Exercise 5.4

1. If A = {a, b} and 5 = {c, d}, then find A × B and B × A.

Solution:

A = {a, b} and B = {c, d}

A × B = (a, b} × {c, d}

= {(a,c), (a,d), (b,c), (b,d)}

B × A = {c, d} × (a, b}

= {(c, a), (c, b), (d, a), (d, b)}

2. If A = {0, 2, 4), B = {-1, 3}, then find A × B, B × A, A × A, B × B.

Solution:

A = {0, 2, 4} and B = {-1, 3}

A × B = {0, 2, 4} × {-1, 3}

= {(0, -1), (0, 3), (2, -1), (4, -1), (4, 3)}

B × A = {-1, 3} × {0, 2, 4}

= {(-1, 0), (-1, 2), (-1, 4), (3, 0), (3, 2), (3, 4)}

A × A = {0, 2, 4} × (0, 2, 4}

= {(0, 0), (0, 2), (0, 4), (2, 0), (2, 2), (2, 4), (4, 0), (4, 2), (4, 4)}

B × B = {-1, 3} × {-1, 3}

= {(-1, -1), (-1, 3), (3, -1), (3, 3)}

3. Find a and b, if

(i) (a - 4, 6 -2) = (2, 1)

Solution:

$\Rightarrow$ a - 4 = 2 and b - 2 = 1

a = 2 + 4 b = 1 + 2

a = 6 b = 3

(ii) (2a + 5, 3) = (7, b - 4) Solution:

$\Rightarrow$ 2a + 5 = 7 and 3 = b - 4

2a = 7 - 5 b = 4 + 3

2a = 12 b = 7

$a = \frac{12}{2}$

a = 6

(iii) (3 - 2a, b - 1) - (a - 7, 2b + 5)

Solution:

$\Rightarrow$ 3 - 2a = a - 7 and b - 1 = 2b + 5

- 2a - a - 7 - 3 b - 2b = 1 + 5

- 3a = - 10 - b = 6

$\Rightarrow$ 3a =10 b = - 6

$a = \frac{10}{3}$

4. Find the sets - X and Y, if X × Y = {(a, a), (b, a), (c, a), (d, a)}

Solution:

X × Y = {(a, a), (b, a), (c, a), (d, a)}

$\Rightarrow$ X = {a, b, c, d} and Y = {a}

5. If X = {a, b, c} and 7= {d, e}, then find the number of elements in

(i) X × Y

Solution:

Since set X has 3 elements and set y has 2 elements.

Hence, product X × Y has 3 × 2 = 6 elements.

(ii) Y × X

Solution

Since set y has 2 elements and set X has 3 elements.

(iii) X ×X

Solution:

Since set X has 3 elements. Hence, product X × X has 9 elements.

Binary relation:

If A and B are any two non-empty sets, then a subset R ⊆ A × B is called binary relation from set A into set B, because there exists some relationship between first and second element of each ordered pair in R.

Domain of relation denoted by Dom R is the set consisting of all the first elements of each ordered pair in the relation.

Range of relation denoted by Rang R is the set consisting of all the second elements of each ordered pair in the relation.

Function or Mapping:

Suppose A and B are two non-empty sets, then relation f: A → B is called a function.

If (i) Dom f = A (ii) every x ∈ A appears in one and only one ordered pair in f.

Alternate Definition:

Suppose A and B are two non-empty sets, then relation f: A → B is called a function if (i) Dom f = A (ii) ∀ x ∈ A we can associate some unique image element y = f (x) ∈ B.

Domain, Co-domain and Range of Function:

If f: A → B is a function, then A is called the domain of f and B is called the co-domain of f.

Domain f is the set consisting of all first elements of each ordered pair in f and range f is the set consisting of all second elements of each ordered pair in f

Example:

