Exercise 5.5

1. If L = {a, b, c}, M = {3, 4), then Find two binary relations of L × M and M × L.

Solution:

L = {a, b, c}, (3, 4)

L × M = {a, b, c} × {3,4}

= {(a, 3), (a, 4),(b, 3), (b, 4), (c, 3), (c, 4)}

Then R_1 = {(a, 3), (b, 4),(c, 3))

R_2 = ((a, 4), (b, 3),(c, 4))

= {(3, a), (3, b), (3, c), (4, a), (4, b), (4, c)}

Here R_1 = {(3, a), (4, a),(4, c)}

R_2 = {(3, b), (4, c))

2. If Y = {-2, 1, 2), then make two binary relations for Y × Y. Also find their domain and range.

Solution:

Y = {-2, 1, 2}

Y×Y = {-2, 1, 2} × {-2, 1, 2}

= {(-2, -2)}, (-2,1), (-2, 2), (1, -2), (1, 1), (1, 2), (2, -2),

(2, 1), (2, 2)}

R_1 = {(-2, -2)}, (-2, 1), (1, 2), (2, 2)

Dom R_1 = {-2,1, 2}

Dom R_1 = {-2,1,2}

Dom R_2 = {-2, 1}

Range R_2 = {1, 2}

3. If L = {a, b, c} and M = {d, e, f, g}, then find two binary relations in each:

(i) L × L (ii) L × M (iii) M × M

Solution

(i) L × L

L = {a, b, c}

Now L×L = {a, b, c} × {a, b, c}

= {(a, a), (a, b), (a, c), (b, a), (b, b), (b, c), (c, a), (c, b), (c, c)}

R_1 = {(a, a)}

R_2 = {(a, b), (b, b), (c, b)}

(ii) L × M

L = {a, b, c}

M = {d, e, f, g} × {a, b, c}

Now L × M = {a, b, c}, {d, e, f, g} ,,

= {(a, d), (a, e), (a, f), (a, g), (b, d), (b, e), (b, f), (b, g), (c, d), (c, e), (c, f), (c, g)}

Here R_1 = {(a, d), (b, f)}

R_2 = {(b, e), (b, g), (c, d), (c, e)}

(iii) M × M

M = {d, e, f, g}

Now M × M = {d, e, f, g} × {d, e, f, g}

$= \{(d, d), (d, e), (d, f), (d, g), (e, d), (e, e), (e, f), (e, g), (f, d), (f, e),$

$(f, f), (f, g), (g, d), (g, e), (g, f), (g, g)\}$

Here $R_1 = \{(d, e), (d, f), (f, f)\}$

$R_2 = \{(d, f), (e, d), (e, e), (g, g)\}$

4. If set M has 5 elements, then find the number of binary relations in M.

Solution:

Number of elements in M = 5

Number of elements in M = 5

Number of elements in $M \times M = 2^{5\times5} = 2^{25}$

5. If $L = \{x \mid x \in N \wedge x \leq 5\}$, $M = \{y \mid y \in P \wedge y < 10\}$, then make the following relations from L to M.

(i) $R_1 = \{(x, y) \mid y < x\}$ (ii) $R_2 = \{(x, y) \mid y = x\}$

(iii) $R_3 = \{(x, y) \mid x + y = 6\}$ (iv) $R_4 = \{(x, y) \mid y - x = 2\}$

Also write the domain and range of each relation.

(i) $R_1 = \{(x, y) \mid Y < x\}$

Solution:

$R_1 = \{x \mid x \in N \wedge x \leq 5\}$,

Thus, $L = \{1, 2, 3, 4, 5\}$

and $M = \{y \mid y \in P \wedge y < 10\}$

Thus, $M = \{2, 3, 5, 7\}$

Now $L \times M = \{1, 2, 3, 4, 5\}, \{2, 3, 5, 7\}$

$= \{(1, 2), (1, 3), (1, 5), (1, 7), (2, 2), (2, 3), (2, 5), (2, 7),$

$(3, 2), (3, 3), (3, 5), (3, 7), (4, 2), (4, 3), (4, 5), (4, 7), (5, 2), (5, 3), (5,$

