Exercise 6.4

1. Find the sum of all the integral multiples of between 4 and 97.

Solution:

Let the integral multiple of 3 is

Therefore 6, 9, 12, ……………, 96.

We know that

$a_n = a_1 + (n-1)d$

$96 = 6 + (n-1)3$

$3(n-1) = 96 - 6 = 90$

$(n-1) = \frac{90}{3} = 30$

$n - 1 = 30$

$n - 30 + 1 = 31$

$S_n = \frac{n}{2}[a + 1]$

$= \frac{31}{2}[6 + 96]$

$S_{31} = \frac{31}{2}[102] = 31 \times 51 = 1581$

Hence, $S_{31} = 1581$

2. Sum the series

i. $-3 + (-1) + 1 + 3 + 5 + \cdots \dots \dots \dots + a_{16}$.

ii. $\frac{3}{\sqrt{2}} + 2\sqrt{2} + \frac{5}{\sqrt{2}} + \cdots \dots \dots + a_{13}$.

iii. $1.11 + 1.41 + 1.71 + \cdots \dots \dots + a_{10}$.

Iv. $-8, -3\frac{1}{2} + 1 + \cdots \dots \dots + a_{11}$.

v. $(x-a)+(x+a)+(x+3a)+\cdots \ldots$ *to n terms.*

vi. $\frac{1}{1-\sqrt{x}}+\frac{1}{1-x}+\frac{1}{1+\sqrt{x}}+\cdots \ldots$ *to n terms.*

vii. $\frac{1}{1+\sqrt{x}}+\frac{1}{1-x}+\frac{1}{1-\sqrt{x}}+\cdots \ldots$ *to n terms.*

i. $-3+(-1)+1+3+5+\cdots \ldots \ldots \ldots + a_{16}.$

Solution:

We know that

$$S_n=\frac{n}{2}[2a+(n-1)d]$$

$$S_{16}=\frac{16}{2}[2(-3)+(16-1)(2)]$$

$$=8[-6+15(2)]$$

$$=8\,[-6+30]$$

$$=8\,[24]=194$$

Hence; $S_{16}=194$

ii. $\frac{3}{\sqrt{2}}+2\sqrt{2}+\frac{5}{\sqrt{2}}+\cdots \ldots \ldots + a_{13}.$

Solution:

We know that

$$S_n=\frac{n}{2}[2a+(n-1)d]$$

$$S_{13}=\frac{13}{2}[2\left(\frac{3}{\sqrt{3}}\right)+(13-1)\left(2\sqrt{2}-\frac{3}{\sqrt{2}}\right)]$$

$$=\frac{13}{2}[3\sqrt{2}+12\left(\frac{4-3}{\sqrt{2}}\right)]$$

$$=\frac{13}{2}\left[\frac{6+12(1)}{\sqrt{2}}\right]$$

$$= \frac{13}{2}\left[\frac{18}{\sqrt{2}}\right]$$

$$= \frac{13\times 18}{2\sqrt{2}} = \frac{13\times 9}{\sqrt{2}} = \frac{117}{\sqrt{2}}$$

Hence: $S_{13} = \frac{117}{\sqrt{2}}$

iii. $1.11 + 1.41 + 1.71 + \cdots \ldots \ldots + a_{10}$.

Solution:

We know that

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{10} = \frac{10}{2}[2(1.11) + (10-1)(1.41 - 1.11)]$$

$$= 5[2.22 + 9(0.30)]$$

$$= 5\,[2.22 + 2.7]$$

$$= 5\,[4.92] = 24.60$$

Hence; $S_{10} = 24.60$

Iv. $-8, -3\frac{1}{2} + 1 + \cdots \ldots \ldots + a_{11}$.

Solution:

$a = -8$ $\quad\quad$ $d = -3\frac{1}{2} - (-8)$

$$= -\frac{7}{2}8 + = \frac{-7+16}{2} = \frac{11}{2} = 4\frac{1}{2}$$

We know that

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{11} = \frac{11}{2}[2(-8) + (11-1)\left(4\frac{1}{2}\right)]$$

$$= 5\left[-16 + 10 \times \frac{9}{2}\right]$$

$$= 5[-16 + 45]$$

$$= 5[45] = \frac{319}{2} = 159\frac{1}{2}$$

Hence; $S_{11} = 159\frac{1}{2}$

v. $(\boldsymbol{x - a}) + (\boldsymbol{x + a}) + (\boldsymbol{x + 3a}) + \cdots ....$

