Exercise 7.4

Q1. Evaluate the following:

i. $^{12}C_5$ **ii.** $^{20}C_{17}$ **iii.** $^{n}C_4$

i. $^{12}C_5$

Solution:

$$= \frac{12!}{3!(12-3)!}$$

$$= \frac{12\times11\times10\times9!}{3\times2\times1\times9!} = 220$$

Hence, $^{12}C_5 = 220$

ii. $^{20}C_{17}$

Solution:

$$= \frac{20!}{17!(20-17)!}$$

$$= \frac{20\times18\times18\times17!}{17 \times 3!}$$

$$= \frac{20\times19\times18}{3\times2\times1} = 1140$$

Hence, $^{20}C_{17} = 1140$

iii. $^{n}C_4$

Solution:

$$= \frac{n!}{4!(x-4)!}$$

$$= \frac{n(n-1)(n-2)(n-3)(n-4)!}{4!(x-4)!}$$

$$= \frac{n(n-1)(n-2)(n-3)}{4\times3\times2\times1}$$

$$= \frac{1}{24}\left(n(n-1)(n-2)(n-2)\right)$$

Hence, $\quad {}^nC_4 \quad = \frac{1}{24}\left[\left(n(n-1)(n-2)(n-2)\right)\right]$

Q2. Find the value of, when

i. $\quad {}^nC_5 = {}^nC_4 \qquad$ *ii.* ${}^nC_{10} = \frac{12\times11}{2!} \qquad$ *iii.* ${}^nC_{12} = {}^nC_6$

i. $\quad {}^nC_5 = {}^nC_4$

Solution:

$${}^nC_5 = {}^nC_4$$

$$\frac{n!}{5!(n-5)!} = \frac{n!}{4!(n-4)!}$$

$$\frac{(n-4)(n-5)!}{(n-5)!} = \frac{5!}{4!}$$

$$\frac{(n-4)(n-5)!}{(n-5)!} = \frac{5\times4!}{4!}$$

$$n-4=5$$

$$n = 5+4=9$$

Hence, n $= 9$

ii. ${}^nC_{10} = \frac{12\times11}{2!}$

Solution:

$${}^nC_{10} = \frac{12\times11\times10!}{2!\times10!}$$

$$^nC_{10} = \frac{12!}{10!(12-10)!}$$

$$n = 12$$

Hence, $n = 12$

iii. $^nC_{12} = {}^nC_6$

Solution:

$$^nC_{12} = {}^nC_6$$

$$n - 12 = 6$$

$$n = 6 + 12 = 18$$

Hence, $n = 18$

Q3. Find the values of n and r, when

i. $^nC_r = 35$ **and** $^nP_r = 210$

ii. $^{n-1}C_{r-1} : {}^nC_r : {}^{n+1}C_{r+1} = 3 : 6 : 11$

i. $^nC_r = 35$ **and** $^nP_r = 210$

Solution:

$$^nC_r = 35$$

$$\frac{n!}{n!(n-r)!} = 35$$

$$\frac{n!}{(n-r)!} = 35\, r!$$

$$^nP_r = 35\, r!$$

$$210 = 35r!$$

$$r! = \frac{210}{35} = 6$$

$$r! = 6$$

$$r! = 3 \times 2 \times 1$$

$$r! = 3!$$

$$r = 3$$

Put the value of 'r' in any equation

$$^{n}C_{3} = 35$$

$$\frac{n!}{3!(n-r)!} = 35$$

$$\frac{n(n-1)(n-2)(n-3)!}{3! \quad (n-3)!} = 35$$

$$n(n-1)(n-2) = 35 \times 3!$$

$$n(n-1)(n-2) = 35 \times 32 \times 1$$

$$n(n-1)(n-2) = 5 \times 7 \times 6$$

Or

$$n(n-1)(n-2) = 7(7-1)(7-2)$$

Therefore, by comparison

$$n = 7$$

Hence, $r = 3 \; and \; n = 7$

$ii.$ $^{n-1}C_{r-1}$: $^{n}C_{r}$: $^{n+1}C_{r+1}$ $= 3:6:11$

Solution:

