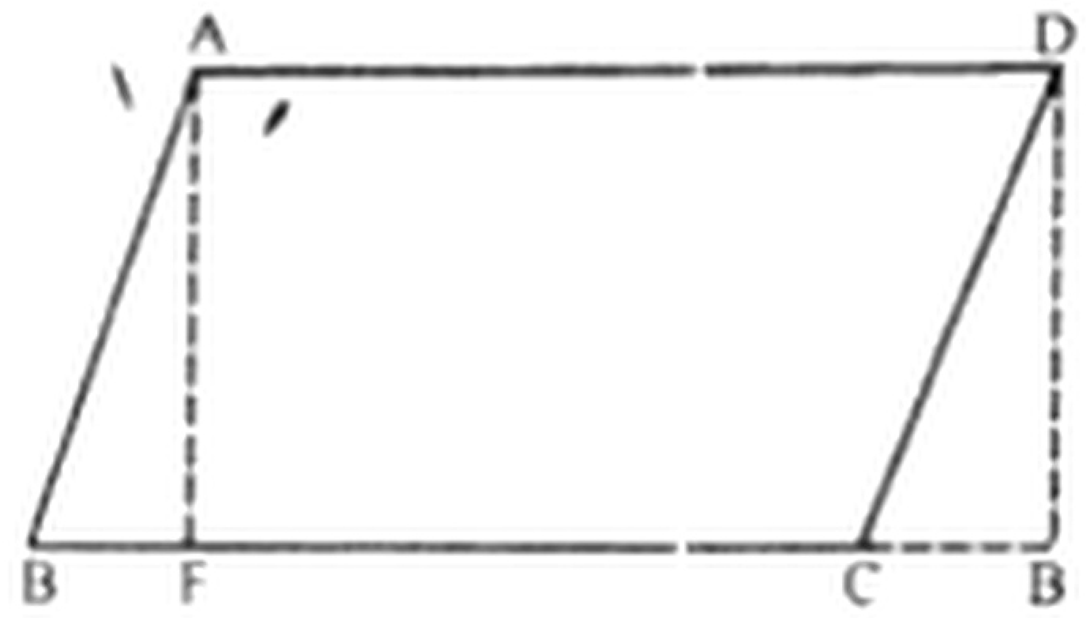
$$2\overline{BD}^2 = 59.5$$

$\Rightarrow$ $\overline{BD}^2 = 29.75$

$\Rightarrow$ $\overline{BD} = \sqrt{29.75} = 5.45 \text{ cm}$

Q3. In a parallelogra ABCD prove that $\left(\overline{AC}\right)^2 + \left(\overline{BD}\right)^2 = 2\left[\left(\overline{AB}\right)^2 + \left(\overline{BC}\right)^2\right]$

Solution:



$$\left(\overline{BD}\right)^2 = \left(\overline{CD}\right)^2 + \left(\overline{BC}\right)^2 + 2\left(\overline{BC}\right)\left(\overline{CE}\right) \qquad (1)$$

$$\left(\overline{AC}\right)^2 = \left(\overline{AB}\right)^2 + \left(\overline{BC}\right)^2 - 2\left(\overline{BC}\right)\left(\overline{BF}\right) \qquad (2)$$

Adding (1) and (2), we get

$$\left(\overline{AC}\right)^2 + \left(\overline{BD}\right)^2 = \left(\overline{CD}\right)^2 + \left(\overline{BC}\right)^2 + 2\left(\overline{BC}\right)\overline{CE} + \left(\overline{AB}\right)^2 + \left(\overline{BC}\right)^2 - 2\left(\overline{BC}\right)\left(\overline{BF}\right)$$

$$= \left(\overline{AB}\right)^2 + \left(\overline{CD}\right)^2 + 2\left(\overline{BC}\right)^2 + 2\left(\overline{BC}\right)\left(\overline{CE}\right)^2 - 2\left(\overline{BC}\right)\left(\overline{BF}\right)$$

In parallelogram opposite sides are congruent, so

$\overline{AB} = \overline{DC}, \overline{AD} = \overline{BC},$ and $\overline{BF} = \overline{CE}$

$$\left(\overline{AC}\right)^2 + \left(\overline{BD}\right)^2 = 2\left(\overline{AB}\right)^2 + \left(\overline{AB}\right)^2 2\left(\overline{BC}\right)^2 + 2\left(\overline{CE}\right) - 2\left(\overline{BC}\right)\overline{CE}$$

$$\left(\overline{AC}\right)^2 + \left(\overline{BD}\right)^2 = 2\left[\left(\overline{AB}\right)^2 + \left(\overline{BC}\right)^2\right]$$

Hence Proved.