Exercise 6.5

1. **A man deposits in a bank RS. 10 in first month; Rs. 15 in second month; Rs. 20 in third month and so on. Find how much will he have deposited in the bank by the 9th month.**

Solution:

Let the man deposit in the first month $= Rs. 10$

Second month $= Rs. 15$

Third month $= Rs. 20$

Therefore, 10, 15, 20, is A.P.

$a_1 = 10;$ $d = 15 - 10 = 5;$ $n = 9$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_9 = \frac{9}{2}[2(10) + (9-1)(5)]$

$= \frac{9}{2}[20 + 40]$

$= \frac{9}{2}[60]$

So, $9[30] = Rs. 270.$

Hence, the man deposited total $Rs. = 270$

2. **378 trees are planted in the shape of an isosceles triangle in a row the number in successive Rose decreasing by one from the base to top. ow many trees are there in the row which forms the base of the triangle?**

Solution:

Total number of trees $(s_9) = 278$

In first row $= 1$

Second row $= 2$

Third row $= 3$

And so on

Therefore, $1, 2, 3, 4.$ Is an AP.

$a = 1$, $d = 2 - 1 = 1$ $;n = ?$

We know that

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$378 = \frac{n}{2}[2(1) + (n-1)(1)]$$

$$378 = \frac{n}{2}[2 + (n-1]$$

$$756 = n[2 + n - 1]$$

$$756 = n[n + 1]$$

$$n^2 + n - 756 = 0$$
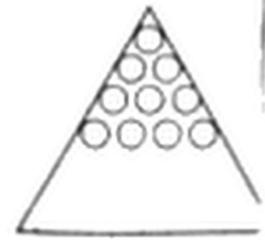
$$n^2 + 28n - 27n - (n - 28) = 0$$

$$n(n + 28) - 27(n - 28) = 0$$

$$(n - 27)(n + 28) = 0$$

Either $n - 27 = 0$ or $n + 28 = 0$

$n = 27$ or $n = -28$ which is not possible

Hence, the total number of rows $= 27$

3. **A man borrows Rs. 1100 and agrees to repay with a total interest of Rs. 230 in 14 installments, each installment being less than the preceding by Rs.10. what should be his first installment?**

Solution:

$$\text{Total repay} \quad = Rs.\,1100 + Rs.\,230$$

$$= Rs.\,1330$$

$$\text{No. of installments} = 14$$

$$\text{Difference} = Rs.\,10$$

$$\text{First installment} \quad = ?$$

We know that

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$1330 = \frac{14}{2}[2a + (14-1)(10)]$$

$$1330 = 7[2a + 13 \times 10]$$

$$\frac{1330}{7} = 2a + 130$$

$$2a + 130 = 190$$

$$2a = 190 - 130$$

$$2a = 60$$

$$a = \frac{60}{2} = 30$$

Hence, 1st instalment $= Rs.\,30$

4. **A clock strikes once when its hour hand is at one, twice when it is at two and so on. How many times does the clock strikes in twelve hours?**

Solution:

Let 1, 2, 3, ...12 ia an A.P.

Where

$a = 1;$ $\qquad\qquad d = 2 - 1 = 1;$ $\qquad n = 12$

Total strikes $(s_n) = ?$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{12}{2}[2(1) + (12-1)(1)]$$

$$= 6[2 + 11]$$

$$= 6[13]$$

So, $\qquad 6[13] = 78.$

Hence, the clock strikes $\qquad = 78 \, times$

5. **A student saves Rs. 12 at the end of the first week and goes on increasing his saving Rs. 4 weekly. After how many weeks will he be able to save Rs. 2100?**

Solution:

1st saving $= Rs. 12$

Difference $= Rs. 4$

Total savings $(s_n) = Rs. 2100$

No. of weeks $(n) = ?$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$2100 = \frac{n}{2}[2a + (12)(n-1)(4)]$$

$$2100 \times 2 = n[24 + 44n - 4]$$

$$4200 = n\,n + [24 + n]$$

$$4200 = 4n[5 + n]$$

$$\frac{4200}{4} = n\,[n + 5]$$

$$1050 = n^2 + 5n$$

$$n^2 + 5n - 1050 = 0$$

$$n^2 + 35n - 30n - 1050 = 0$$

$$n(n + 35) - 30(n + 35) = 0$$

$$(n + 35)(n - 30) = 0$$

Either

$$n + 35 = 0 \quad \text{or} \quad n - 30 = 0$$

$$n = -35 \quad \text{(not possible)}$$

And

$$n = 30$$

Hence;   no of weeks $= 30$

6. **An object falling form rest, falls 9 meters during the first second, 27 meters during the next second, 45 meters during the third second and so on.**

Solution:

   i.

let the object falls in first second $= 9m$

$$\text{second} \quad = 27m$$

$$\text{third second} \quad = 45m \text{ and so on}$$

So, 9, 27, 45, ... is an A.P.