Solution:

We know that

$$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

$$= \frac{n}{2}[2(x-a) + (n-1)2a]$$

$$= \frac{n}{2}[2x - 2a + 2an - 2a]$$

$$= \frac{n}{2}[2x + 2an - 4a]$$

$$= n[c + a(n-2)]$$

Hence; $S_n = n[c + a(n-2)]$

vi. $\frac{1}{1-\sqrt{x}} + \frac{1}{1-x} + \frac{1}{1+\sqrt{x}} + \cdots ....$

Solution:

$a = \frac{1}{1-\sqrt{x}}$; $d = \frac{1}{1-x} + \frac{1}{1+\sqrt{x}}$

$$= \frac{1-\sqrt{x}-1+x}{(1-x)(1-\sqrt{x})}$$

$$= \frac{x-\sqrt{x}}{(1-x)(1-\sqrt{x})}$$

$$= \frac{-\sqrt{x}(1-\sqrt{x})}{(1-x)(1-\sqrt{x})}$$

$$= \frac{-\sqrt{x}}{1-x}$$

We know that

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}[\frac{2\times 1}{1-\sqrt{x}} + (n-1)\left(\frac{-\sqrt{x}}{1-x}\right)]$$

$$= \frac{n}{2}\left[\frac{2}{1-\sqrt{x}} - \frac{\sqrt{x}(n-1)}{1-x}\right]$$

$$= \frac{n}{2}\left[\frac{2(1+\sqrt{x})-\sqrt{x}(n-1)}{1-x}\right]$$

$$= \frac{n}{2}\left[\frac{2+2\sqrt{x}-n\sqrt{x}+\sqrt{x}}{1-x}\right]$$

$$= \frac{n}{2}\left[\frac{3\sqrt{x}-n\sqrt{x}+2}{1-x}\right]$$

$$= \frac{n}{2}\left[\frac{(3-1)\sqrt{x}+2}{1-x}\right]$$

Hence; $S_n = \frac{n}{2}\left[\frac{(3-1)\sqrt{x}+2}{1-x}\right]$

vii. $\frac{1}{1+\sqrt{x}} + \frac{1}{1-x} + \frac{1}{1-\sqrt{x}} + \cdots ....$

Solution:

$$a = \frac{1}{1+\sqrt{x}}$$

$$d = \frac{1}{1-x} + \frac{1}{1-\sqrt{x}}$$

$$= \frac{1-(1-\sqrt{x})}{(1-x)}$$

$$= \frac{\sqrt{x}}{1-x}$$

We know that

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}[2\left(\frac{1}{1+\sqrt{X}}\right) + (n-1)\left(\frac{\sqrt{x}}{1-x}\right)]$$

$$= \frac{n}{2}\left[\frac{2}{1+\sqrt{X}} + \frac{\sqrt{x}(n-1)}{1-x}\right]$$

$$= \frac{n}{2}\left[\frac{2(1-\sqrt{x})+\sqrt{x}(n-1)}{1-x}\right]$$

$$= \frac{n}{2}\left[\frac{2-3\sqrt{x}-x\sqrt{n}}{1-x}\right]$$

$$= \frac{n}{2}\left[\frac{2-\sqrt{x}(3-n)}{1-x}\right]$$

Hence; $S_n = \frac{n}{2}\left[\frac{2-\sqrt{x}(3-n)}{1-x}\right]$

3. How many terms of the series.

i. -7+(-5) +(-3) +…… amount to 65?

ii. -7+(-4) +(-1) + …… amount to 114?

i. -7+(-5) +(-3) +…… amount to 65?

Solution:

a = -7

$$\boldsymbol{d} = -5 - (-7)$$

$$= -5 + 7 = 2$$

We know that

$$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

$$65 = \frac{n}{2}[2(-7) + (n-1)(2)]$$

$$130 = n[-14 + 2n - 2]$$

$$130 = n\,[2n - 16]$$

$$130 = 2n^2 - 16n$$

$$2n^2 - 16n - 130 = 0$$

$$2(n^2 - 8n - 65) = 0$$

$$n^2 - 8n - 65 = 0$$

$$n^2 - 13n + 5n - 65 = 0$$

$$n(n - 13) + 5(n - 13) = 0$$

$$(n + 5)(n + 13) = 0$$

Either

$n + 5 = 0$ or $n - 13 = 0$

$n = -5$ (not possible); $n = 13$ Hence; $n = 13$

ii. -7+(-4) +(-1) + ... amount to 114?