$= \{(3, 2)\ (4, 2),\ (4, 3),),(5, 2),\ (5, 3)\}$

$\text{Dom}R_1 = \{3, 4, 5\}$

$\text{Range } R_1 = \{2, 3\}$

$R_2 = \{(x, y) \mid y = x\}$

(ii) $R_2 = \{(x, y) \mid y = x\}$

$R_2 = \{(2, 2),\ (3, 3),\ (5, 5)\}$

$\text{Dom } R_2 = \{2, 3, 5\}$

$\text{Range } R_2 = \{2, 3, 5\}$

(iii) $R_3 = \{(x, y\} \mid x + y = 6\}$

$R_3 = \{(1, 5),\ (3, 3),\ (4, 2)\}$

$\text{Dom } R_3 = \{1, 3, 4\}$

$\text{Range } R_3 = \{5, 3, 2\}$

(iv) $R_4 = \{(x, y) \mid y - x = 2\}$

$R_4 = \{(1, 3),\ (3, 5),\ (5, 7)\}$

$\text{Dom } R_4 = \{1, 3, 5\}$

$\text{Range } R_4 = \{3, 5, 7\}$

Also write the domain and range of each relation.

6. Indicate relations, into function, one-one function, onto function and bijective function from the following. Also find their domain and the range.

Solution:

(i) $R_1 = \{(1,1),\ (2, 2),\ (3, 3),\ (4, 4)\}$

Dom R_1 = {1, 2, 3, 4}

Range R_1 = {1, 2, 3, 4}

(ii) R_2 = {(1, 2), (2, 1), (3, 4), (3, 5)}

Relation

Dom R_2 = {1, 2, 3}

Range R_2 = {1, 2, 4, 5}

(iii) R_3 = {(b, a), (c, a), (d, a)}

Function

Dom R_3 = {b, c, d}

Range R_3 = {a}

(iv) R_4 = {(1, 1), (2, 3), (3, 4), (5, 4)}

On to function

Dom R_4 = {1, 2, 3, 4, 5}

Range R_4 = {1, 3, 4}

(v) R_5 = {(a, b), (b, a), (c, d), (d, e)}

One-one function

Dom R_5 = {a, b, c, d}

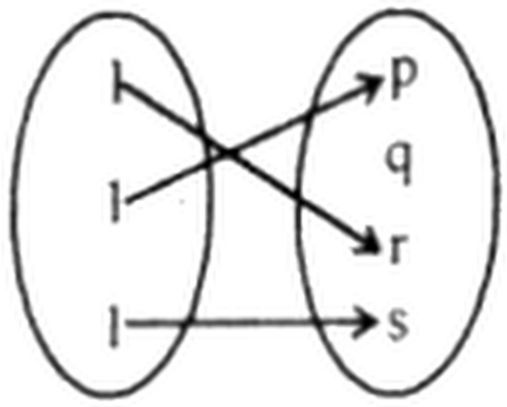
Dom R_5 = {a, b, d, e}

(vi) R_6 = {(1, 2), (2, 3), (1, 3), (3, 4)}

Relation

Dom R_6 = {2, 3, 4}

(vii) R_7 =

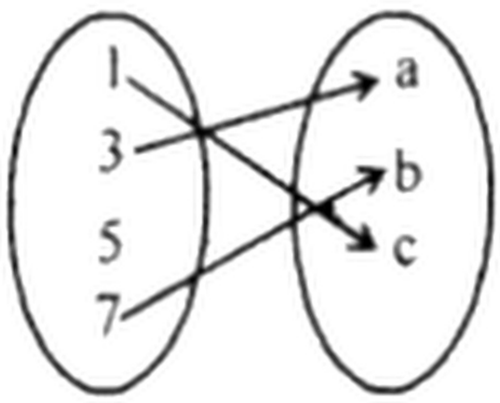


one-one function

Dom R_7 = {1, 2, 3}

Dom R_7 = {r, p, s}

(viii) R_8 =



Relation

Dom R_8 = {1, 3, 7}

Dom R_8 = {c, a, b}