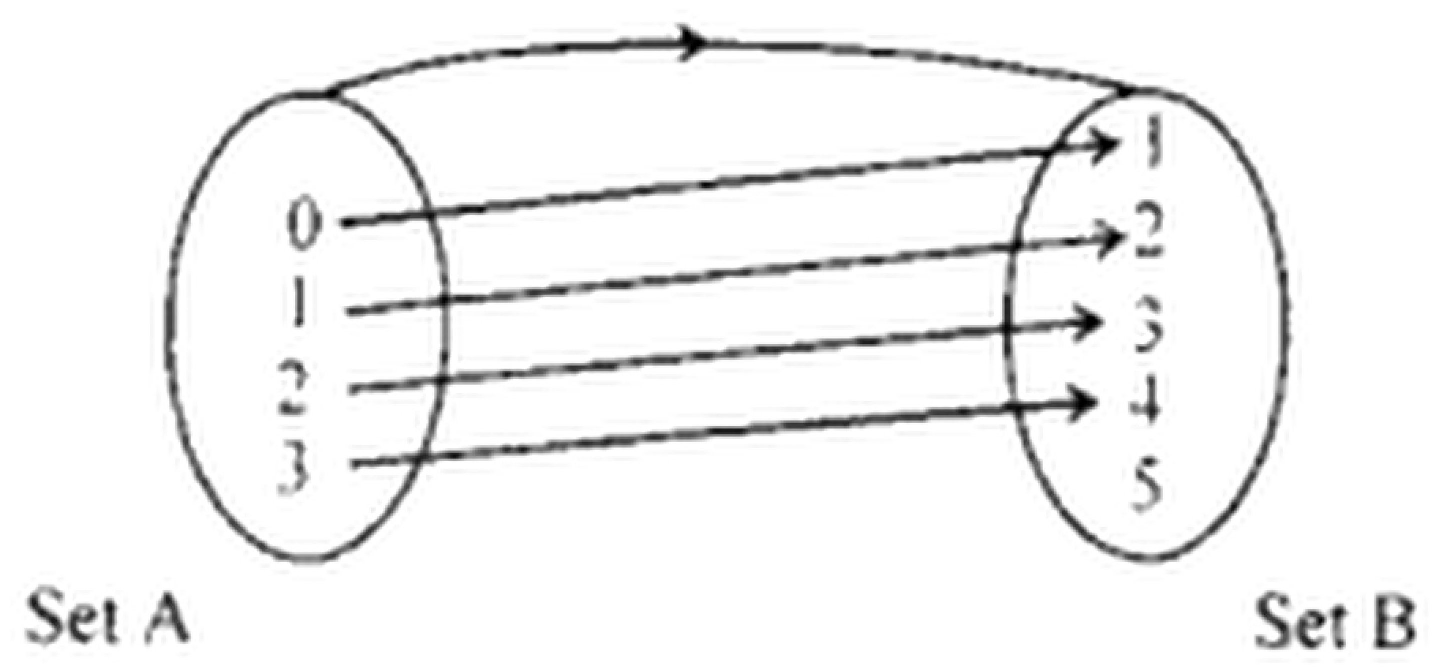
Suppose A = {0, 1, 2, 3} and B = {-1, 2, 3, 4, 5}