Solution:

a = -7

$$\boldsymbol{d} = -\mathbf{4} - (-\mathbf{7})$$

$$= -\mathbf{4} + \mathbf{7} = \mathbf{3}$$

We know that

$$S_n = \frac{n}{2}[2a_1 + (n - 1)d]$$

$$114 = \frac{n}{2}[2(-7) + (n - 1)(3)]$$

$$228 = n[-14 + 3n - 3]$$

$$228 = n\,[3n - 17]$$

$228 = 3n^2 - 17n$

$3n^2 - 17n - 228 = 0$

$3n^2 - 36n + 19n - 228 = 0$

$3n(n - 12) + 19(n - 12) = 0$

$(3n + 19)(n - 12) = 0$

Either

$3x + 9 = 0$

$3n = -19$

$n = \frac{-19}{3}$ (not possible)

Or $n - 12 = 0$

$n = 12$

Hence; $n = 12$

4. Sum the series

i. $3 + 5 - 7 + 9 + 11 - 13 + 15 + 17 - 19 + \cdots \ldots$ *to 3n terms*

ii. $1 + 4 - 7 + 10 + 13 - 16 + 19 + 22 - 25 + \cdots \ldots$ *to 3n terms*

i. $3 + 5 - 7 + 9 + 11 - 13 + 15 + 17 - 19 + \cdots \ldots$ *to 3n terms*

Solution:

$= (3 + 9 + 15 + \cdots \ldots n\ terms) + (5 + 11 + 17 + \cdots n\ terms) - (7 + 13 + 19 + \cdots n\ terms)$

$= \left[\frac{n}{2}[2(3) + (n - 1)(6)]\right] + \left[\frac{n}{2}[2(5) + (n - 1)(6)]\right]\left[\frac{n}{2}[2(7) + (n - 1)(6)]\right]$

$= \frac{n}{2}[(6 + 6(n - 1)) + (10 + 6(n - 1)) - 14 - 6(n - 1))]$

$= \frac{n}{2}[6 + 6(n - 1) + 10 + 6(n - 1) - 14 - 6(n - 1)]$

$= \frac{n}{2}[2 + 6(n - 1)]$

$= \frac{n}{2}\ 2[1 + 3(n - 1)]$

$= n[1 + 3(n - 1)]$

$= n[1 + 3n - 3]$

$= n[3n - 2]$

Hence, sum of series $= n[3n - 2]$

ii. $\mathbf{1 + 4 - 7 + 10 + 13 - 16 + 19 + 22 - 25 + \cdots \ldots to\ 3n\ terms}$

solution:

$$= (\mathbf{1} + 10 + 19 + \cdots \ldots n\ terms) + (4 + 13 + 19 + \cdots n\ terms) - (7 + 16 + 25 + \cdots n\ terms)$$

$= \left[\frac{n}{2}[2(1) + (n - 1)(9)]\right] + \left[\frac{n}{2}[2(4) + (n - 1)(9)]\right]\left[\frac{n}{2}[2(7) + (n - 1)(9)]\right]$

$= \frac{n}{2}[(2 + 9(n - 1)) + (8 + 9(n - 1)) - 14 + 9(n - 1))]$

$= \frac{n}{2}[2 + 9(n - 1) + 8 + 9(n - 1) - 14 + 9(n - 1)]$

$= \frac{n}{2}[-4 + 9(n - 1)]$

$= n[-4 + 9n - 9]$

$= n[9n - 13]$

Hence, sum of series $= n[9n - 13]$

5. Find the sum of 20 terms of the series whose rth term is 3r+1.

Solution:

$\boldsymbol{n = 20}$

$a_1 = 3(1) + 1 = 3 + 1 = 4$

$a_2 = 3(2) + 1 = 6 + 1 = 7$

$a_3 = 3(3) + 1 = 9 + 1 = 10$

$a_{20} = 3(20) + 1 = 60 + 1 = 61$

We know that

$S_n = \frac{n}{2}[\, a + a_{20}]$

$S_{20} = \frac{20}{2}[\, 4 + 61]$

$= 10[65] = 650$

$hence; \; S_{20} = 650$

6. If $S_n = n(2n - 1)$, *then find the series.*

Solution:

