Exercise 5.5

1. If L = {a, b, c}, M = {3, 4), then Find two binary relations of L × M and M × L.

Solution:

L $\qquad$ = {a, b, c}, $\quad$ (3, 4)

L × M = {a, b, c} × {3,4}

$\qquad$ = {(a, 3), (a, 4),(b, 3), (b, 4), (c, 3), (c, 4)}

Then R₁ = {(a, 3), (b, 4),(c, 3))

R₂ $\qquad$ = ((a, 4), (b, 3),(c, 4))

$\qquad$ = {(3, a), (3, b), (3, c), (4, a), (4, b), (4, c)}

Here R₁ = {(3, a), (4, a),(4, c)}

R₂ $\qquad$ = {(3, b), (4, c))

2. If Y = {-2, 1, 2), then make two binary relations for Y × Y. Also find their domain and range.

Solution:

Y $\qquad$ = {-2, 1, 2}

Y×Y $\quad$ = {-2, 1, 2} × {-2, 1, 2}

$\qquad$ = {(-2, -2)}, (-2,1), (-2, 2), (1, -2), (1, 1), (1, 2), (2, -2),

$\qquad$ (2, 1), (2, 2)}

R₁ $\qquad$ = {(-2, -2)}, (-2, 1), (1, 2), (2, 2)

Dom R₁ = {-2,1, 2}

Dom R₁ = {-2,1,2}

Dom R_2 = {-2, 1}

Range R_2 = {1, 2}

3. If L = (a, b, c} and M = {d, e, f, g}, then find two binary relations in each:

(i)   L × L   (ii) L × M   (iii) M × M

Solution

(i)   L × L

L   = {a, b, c}

Now LxL   = {a, b, c} × {a, b, c}

= {(a, a), (a, b), (a, c), (b, a), (b, b), (b, c), (c, a), (c, b), (c, c)}

R_1 = {(a, a)}

R_2 = {(a, b), (b, b), (c, b)}

(ii)   L × M

L   = {a, b, c}

M   = {d, e, f, g} × {a, b, c}

Now L × M   = {a, b, c}, {d, e, f, g}   „

= {(a, d), (a, e), (a, f), (a, g), (b, d), (b, e), (b, f), (b, g), (c, d), (c, e), (c, f), (c, g)}

Here R_1 = {(a, d), (b, f)}

R_2 = {(b, e), (b, g), (c, d), (c, e)}

(iii)   M × M

M   = {d, e, f, g}

Now M × M = {d, e, f, g} × {d, e, f, g}

= {(d, d), (d, e), (d, f), (d, g), (e, d), (e, e), (e, f), (e, g), (f, d), (f, e),

(f, f), (f, g), (g, d), (g, e), (g, f), (g, g)}

Here R_1 = {(d, e), (d, f), (f, f)}

      R_2 = {(d, f), (e, d), (e, e), (g, g)}

4. If set M has 5 elements, then find the number of binary relations in M.

Solution:

Number of elements in M = 5

Number of elements in M = 5

Number of elements in M × M = $2^{5×5}$ = 2^{25}

5. If L = {x | x ∈ N ∧ x ≤ 5}, M = {y | y ∈ P ∧ y < 10}, then make the following relations from L to M.

(i) R_1 = {(x, y) | y < x}   (ii) R_2 = {(x, y) | y = x}

(iii) R_3 = {(x, y} | x + y = 6}   (iv) R_4 = {(x, y) | y - x = 2}

Also write the domain and range of each relation.

(i) R_1 = {(x, y) | Y < x}   •
Solution:

       R_1 = {x | x ∈ N ∧ x ≤ 5},

Thus,   L   = {1, 2, 3, 4, 5}

and M   = {y | y ∈ P ∧ y < 10}
Thus, M   = {2, 3, 5, 7}

Now L × M = {1, 2, 3, 4, 5}, {2, 3, 5, 7}

               = {(1, 2), (1, 3), (1, 5), (1, 7), (2, 2), (2, 3), (2, 5), (2, 7),

               (3, 2), (3, 3), (3, 5), (3, 7), (4, 2), (4, 3), (4, 5), (4, 7),(5, 2), (5, 3), (5,