Define a function f: A → B

f = {(x, y) | y = x + 1 ∀ x ∈ A, y ∈ B} f = {(0, 1), (1, 2), (2, 3), (3, 4)}

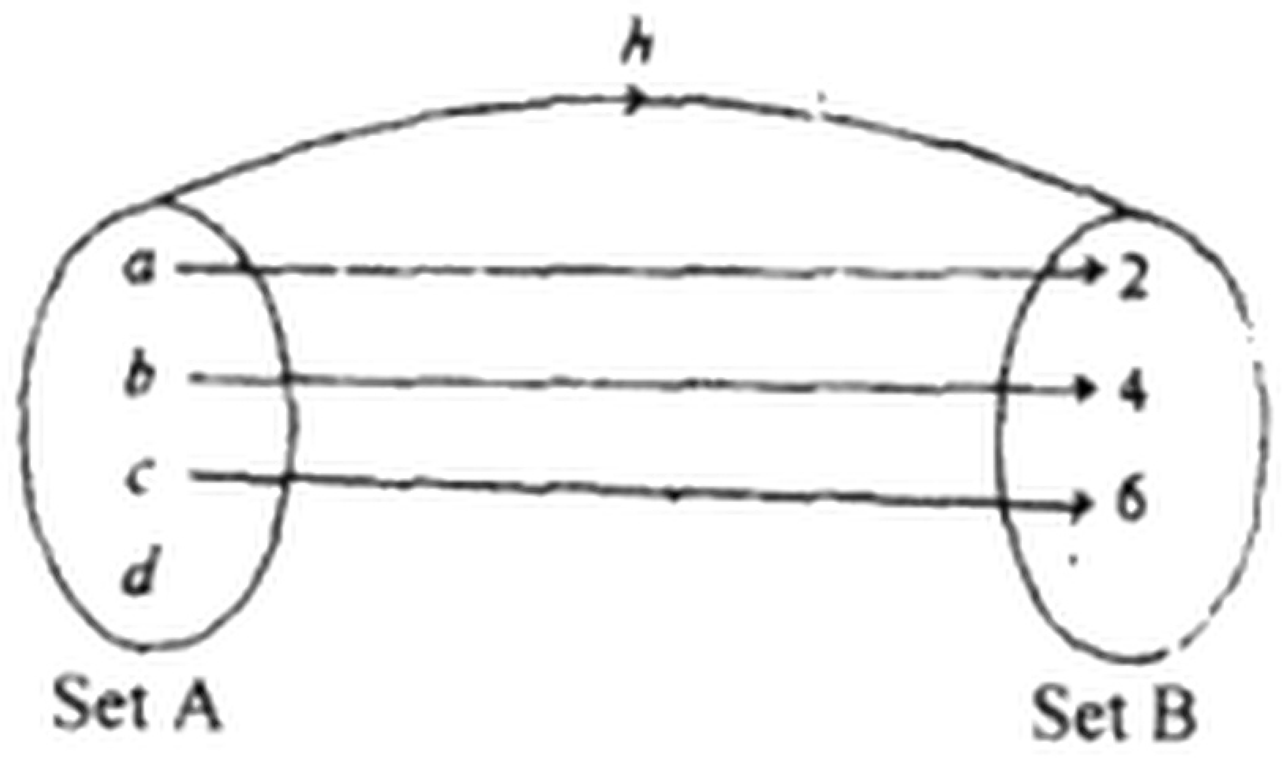
Dom f = {0, 1, 2, 3} = A

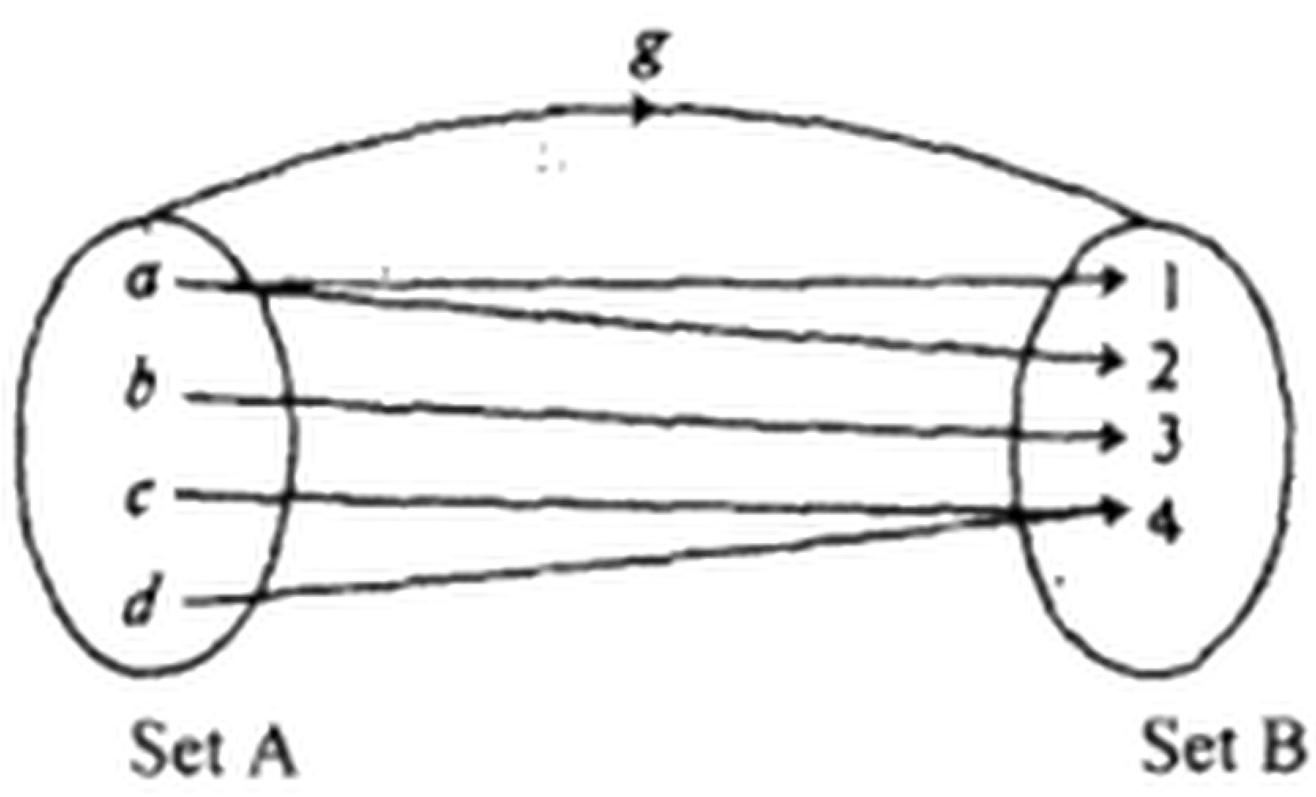
Rang f = {1, 2, 3, 4} ⊆ = B



The following are the examples of relations but not functions.