$S_n = n(2n - 1)$

$S_{n-1} = (n - 1)[2(n - 1) - 1]$

$= (n - 1)[2n - 2 - 1]$

$= (n - 1)[2n - 3] = 2n^2 - 3n - 2n + 3$

$S_{n-1} = 2n^2 - 5n + 3$

$a_n = S_n - S_{n-1}$

$= 2n^2 - n - [2n^2 - 5n + 3]$

$= \; 2n^2 - n - 2n^2 + 5n - 3$

$a_n = 4n - 3$

So, $a_1 = 4(1) - 1 = 4 - 3 = 1$

$a_2 = 4(2) - 1 = 8 - 3 = 5$

$a_3 = 4(3) - 1 = 12 = 9$

Hence, series 1, 5, 9, ………, $(4n - 3), \ldots \ldots$

7. The ratio of the sums of n term of two series in A.P. is 3n+2 ; n+1. Find the Ratio of their 8th term.

Solution:

Let the first term

Where

First term = a & difference = d ; $sum = S_n$

$$\frac{Sn}{s_a} = \frac{3n+2}{n+1}$$

$$\frac{\frac{n}{2}[2a+(n-1)d]}{\frac{n}{2}[2a'+(n-1)d']} = \frac{3n+2}{n+1}$$

$$\frac{2a+(n-1)d}{2a'+(n-1)d'} = \frac{3n+2}{n+1}$$

We know that A.P. to 8th term = $a + 7d$

And $\frac{a+\left(\frac{(n\ 1)}{2}\right)d}{a+\left(\frac{(n\ 1)}{2}\right)d} = \frac{3n+2}{n+1}$

Then $a + \left(\frac{(n-1)}{2}\right) d = a + 7d$

By comparison

$$\frac{n-1}{2} = 7$$

$$n - 1 = 14$$

$$n = 14 + 1 = 15$$

$so, value\ of\ n\ is\ 15.$

The required ration

$$\frac{a+7d}{a'+7d} = \frac{3(15)+2}{15+1} = \frac{45+2}{15+1} = \frac{47}{16}$$

hence; $\frac{a+7d}{a'+7d} = \frac{47}{16}$

8. If $S_{2,}S_{3,}S_{5,}$ are the sums of 2n, 3n, 5n, terms of an A.P., show that $S_5 = 5(S_{3,} - S_{2,})$

Solution:

We know that

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{2,} = \frac{2n}{2}[2a + (2n-1)d]$$

$$S_{3,} = \frac{3n}{2}[2a + (3n-1)d]$$

$$S_{5,} = \frac{5n}{2}[2a + (5n-1)d]$$

So, $S_5 = 5(S_{3,} - S_{2,})$

R.H.S $= 5(S_{3,} - S_{2,})$

$$= 5\left[\frac{3n}{2}[2a + (3n-1)d] - \frac{2n}{2}[2a + (2n-1)d\right]$$

$$= 5\left[\frac{6an}{2} + \frac{9n^2d}{2} - \frac{3n}{2}d - \frac{4an}{2} - \frac{4n^2d}{2} + \frac{2}{2}nd\right]$$

$$= 5\left[\frac{6an-4an}{2} + \frac{9n^2d-4n^2d}{2} - \frac{3nd-2nd}{2}\right]$$

$$= 5\left[\frac{2an}{2} + \frac{5n^2d}{2} - \frac{nd}{2}\right]$$

$$= \frac{5}{2}[2an + 5n^2d + nd]$$

$$= \frac{5n}{2}[2a + (5n-1)d]$$

$$= R.H.S.$$

Hence proved

$$S_5 = 5(S_{3,} - S_{2,})$$

9. obtain the sum of all integers in the first 1000 integers which are neither divisible by 5 nor by 2.

Solution:

Let the integers are

1, 3, 7, 9, 11, 13, 17, 19, 21, 23,……… upto 400 terms.

