Measures of Dispersion:

Statistically, Dispersion means the spread or scatterness of observations in a data set. The spread or scatterness in a data set can *be seen* in two ways:

(i) The spread between two extreme observations in a data set.

(ii) The spread of observations around an average say their arithmetic mean.

The purpose of finding Dispersion is to study the behavior of each unit of population around the average value. This also helps in comparing two sets of data in more detail.

The measures that are used to determine the degree or extent of variation in a data set are called Measures of Dispersion.

We shall discuss only some important absolute measures of dispersion now.

(i) Range:

Range measures the extent of variation between two extreme observations of a data set. It is given by the formula:

Range = $X_{max} - X_{min} = X_m - X_0$

where $X_{max} = X_m$ = the maximum, highest or largest observation.

$X_{min} = X_0$ = the minimum, lowest or smallest observation.

The formula to find range for grouped continuous data is given below:

Range = (Upper class boundary of last group) - (lower class boundary of first group).

(ii) Variance:

Variance is defined as the mean of the squared deviations of x_i (i =1, 2, ……. n)

$$\text{Variance of X} = \text{Var}(X) = S^2 = \frac{\sum(X-\overline{X})^2}{n}$$

(iii) Standard Deviation:

Standard deviation is defined as the positive square root of mean of the squared deviations of X_i (i = 1, 2, …., n) observations from their arithmetic mean. In symbols we write.

$$\text{Standard Deviation of X} = \text{S.D}(X) = S = \sqrt{\frac{\sum(X-\overline{X})^2}{n}}$$

Computation of Variance and Standard Deviation:

We use the following formulae to compute Variance and Standard Deviation for Ungrouped and Grouped Data.

Ungrouped Data

The formula of Variance is given by:

$$\text{Var}(X) = S^2 = \frac{\sum X^2}{n} - \left(\frac{\sum X}{n}\right)^2$$

And Standard Deviation is given by:

$$\text{S.D}(X) = S = \sqrt{\left[\frac{\sum X^2}{n} - \left(\frac{\sum X}{n}\right)^2\right]}$$

Grouped Data:

$$\text{Var}(X) = S^2 = \frac{\sum fX^2}{\sum f} - \left(\frac{\sum fX}{\sum f}\right)^2$$

And Standard Deviation is given by:

$$\text{S.D}(X) = S = \sqrt{\left[\frac{\sum fX^2}{\sum f} - \left(\frac{\sum fX}{\sum f}\right)^2\right]}$$

Exercise 6.3

1. What do you understand by Dispersion?

Solution:

Dispersion:

Dispersion means the spread or scatterness of observations in a data set. By dispersion we mean the extent to which the observations in a sample or in a population are spread out. The main measures of dispersion are range, variance and standard deviation.

2. How do you define measure of dispersion?

Solution:

The measures that are used to determine the degree or extent of variation in a data set are called measure of dispersion.

Solution:

Range:

Range measures the extent of variation between two extreme observations of a data set. It is given by the formula:

Range = $X_{max} - X_{min}$

Range = (upper C. B of the last group) - (lower C. B of first group)

Variance:

The mean of the squared deviations of x_i (i = 1, 2, ..., n) observations from their arithmetic mean.

$$\text{Variance} = S^2 = \frac{\sum\left(X-\overline{X}\right)^2}{n}$$

$$= S^2 = \frac{\sum X^2}{n} - \left(\frac{\sum X}{n}\right)^2$$

Standard Deviation:

The positive square root of the squared deviations of x_i (i = 1, 2, 3, ..., n) observations from their mean.

$$\text{Standard Deviation} = S = \sqrt{\frac{\sum\left(X-\overline{X}\right)^2}{n}}$$

$$= S = \sqrt{\frac{\sum X^2}{n} - \left(\frac{\sum X}{n}\right)^2}$$

4. The salaries of five teachers in Rupees are as follows.

11500, 12400, 15000, 14500, 14800.

Find Range and standard deviation.