g is not a function, because an element a e A has two images in set B and A is not a function because an element d G A has no image in set B.



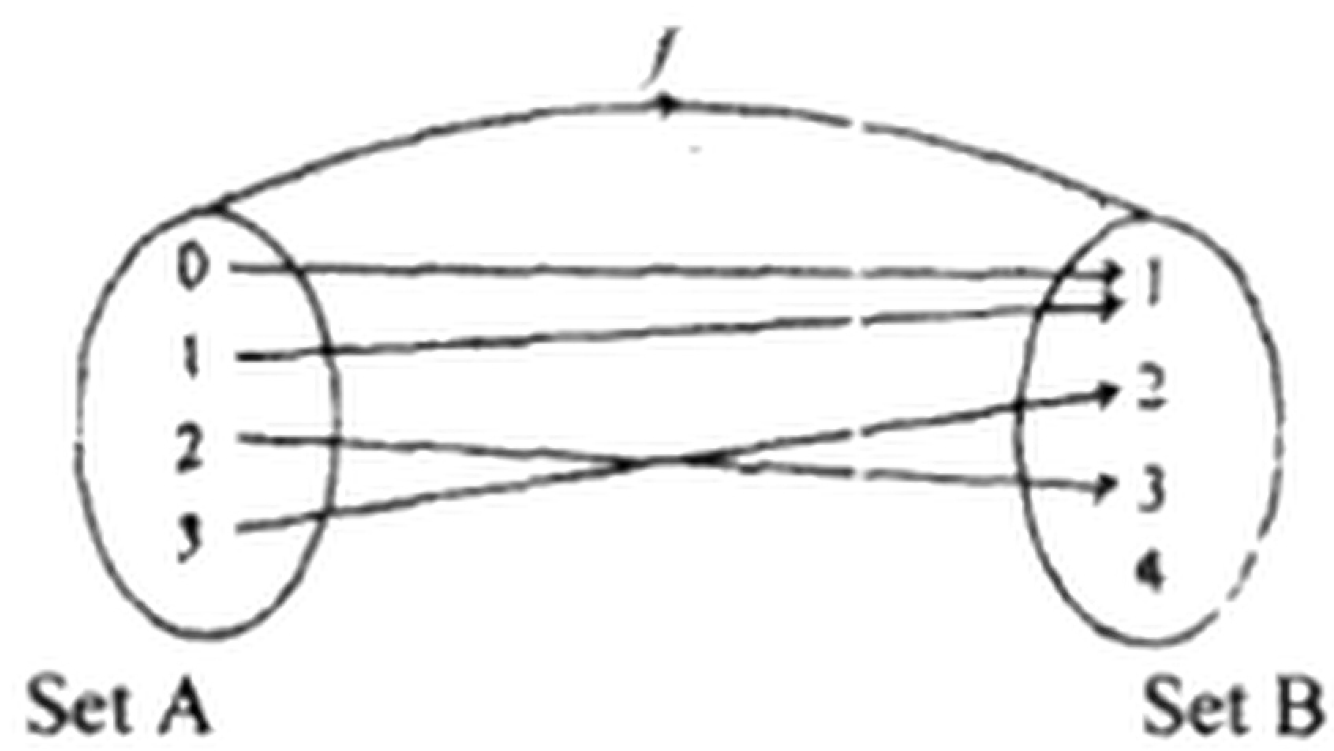


Demonstrate the following:

(a) Into function:

A function f: A → B is called an into function, if at least one element in B is not an image of some element of set A i.e.,

Range of f ⊂ set R.



For example, we define a function f : A → S such that

f = {(0, 1), (1, 1), (2, 3), (3, 2)}

where A = {0, 1, 2, 3} and B = {1, 2, 3, 4} f is an into function.