Sum of the terms

$$= (\mathbf{1} + 11 + 21 + \cdots \ldots 100\ terms) + (3 + 13 + 23 + \cdots 100\ terms)$$

$$-(7 + 17 + 27 + \cdots 100\ terms)(9 + 19 + 29 + \cdots 100\ term)$$

$$= \left[\frac{n}{2}[2 + (n-1)(10)]\right] + \left[\frac{n}{2}[6 + (n-1)(10)]\right] + \left[\frac{n}{2}[14(n-1)(10)]\right]$$

$$+ \left[\frac{n}{2}[18 + (n-1)(10)]\right]$$

$$= \left[\frac{n}{2}[2 + 10n - 10]\right] + \left[\frac{n}{2}[6 + 10n - 10]\right] + \left[\frac{n}{2}[14 + 10n - 10]\right] +$$

$$\left[\frac{n}{2}[18 + 10n - 10]\right]$$

$$= \left[\frac{100}{2}[10n - 8]\right] + \left[\frac{100}{2}[10n - 4]\right] + \left[\frac{100}{2}[10n + 4]\right] + \left[\frac{100}{2}[10n + 8]\right]$$

$$= 50[10n - 8 + 10n - 4 + 10n + 4 + 10n + 8]$$

$$= 50\ [40n]$$

$$= 50\ [40 \times 100]$$

$$= 50 \times 4000$$

$$s_n = 200000$$

Hence, sum of series = 200000

10. s_8 *and* s_9 are the sums of the first eight and nine terms of an A.P., find s_9 *if* 50 $s_9 = 63\ s_8$ *and* $a_1 = 2$.

Solution:

$50\ s_9 = 63\ s_8$

$50(s_8 + s_9) = 63\ s_8$

$50s_8 + 50s_9 = 63\ s_8$

$63s_8 - 50s_8 = 50s_9$

$13s_8 = 50s_9$

$13\left[\frac{8}{2}[2a_1 + (8-1)d\right] = 50[a_1 + (9-1)d]$

$13[4(2a_1 + 7d)] = 50[a_1 + 8d$

$52(2a_1 + 7d) = 50[a_1 + 8d]$

$52(2(2) + 7d) = 50(2 + 8d)$

$52(4 + 7d) = 50(2 + 8d)$

$208 + 364d = 100 + 400d$

$400d - 364d = 208 - 100$

$36d = 108$

$d = \frac{108}{36} = 3$

Therefore, $a_1 = 2$ and $d = 3$

$a_1 = a = 2$

$a_2 = a + d = 2 + 3 = 5$

$a_3 = a + 3d = 2 + 2(3) = 2 + 6 = 8$

$a_n = a + 3d = 2 + 3(3) = 2 + 9 = 11$

Hence; series 2, 5, 8, 11, …………

11. The sum of 9 terms of an A.P is 171 and the eight terms is 31. Find the series.

Solution:

$s_a = 171$ and $a_s = 31$

$\frac{9}{2}[2a + 8d] = 171$

$9[a + 4d] = 171$

$a + 4d = \frac{171}{9} = 19$

$a + 4d = 19$

And $\boldsymbol{a_s = a + 7d = 31}$

$a + 7d + 31$

Therefore $a + 7d + 31$

$\frac{\pm a \pm 4d = \pm 19}{3d = 12}$ (by subtracting)

$d = \frac{12}{3} = 4$

So, $a + 4d = 19$

$a + 4(4) = 19$

$a + 16 = 19$

$a = 19 - 16 = 3$

So, $a = 4$ and $d = 4$

Therefore series

$a_1 = a = 2$

$a_2 = a + d = 3 + 4 = 7$

$a_3 = a + 3d = 3 + 8 = 11$

$a_n = a + 3d = 3 + 12 = 15$

Hence series 3, 7, 11, 15, …………

12. The sum of s_9 and s_7 is 203 and $s_9 - s_7 = 49, s_9$ and s_7 being the sums of the first 7 and 9 terms of an A.P. respectively. Determine the series.

Solution:

Let the sum of first 7 terms of A.P. = s_7

let the sum of first a numbers of A.P. = s_9

$s_9 - s_7 = 49,$

And $\frac{s_9 + s_7 = 203}{2s_9 \quad = 252}$ (by adding)

$s_9 = \frac{252}{2} = 126$

$\frac{9}{2}[2a + 8d] = 126$

$9[2a + 8d] = 126 \times 2 = 252$

$18a + 72d = 252$

$18(a + 4d) = 252$ ……………. (i)

$a + 4d = \frac{252}{18} = 14$

$s_9 + s_7 = 203$

$\pm s_{9m} s_7 = \pm 49$ (by subtracting)

$2s_7 \quad = 154$

$s_7 = \frac{154}{2} = 77$

$\frac{7}{2}[2a + 6d] = 77$

$7[a + 3d] = 77$

$a + 3d = 11$

So,

$a + 4d = 14$

$\pm a \pm 3d = \pm 11$ (by subtracting)