$$^{n-1}C_{r-1} : {}^{n}C_{r} = 3:6$$

$$\frac{(n!)}{(r-1)![(n-1)-(r-1)]!} + \frac{n!}{r!(n-r)!} = 3 + 6$$

$$\frac{(n!)}{(r-1)![(n-1)-(r-1)]!} \times \frac{n!(n-r)!}{n!} = \frac{3}{6}$$

$$\frac{(n-1)!}{(r-1)![(n-1)-(r-1)]!} \times \frac{r!(r-1)!(n-r)!}{r!(n-r)!} = \frac{3}{6}$$

$$\frac{1}{(n-r)!} \times \frac{(n-r)!}{n} r = \frac{3}{6}$$

$$\frac{r}{n} = \frac{1}{2}$$

$$n = 2r$$

And

$^{n}C_{r}$ $^{n+1}C_{r+1}$ $6:11$

$$\frac{n!}{r!(n-r)!} + \frac{(r+1)!}{(r+1)![(n+1)-(r+1)]!} = 6 + 11$$

$$\frac{n!}{r!(n-r)!} \times \frac{(r+1)!|n-r|!}{(r+1)!} = \frac{6}{11}$$

$$\frac{n!}{r!} \times \frac{(r+1)r!}{(n+1)n!} = \frac{6}{11}$$

$$\frac{r+1}{n+1} = \frac{6}{11}$$

Put the value of n in equation

$$\frac{r+1}{2r+1} = \frac{6}{11}$$

$$11(r+1) = 6(2r+1)$$

$$11r + 11 = 12r + 6$$

$$12r - 11r = 11 - 6$$

$$r = 5$$

So,

$$n = 2r$$

$$= 2(5) = 10$$

Hence,

$r = 5$ and $n = 10$

Q4. How many (a) diagonals and (b) triangles can be formed by joining the verities of the polygon having:

i. 5 sides ii. 8 sides ii. 12 sides

Solution:

***i.* number of diagonals of sided polygon $^5C_2 - 5$**

$$= \frac{5!}{2!(5-2)!} - 5$$

$$= \frac{5\times4\times3!}{2\times1\times3!} - 5$$

$$= \frac{20}{2} - 5$$

$$= 10 - 5 = 5$$

Hence, number of diagonals of 5-sided polygon

$$= 5$$

Number of diagonals of 8-sided polygon

$$^8C_2 - 5$$

$$= \frac{8!}{2!(8-2)!} - 8$$

$$= \frac{8\times7\times6!}{2! \quad 6!} - 8$$

$$= \frac{56}{2} - 8$$

$$= 28 - 8 = 20$$

Hence, number of diagonals of 12-sided polygon

$$= 20$$

Number of diagonals of 12-sided polygon

$^{12}C_2 \; -12$

$= \frac{12!}{2!(12-2)!} - 12$

$= \frac{11\times11\times10!}{2\times1\times10!} - 12$

$= 66 - 12 = 54$

Hence, number of diagonals of 12-sided polygon

$= 54$

ii. **Number of triangles of 5-sided polygon**

Solution:

$= {}^{5}C_3 = \frac{5!}{3!(5-3)!}$

$= \frac{5\times4\times3!}{3!(2\times1)}$

$= 10$

Hence the number of triangles of side polygon

$= 10$

Number of triangles of 8-sided polygon

$= {}^{8}C_5 = \frac{8!}{3!(8-3)!}$

$= \frac{8\times7\times6\times5!}{(3\times2\times1)\times5!} = 56$

Hence the number of triangles of 8 side polygon

$= 56$

Number of triangles of 12-sided polygon

$^{12}C_3 = \frac{12!}{3!(12-3)!}$

$$= \frac{12\times11\times10\times9!}{(3\times2\times1)\times9!}$$

$$= 220$$

Hence, Number of triangles of 12-sided polygon

$$= 220$$

Q5. The members of a club are 12 boys and 8 girls. In how many ways can a committee of 3 boys and 2 girls be formed?

