# Exercise 6.7

**1. Find G.M. between**

**(i)** $-2 \text{ and } 8$ **(ii)** $-2i \text{ and } 8i$

**Solution:**

**i.** $-2 \text{ and } 8$

$$\text{G.M.} = \sqrt{ab}$$
$$= \sqrt{-(2)(8)} = \sqrt{-16} = \sqrt{-1 \times 16} = \sqrt{i^2 16} = 4i$$

Hence, G.M. $= 4i$

**ii.** $-2i \text{ and } 8i$

$$\text{G.M} = \sqrt{-2i \times 8i}$$
$$= \sqrt{-16i^2}$$
$$= \sqrt{(-1)16(-1)}$$
$$= \sqrt{16} = 4$$

Hence, G.M. $= 4$

**2. Insert two G.Ms between**

**(i)** **1 and 8** **(ii)** **2 and 16**

**Solution:**

**i.** **Let the two G.Ms 1 and 8 is $G_1, G_2$**
**1, $G_1, G_2$, 8**

We know that

$$a_n = ar^{n-1}$$

$$a_n = (1)r^{4-1}$$

$8 = r^3$

$2^3 = r^3$

$r = 2$

$G_1 = ar = (1)(2) = 2$

$G_2 = G_1 r = (2) \times (2) = 4$

Hence, $G_1 = 2, G_2 = 4$

**ii. Let the two G.Ms 2 and 16 is $G_1, G_2$**
**2, $G_1, G_2$, 16**

We know that

$a_n = ar^{n-1}$

$16 = 2r^{8-1}$

$8 = r^{4-1}$

$r^3 = 8$

$r = \sqrt[3]{8}$

$r = 2$

$G_1 = ar = 2(2) = 4$

$G_2 = G_1 r = 4(2) = 8$

Hence, $G_1 = 4$

$G_2 = 8$

**3. Insert three G.Ms between**

**(i) 1 and 16 (ii) 2 and 32**

**Solution:**

**i. 1 and 16**

Let the two G.Ms between 1 and 16 is $G_1, G_2, G_3$

**1, $G_1, G_2, G_3$, 8**

We know that

$a_n = ar^{n-1}$

$a_5 = ar^4$

$16 = (1)r^4$

$r^4 = 16$

$r^4 = 2^4$

$r = 2$

$G_1 = ar = (1)(2) = 2$

$G_2 = G_1 \times r = (2)(2) = 4$

$G_3 = G_2 \times r = (4)(2) = 8$

Hence,

$G_1 = 2, G_2 = 4$ & $G_3 = 8$

**ii. 2 and 32**

Let the two G.Ms between 2 and 32 is $G_1, G_2, G_3$

2, $G_1, G_2, G_3$, 32

We know that

$a_n = ar^{n-1}$

$a_5 = ar^{5-1}$

$16 = (1)r^4$

$r^4 = 16$

$r^4 = 2^4$

$r = 2$

$G_1 = ar = 2(2) = 4$

$G_2 = G_1 \times r = 4(2) = 8$

$G_3 = G_2 \times r = 8(2) = 16$

Hence, $G_1 = 4, G_2 = 8$ & $G_3 = 16$

**4. Insert four real geometric means between 3 and 96.**

**Solution:**

Let four real geometric means between 3 and 96 is $G_1, G_2, G_3, G_4$ *then* $3, G_1, G_2, G_3, G_4, 96.$

We know that

$a_n = ar^{n-1}$

$a_6 = (3)r^{6-1}$

$96 = 3r^5$

$32 = r^5$

$r^5 = 2^5$

$r = 2$

$G_1 = ar = (3)(2) = 6$

$G_2 = G_1 \times r = (6)(2) = 12$

$G_3 = G_2 \times r = (12)(2) = 24$

$G_4 = G_3 \times r = (24)(2) = 48$

Hence, $G_1 = 6, G_2 = 12, G_3 = 24$ & $G_4 = 48$

**5. If both x and y are positive distinct real numbers, show that the geometric mean between x and y is less than their arithmetic mean.**

**Solution:**

Let $a = x$ & $b = y$ $\begin{bmatrix} \therefore x > 0 \\ y > 0 \end{bmatrix}$

Then G.M. $= \sqrt{xy}$

And A.M. $= \frac{x+y}{2}$

We want to prove

A.M. G.M. $> 0$

$$\frac{x+y}{2} - \sqrt{xy} > 0$$

$$\frac{x^2 + y^2 - 2\sqrt{xy}}{2} > 0$$

$$\frac{(\sqrt{x})^2 + (\sqrt{y})^2 - 2\sqrt{x}\sqrt{y}}{2} > 0$$

$$\frac{\left(\sqrt{x} - \sqrt{y}\right)^2}{2} > 0$$

$$\left(\sqrt{x} - \sqrt{y}\right) > 0$$

Which shows A.M > G.M

Hence, proved

**6. For what value of 'x' is $\frac{a^2+b^2}{a^{n\ 1}+b^{n\ 2}}$, the geometric mean between 'a' and 'b'?**

**Solution:**

We know that the geometric mean between 'a' and 'b'.