$d = 3$

$a + 3d = 11$

$a + 3(3) = 11$

$a + 9 = 11$

$a = 11 - 9 = 2$

So, $a = 2;$ $d = 3$

$a_1 = a = 2$

$a_2 = a + d = 2 + 3 = 5$

$a_3 = a + 3d = 2 + 6 = 8$

$a_n = a + 3d = 2 + 3(3) = 11$

Hence series 2, 5, 8, 11, ……………….,

13. s_7 *and* s_9 are the sums of the first 7 and 9 an A.P. respectively. If $\frac{s_9}{s_7} = \frac{18}{11}$ and $a_7 = 20$, find the series.

Solution:

Let $s_7 =$ sum of first 7 terms of A.P.

$s_9 =$ sum of the first 9 terms of A.P.

$s_7 = \frac{7}{2}[2a + 6d]$

$s_7 = 7[2a + 3d = 7a + 21d$

And $s_9 = \frac{9}{2}[2a + 8d]$

$= 9[a + 4d] = 9a + 36d$

So, $\frac{s_9}{s_7} = \frac{9a+36d}{7a+21d} = \frac{18}{11}$

$\frac{9(a+4d)}{7a+21d} = \frac{18}{11}$

$\frac{a+4d}{7a+21d} = \frac{2}{11}$

$11(a + 4d) = 2(7a + 21d)$

$11a + 44d = 14a + 42d$

$14a - 11a + 42d - 44d = 0$

$3a - 2d = 0$

$3a - 2d = 0$ .... (i)

And $a_7 = a + 6d = 20$ .... (ii)

So, $a + 6d = 20$

$3a - 2d = 0$

$\underline{\pm 3a \pm 18d = \pm 60}$ (by subtracting)

$-20d = -60$

$d = \frac{-60}{-20} = 3$

So, $3a - 2d = 0$

$3a - 2(3) = 0$

$3a - 6 = 0$

$3a = 6$

$a = \frac{6}{3} = 2$

Therefore,

$a = 2; \qquad d = 3$

$a = 2$

$a + 2d = 2 + 2(3) = 5$

$a + 2d = 2 + 2(3) = 8$

$a + 3d = 2 + 3(3) = 11$

Hence series 2, 5, 8, 11, ……………….,

14. The sum of three numbers is an A.P. is 24 and their product is 440. Find the numbers.

Solution:

Let the numbers of A.P.

$a - d, a, a + d$

Then, $a - d + a + a + d = 24$

$3a = 24$

$a = \frac{24}{3} = 8$

And $(a - d)(a)(a + d) = 440$

$a[a^2 - d^2] = 440$

$8[(8)^2 - d^2] = 440$

$64 - d^2 = \frac{440}{8} = 55$

$64 - 55 = d^2$

$d^2 = 9$

$d = \sqrt{9} = \pm 3$

Therefore,

$a = 8;$ $a = 3$

$a - d = 8 - 3 = 5$

$a = 8$

$a + d = 8 \quad (-3) = 8 + 3 = 11$

$a = 8$

$a + d = 8 + 3 = 8 - 3 = 5$

Hence A.P. 5, 8, 11,

Or 11, 8, 5

15. Find the numbers in A.P. whose sum is 32 and the sum of whose squares is 276.

Solution:

Let the four numbers of A.P.

$a - 3d, \; a - d, \; a + d, \; a + 3d$

Then, $a - 3d + a - d + a + d + a + 3d = 32$

$4a = 32$

$a = \frac{32}{4} = 8$

And $(a-3d)^2+(a-d)^2+(a+3d)^2+(a+3d)^2=276$

$$a^2-6d+9d^2+a^2-2d+d^2+a^2+2d+d^2+a^2+6d+9d^2=276$$

$$4a^2+20d^2=276$$

$$4(a^2+5d^2)=276$$

$$(8)^2+5d^2=\frac{276}{4}$$

$$64+5d^2=69$$

$$5d^2=69-64=5$$

$$5d^2=5$$

$$d^2=\frac{5}{5}=1$$

$$d^2=1$$

$$d=\sqrt{1}=\pm 1$$

So,

$a=8;\quad d=1$ and $a=8;\quad d=-1$

$a-3d=8-3=5$ $\qquad$ $a-3d=8-(-3)=8+3=11$

$a-d=8-1=7$ $\qquad$ $a-d=8-(-1)=8+1=9$

$a+d=8+1=9$ $\qquad$ $a+3d=8+(-3)=8-1=7$

$a+3d=8+3=11$ $\qquad$ $a+3d=8+(-1)=8-3=5$

Hence; A.P. 5, 7, 9, 11

Or 11, 9, 7, 5.