Solution:

Total no. of boys = $12\ boys$

Total no. of girls = $8\ girls$

No. of boys in the committee = $3\ boys$

Members can be set = $^{12}C_3 \times {}^8C_2$

$$= \frac{12!}{3!''(12-3)!} \times \frac{8!}{2!(8-2)!}$$

$$= \frac{12\times11\times10\times9!}{(3\times2\times1)\times9!} \times \frac{8\times7\times6\times!}{(2\times1)(6!)}$$

$= 220 \times 28 = 6160$ ways.

Hence, the number of clubs can be formed

$= 6160$ ways

Q6. How many committees of 5 members can be chosen from a group of 28 persons when each committee must include 2 particular persons?

Solution:

Total no of persons= 8

Each committee include 2 particular people out of 8

It means we have to select 3 persons (members) out of 6 persons

$$^6C_3 = \frac{6!}{3!(6-3)!}$$

$$= \frac{6\times5\times4\times3!}{3!\times3!}$$

$$= \frac{6\times5\times4}{3\times2\times1} = 20$$

Hence, the committees od 5 members can be chosen = 20 ways

Q7. In how many ways can hockey team of 11 players be selected out of 15 players? How many of them will include a particular player?

Solution:

Total no of players = 15 players

Select from total number of players = 11 players

Thus, $^{15}C_{11} = \frac{15!}{11!(15-11)!}$

$$= \frac{15\times14\times13\times12\times11}{11!(4!)}$$

$$= \frac{15\times14\times13\times12}{4\times3\times2\times1} = 1365 \text{ ways}$$

Hence, no ways can be selected

= 1365 ways

Let one particular player in every team

Then

Total members = 15 − 1 = 14 players

Team members = 11 − 1 = 10 players

Thus

$$^{14}C_{10} = \frac{14!}{10!(14-10)!}$$

$$= \frac{14\times13\times12\times11\times10!}{10!\times4!}$$

$$= \frac{14\times13\times12\times11}{4\times3\times2\times1} = 1365 \text{ ways}$$

Hence, no ways can be selected

$$= 1365 \text{ ways}$$

Q8. Show that: $^{16}C_{11} + {}^{16}C_{10} = {}^{17}C_{11}$

Solution:

$$\text{L.H.S} = {}^{16}C_{11} + {}^{16}C_{10}$$

$$= \frac{16!}{11!(16-11)!} + \frac{16!}{10!(16-10)!}$$

$$= \frac{16!}{10!6!}\left[\frac{1}{11} + \frac{1}{6}\right]$$

$$= \frac{16!}{10!6!}\left[\frac{6+11}{66}\right]$$

$$= \frac{16!}{10!6!}\left[\frac{17}{66}\right]$$

$$= \frac{17!}{10!5!11!\times6}$$

$$= \frac{17!}{(11!10!)\times(6\times5!)}$$

$$= \frac{17!}{11!\times6!} = \frac{17!}{11!(17-11)!}$$

$$= {}^{17}C_{11} = \text{R.H.S}$$

Hence, proved

$^{16}C_{11} + {}^{16}C_{10} = {}^{17}C_{11}$

Q9. There are 8 men and 10 women members of a club. How many committees of them can be formed, having:

i. **4 women** *ii.* **At the most 4 women** *iii.* **atleast 4 women ?**

Solution:

i.