Where $\quad a = 9$ $\qquad\qquad d = 27 - 9; n = 5; a_5 = ?$

We know that,

$$a_n = a_1 + (n - 1)d$$

$$a_5 = 9 + (5 - 1)18$$

$$= 9 + 4(18)$$

$$= 9 + 27 = 81$$

$$a_5 = 81$$

Hence, the object falls during fifth second $= 81\ meters$

ii.

$$s_n = ? \qquad a_5 = 81; \qquad a_1 = 9$$

$$s_n = \frac{n}{2}[a_1 + a_1]$$

$$s_n = \frac{5}{2}[81 + 9] = \frac{5}{2}[90] = 5 \times 45$$

$$s_n = 225m$$

Hence, the object will fall up to the fifth second $= 225m$

7. **An investor earned Rs. 6000 for year 19080 and Rs. 12000 for year 1990 on the same investment. If his earning have increased by the same amount each year, how much income he has received from the investment over the past eleven years?**

Solution:

Let the earnings of investor $=$ Rs. 6000

And in 1990 he invested $= Rs. 12000$

$$n = 1$$

Total investment $(S_n) = ?$

$$S_n = \frac{11}{2} [6000 + 12000]$$

$$= \frac{11}{2} (1800)$$

$$= 11 \times 9000$$

$$S_{11} = Rs. 9900$$

Hence, total investment $= Rs. 99000$

8. The sum of interior angles of polygon having sides 3, 4, 5, ... etc. from an A.P. Find the sum of the interior angles for a sided polygon.

Solution:

Sum of angles of polygon having 3 sides $= \pi$

Sum of angles of polygon having 4 sides $= 2\pi$

Sum of angles of polygon having 5 sides $= 3\pi$

Thus, $1\pi, 2\pi, 3\pi, \dots \dots a_{14}$

So,

$$a = \pi; \qquad\qquad d = 2\pi - \pi; \quad n = 14$$

And

$$a_0 = a_1 + (n - 1)d$$

$$a_{14} = \pi + (14 - 1)\pi$$

$$= \pi + (13)\pi$$

$$= \pi + 13\pi = 14\pi$$

$$a_{14} = 14\pi$$

Hence, the sum of interior angles for a 16-sided polygon $= 14\pi$.

9. **The prize money Rs. 60,000 will be distributed among the eight teams according to their positions determined in the match-series. The award increases by the same amount by each higher position. If the last place team is given Rs. 4000, how much will be awarded to the first-place team?**

Solution:

Total amount of prize $= Rs.\,60,000$

No. of terms $= 8$

Last place in the team $= Rs.\,4000$

Amount received by the first position $= ?$

We know that

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$60000 = \frac{8}{2}[2(4000) + (8 - 1)d]$$

$$60000 = 4[8000 + 7d]$$

$$\frac{60000}{4} = 8000 + 7d$$

$$7d = 15000 - 8000 = 7000$$

$$d = \frac{7000}{7} = 1000$$

$$a_n = a_1 + (n - 1)d$$

$$a_8 = 4000 + (8 - 1)(1000)$$

$$= 4000 + 7(1000)$$

$$= 4000 + 7000$$

$a_8 = Rs. 11000$

Hence, 1st position received = $Rs. 11000$

10. **An equilateral triangular base is filled by placing eight balls in the first row, 7 balls in the second row and so on with one ball in the last row. After this base layer, second layer is formed by placing 7 balls in the first row, 6 balls in its second row and so on with one ball in its last row. Continuing this process, a pyramid of balls is formed with one ball on top. How many balls are there in the pyramid?**

Solution:

Let us demote the sum of ball on 1st , 2nd, 3rd, ... and 8th layer by

$S_1, S_2, S_3, \ldots \ldots S_n$ respectively.

We know that

$$S_n = \frac{n}{2}(n + l)$$

$$S_8 = \frac{8}{2}(8 + 1) = 4(9) = 36$$

$$S_7 = \frac{7}{2}(7 + 1) = \frac{7}{2} \times 8 = 28$$

$$S_6 = \frac{6}{2}(6 + 1) = 3 \times 7 = 21$$

$$S_5 = \frac{5}{2}(5 + 1) = \frac{5}{2} \times 6 = 15$$

$$S_4 = \frac{4}{2}(4 + 1) = 2 \times 5 = 10$$

$$S_3 = \frac{3}{2}(3 + 1) = \frac{3}{2} \times 4 = 6$$

$$S_2 = \frac{2}{2}(2 + 1) = 3$$

$$S_1 = \frac{1}{2}(1 + 1) = \frac{1}{2} \times 2 = 1$$

Thus, the total no. of balls = 36 + 28 + 21 + 15 + 10 + 6 + 3 + 1 = 120

Hence, the no. of balls in pyramid = 120