16. Find the five numbers in A.P. whose sum is 25 and the sum of whose square is 135.

Solution:

Let the five numbers in A.P. be

$$a - 2d,\ \ a - d,\ \ a,\ \ a + d,\ \ a + 2d$$

Then, $a - 2d + a - d + a + a + d + a + 2d = 25$

$$5a = 25$$

$$a = \frac{25}{5} = 5$$

And $(a - 2d)^2 + (a - d)^2 + (a)^2 + (a + d)^2 + (a + 2d)^2 = 135$

$$a^2 - 4d + 4d^2 + a^2 - 2d + d^2 + a^2 + a^2 + 2d + d^2 + a^2 + 4d + 4d^2 = 276$$

$$5a^2 + 10d^2 = 135$$

$$5(a^2 + 10d^2) = 135$$

$$5(5)^2 + 10d^2 = 135$$

$$125 + 10d^2 = 135$$

$$10d^2 = 135 - 125 = 10$$

$$d^2 = \frac{10}{10} = 1$$

$$d^2 = 1$$

$$d^2 = \sqrt{1} = \pm 1$$

therefore,

$a = 5;\ \ d = 1$ and $a = 8;\ \ d = -1$

$a - 2d = 5 - 2(1) = 5 - 2 = 5$	$a - 2d = 5 - (-1) = 5 + 2 = 7$
$a - d = 5 - 1 = 4$	$a - d = 5 - (-1) = 5 + 1 = 6$
$a = 5$	$a = 5$
$a + d = 5 + 1 = 6$	$a + d = 5 + 2(-1) = 5 - 2 = 3$

Hence; A.P. 3, 4, 5, 6, 7

Or 7, 6, 5, 4, 3

17. The sum of the 6th and 8th terms of an A.P. is 40 and the product of 4th and 7th terms is 220. Find the A.P.

solution:

$\boldsymbol{a_6 + a_8 = 40}$ **and** $\boldsymbol{a_4 a_7 = 220}$

$a_1 + 5d + a_1 + 7d = 40$

$2a_1 + 12d = 40$

$2(a_1 + 6d) = 40$

$a_1 + 6d = \frac{40}{2} = 20$

$a_1 = 20 - 6d$

and $(a_4)(a_7) = 220$

$(a_3 + 3d)(a_1 + 6d) = 220$

$a_s^2 + 6a_1 d + 3a_1 d + 18d^2 = 220$

$a_1^2 + 9a_1 d + 18d^2 = 220$

$(20 - 6d)^2 + 9(20 - 6d)d + 18d^2 = 220$

$400 + 36d^2 - 240d + 180d - 54d^2 + 18d^2 = 220$

$-60d = 220 - 400$

$-60d = 220 - 400$

$-60d = -180$

$d = \frac{-180}{-60} = 3$

Therefore, $d = 3$

Put the value of 'd' in equation 'ii'

$a_1 = 20 - 6d$

$= 20 - 6(3)$

$= 20 - 18 = 2$

$a_1 = 2$

So,

$a_1 = 2$

$a_2 = a_1 + d = 2 + 3 = 5$

$a_3 = a_1 + 2d = 2 + 6 = 8$

$a_4 = a_1 + 3d = 2 + 9 = 11$

Hence series 2, 5, 8, 11, ………………..

18. If $a^2, b^2 \, and \, c^2$ are in A.P., show that $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P.

Solution:

If $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P.

The common difference between the two terms are same.

$$\frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a}$$

$$\frac{b+c-c-a}{(c+a)(b+c)} = \frac{c+a-a-b}{(c+b)(c+a)}$$

$$\frac{b-a}{(c+a)(b+c)} = \frac{(c-b)}{(c+b)(c+a)}$$

$$\frac{b-a}{b+c} = \frac{c-b}{a+b}$$

$$(b-a)(b+a) = (c-b)(c+b)$$

$$b^2 - a^2 = c^2 - b^2$$

It is a common difference of $a^2 - b^2$. which is given

Therefore $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ is A.P.

Hence, proved.