Total men = 8 men

Total women = 10 women

4 men are selected in 7 member committees

Thus,

$$^{8}C_{13} + {}^{10}C_{4} = \frac{8!}{3!(8-3)!} \times \frac{10!}{4!(10-4)!}$$

$$= \frac{8\times7\times6\times5!}{(3\times2\times1)(5!)} \times \frac{10\times9\times8\times7\times6!}{(4\times3\times2\times1)\times6!}$$

$$= 8 \times 7 \times 10 \times 7 \times 3$$

$$= 11760$$

Hence, committees can be formed

$$= 11760 \text{ ways}$$

ii. **At the most 4 women**

Solution:

He committee should have following combination (no women, 7 men)

(1 women, 6 men), (2 women, 5 men)

(3 women, 4 men), (4 women, 3 men)

$$= ({}^{8}C_{7} + {}^{10}C_{0}) + ({}^{8}C_{6} + {}^{10}C_{1}) + ({}^{8}C_{5} + {}^{10}C_{2}) + ({}^{8}C_{4} + {}^{10}C_{3}) + ({}^{8}C_{3} + {}^{10}C_{4})$$

$$= \left[\frac{8!}{7!1!} \times \frac{10!}{10!0!}\right] + \left[\frac{8!}{6!2!} \times \frac{10!}{9!1!}\right] + \left[\frac{8!}{5!3!} \times \frac{10!}{8!2!}\right] + \left[\frac{8!}{4!14!} \times \frac{10!}{7!3!}\right] + \left[\frac{8!}{3!5!} \times \frac{10!}{6!4!}\right]$$

$$= 8 + \frac{10\times8\times7}{2\times1} + \frac{10\times9}{2\times1} + \frac{8\times7\times6}{3\times2\times1} + \frac{10\times9\times8}{3\times2\times1} + \frac{8\times7\times6\times5}{4\times3\times2\times1}$$

$$= 10 + 280 + 2520 + 8400 + 11760 = 22968$$

Hence committee can formed = 22968 ways

***iii.* Atleast 4 women**

Solution:

The committee should have following combinations.

(4 women, 3 men), (5 women, 2 men)

(6 women, 3 men), (7 women, 0 men)

$$= ({}^{10}C_4 + {}^{8}C_3) + ({}^{10}C_5 + {}^{8}C_2) + ({}^{10}C_6 + {}^{8}C_1) + ({}^{10}C_7 + {}^{8}C_9)$$

$$= \left[\frac{10!}{4!6!} \times \frac{8!}{2!5!}\right] + \left[\frac{10!}{5!5!} \times \frac{8!}{6!2!}\right] + \left[\frac{10!}{4!6!} \times \frac{8!}{7!1!}\right] + \left[\frac{10!}{3!7!} \times \frac{8!}{0!8!}\right]$$

$$= \left[\frac{10\times9\times8\times7\times6\times1}{4\times3\times2\times1\times6!} \times \frac{8\times7\times6\times5\times1}{(3\times2\times1)}\right] + \left[\frac{10\times9\times8\times7\times6\times1}{5\times4\times3\times2\times!} \times \frac{8!\times7\times6!}{6!(2\times1)}\right]$$

$$+ \left[\frac{10\times9\times8\times7\times6\times1!}{(4\times3\times2\times1)6\times!} \times \frac{8!7!}{7!}\right] + \left[\frac{10\times9\times8\times7!}{(3\times2\times1)7!} \times 1\right]$$

$$[10 \times 3 \times 7 \times 8 \times 7 \; + [2 \times 9 \times 2 \times 7 \times 4 \times 7 \; + [10 \times 3 \times 7 \times 8 \; + [5 \times 3 \times 8$$

$$= 11760 + 7056 + 1680 + 120 = 20616$$

Hence committee can form $= 20616$ ways

Q10. Prove that ${}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$

Solution:

L.H.S $= {}^{n}C_{r} + {}^{n}C_{r-1}$

$$= \frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!}$$

$$= \frac{n!}{(r-1)!(n-r)!}\left[\frac{1}{r} + \frac{1}{n-r+1}\right]$$

$$= \frac{n!(n+r)!}{(r-1)!r\times(n+r)!(n-r+1)}$$

$$= \frac{(n+1)!}{r!(n+1-r)!}$$

$= {}^{n-1}C_r$

$=$ R.H.S

Hence proved

${}^{n}C_r + {}^{n}C_{r-1} = {}^{n-1}C_r$