G.M. $= \sqrt{ab}$

And G.M. $= \frac{a^2+b^2}{a^{n\ 1}+b^{n\ 2}}$

Therefore

$$\sqrt{ab} = \frac{a^2+b^2}{a^{n\ 1}+b^{n\ 2}}$$

$$a^{1/2}\,b^{1/2} = \frac{a^2+b^2}{a^{n\ 1}+b^{n\ 2}}$$

$$a^{1/2}\,b^{1/2}\,(a^{n-1}+b^{n-2}) = a^n + b^n$$

$$a^{n-1+1/2}\,b^{1/2} + b^{n-1+1/2}\,a^{1/2} = a^n + b^n$$

$$a^{n+1/2}\,b^{1/2} - a^n = b^n - a^{1/2}\,b^{n+1/2}$$

$$a^{n+1/2}\left(b^{1/2} - a^{1/2}\right) = b^{n+1/2}\left(b^{1/2} - a^{1/2}\right)$$

$$\frac{a^{n+1/2}}{b^{n+1/2}} = \frac{b^{1/2} - a^{1/2}}{b^{1/2} - a^{1/2}}$$

$$\left(\frac{a}{b}\right)^{n+1/2} = 1$$

$$\left(\frac{a}{b}\right)^{n+1/2} = \left(\frac{a}{b}\right)^0$$

by comparison

$$n + 1/2 = 0$$

$$n = -1/2$$

Hence proved value of $n = -1/2$

**7. The A.M. of two positive integral numbers exceeds their (positive) G.M by 1and their sum is 20. Find the number.**

**Solution:**

Let the two positive integral numbers 'a' and 'b'

Then $a + b = 20$

And $\frac{a+b}{2} = \sqrt{ab} + 2$

$\frac{20}{2} = \sqrt{ab} + 2$

$\sqrt{ab} + 2 = 10$

$\sqrt{ab} = 10 - 2 = 8$

$\sqrt{ab} = 8$

$\left(\sqrt{ab}\right)^2 = (8)^2$

$ab = 64$

$a = \frac{64}{b}$

Put the value of a in equation

$$\frac{64}{b} + b = 20$$

$$64 + b^2 = 20b$$

$$b^2 - 20b + 64 = 0$$

$$b^2 - 16b - 4b + 64 = 0$$

$$b(b - 16b) - 4(b - 16b) = 0$$

$$(b - 4)(b - 16b) = 0$$

Therefore

$b - 4 = 0$ or $b - 16 = 0$

$b = 4$ $b = 16$

So, the value of 'a'

$a = \frac{64}{b} = \frac{64}{4} = 16$

or $a = \frac{64}{b} = \frac{64}{16} = 4$

hence, $a = 16$ or $b = 4$

$a = 4$ or $b = 16$

**8. The A.M. between two numbers is 5 and their (positive) G.M. is 4. Find the numbers. Let the two number is a, b the**

**A.M.** $= \frac{a+b}{2} = 5$

**And** **G.M.** $= \sqrt{ab} = 4$

$\left(\sqrt{ab}\right)^2 = (4)^2$

$ab = 16$

From $\frac{a+b}{2} = 5$

$a + b = 10$ & $a = \frac{16}{b}$

Put the value of a in equation.

$\frac{16}{b} + b = 10$

$16 + b^2 = 10b$

$b^2 - 10b + 16 = 0$

$b^2 - 8b - 2b + 16 = 0$

$b(b-8) - 2(b-8) = 0$

$(b-2)(b-8) = 0$

Therefore

$b - 2 = 0$ or $b - 8 = 0$

$b = 2$ $b = 8$

Put the value in equation.

$a = \frac{16}{2} = 8$ or $a = \frac{16}{8} = 2$

Hence $a = 8 \ or \ 2$

$b = 2 \ or \ 8$