X = 11500, 12400, 15000, 14500, 14800

Here, X_{max} = 15000, X_{min} = 11500

Range = X_{max} - X_{min}

= 15000 - 11500 = 3500

$$\overline{X} = \frac{\sum X}{n}$$

$$= \frac{11500 + 12400 + 15000 + 14500 + 14800}{5}$$

$$= \frac{68200}{5} = 13640$$

X	$X - \overline{X}$	$(X - \overline{X})^2$
11500	-2140	4579600
12400	-1240	1537600
15000	1360	1849600
14500	860	739600
14800	1160	1345600

$$\sum(X - \overline{X})^2 = 10052000, n = 5$$

$$S.D = S = \sqrt{\frac{\sum(X - \overline{X})^2}{n}} = \sqrt{\frac{10052000}{5}}$$

$$= \sqrt{2010400} = 1417.88$$

5.

a. Find the standard deviation "S" of each set of numbers:

(i) 12, 6, 7, 3, 15, 10, 18, 5

(ii) 9, 3, 8, 8, 9, 8, 9, 18.

Solution:

(i)

X	$X-\overline{X}$	$\left(X-\overline{X}\right)^2$
12	2.5	6.25
6	-3.5	12.25
7	-2.5	6.25
3	-6.5	42.25
15	5.5	30.25
10	0.5	0.25
18	8.5	72.25
5	-4.5	20.25

$\sum x = 76$ $\sum\left(X-\overline{X}\right)^2 = 190, n = 8$

$$\overline{X} = \frac{76}{8} = 9.5$$

$$S.D = S = \sqrt{\frac{\sum\left(X-\overline{X}\right)^2}{n}} = \sqrt{\frac{190}{8}}$$

$$= \sqrt{23.75} = 4.87$$

(ii)

X	$X-\overline{X}$	$\left(X-\overline{X}\right)^2$
9	0	0
3	-6	36
8	-1	1
8	-1	1
9	0	0
8	-1	1
9	0	0
18	9	81

$\sum x = 72$ $\sum\left(X-\overline{X}\right)^2 = 120$

$$\overline{X} = \frac{\sum x}{n} = \frac{72}{8} = 9$$

$$S.D = S = \sqrt{\frac{\sum\left(X-\overline{X}\right)^2}{n}} = \sqrt{\frac{120}{8}}$$

$$= \sqrt{15} = 3.87$$

b. Calculate variance for the data: 10, 8, 9, 7, 5, 12, 8, 6, 8, 2.

Solution:

(i)

X	$X-\overline{X}$	$\left(X-\overline{X}\right)^2$
10	2.5	6.25
8	0.5	25
9	1.5	2.25
7	- 0.5	.25
5	- 2.5	6.25
12	4.5	20.25
8	0.5	.25
6	-1.5	2.25
8	0.5	.25
2	- 5.5	30.25

$\sum x = 75$ $\sum\left(X-\overline{X}\right)^2 = 68.5$

n = 10

$$\overline{X} = \frac{\sum x}{n} = \frac{75}{10} = 7.5$$

$$Variance = S^2 = \frac{\sum\left(X-\overline{X}\right)^2}{n}$$

$$= \frac{68.5}{10} = 6.85$$

6. The length of 32 items are given below. Find the mean length and standard deviation of the distribution.

Length	20-22	23-25	26-28	29-31	32-34
Frequency	3	6	12	9	2

Solution:

C.I	f	Mid-point (x)	fx	$X-\overline{X}$	$\left(X-\overline{X}\right)^2$	$f\left(X-\overline{X}\right)^2$
20 – 22	3	21	63	–6	36	108
23 – 25	6	24	144	–3	9	54
26 – 28	12	27	324	0	0	0
29 – 31	9	30	270	3	9	81
32 – 34	2	33	66	6	36	72
	$\sum f$ =n=32		$\sum fx$ = 867		$\sum\left(X-\overline{X}\right)^2$ =90	$\sum f\left(X-\overline{X}\right)^2$ =315

$$\overline{X}=\frac{\sum fx}{n}=\frac{667}{32}=27.093=27\,\text{approx}$$

$$S.D=S^2=\sqrt{\frac{\sum f\left(X-\overline{X}\right)^2}{n}}=\sqrt{\frac{315}{32}}$$

$$=\sqrt{9.84375}=3.137$$

7. **For the following distribution of marks calculate Range.**

Marks in percentage	Frequency/ (No of Students)
33 - 40	28
41 - 50	31
51 - 60	12
61 - 70	9
71 - 75	5

Solution:

C.I	Class Boundaries	f
33 - 40	32.5 – 40.5	28
41 - 50	40.5 – 50.5	32
51 - 60	50.5 – 60.5	12
61 - 70	60.5 – 70.5	9
71 - 75	70.5 – 75.5	5

Here, $X_{max} = 75.5$

$X_{min} = 32.5$

Range = $X_{max} - X_{min}$

$= 75.5 - 32.5$

$= 43$