(b) One-one function:

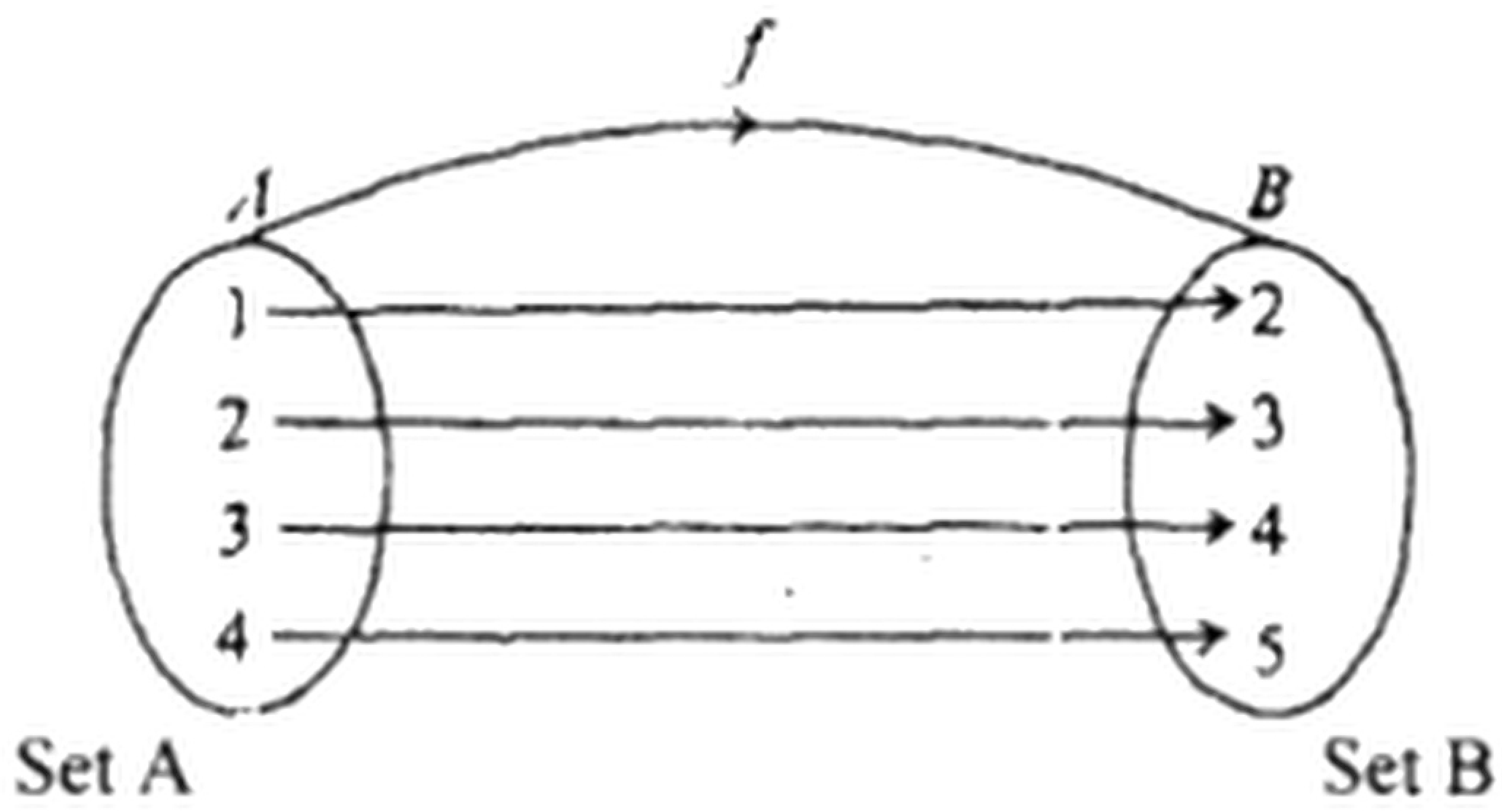
A function f: $A \to B$ is called one-one function, if all distinct elements of A. have distinct images in B, i. e., $f(x_1) = f(x_2) = \Rightarrow x_1 = x_2 \cong A$ or $\forall x_1 \neq x_2 \in A \Rightarrow f(x_1) \neq f(x_2)$

For example,, if A = {0, 1, 2, 3}

and B = {1,2, 3, 4, 5}, then we define a function f: $A \to B$ such that

$f = \{(x, y) \mid y = x + 1, \forall x \in A, y \in B\}$.

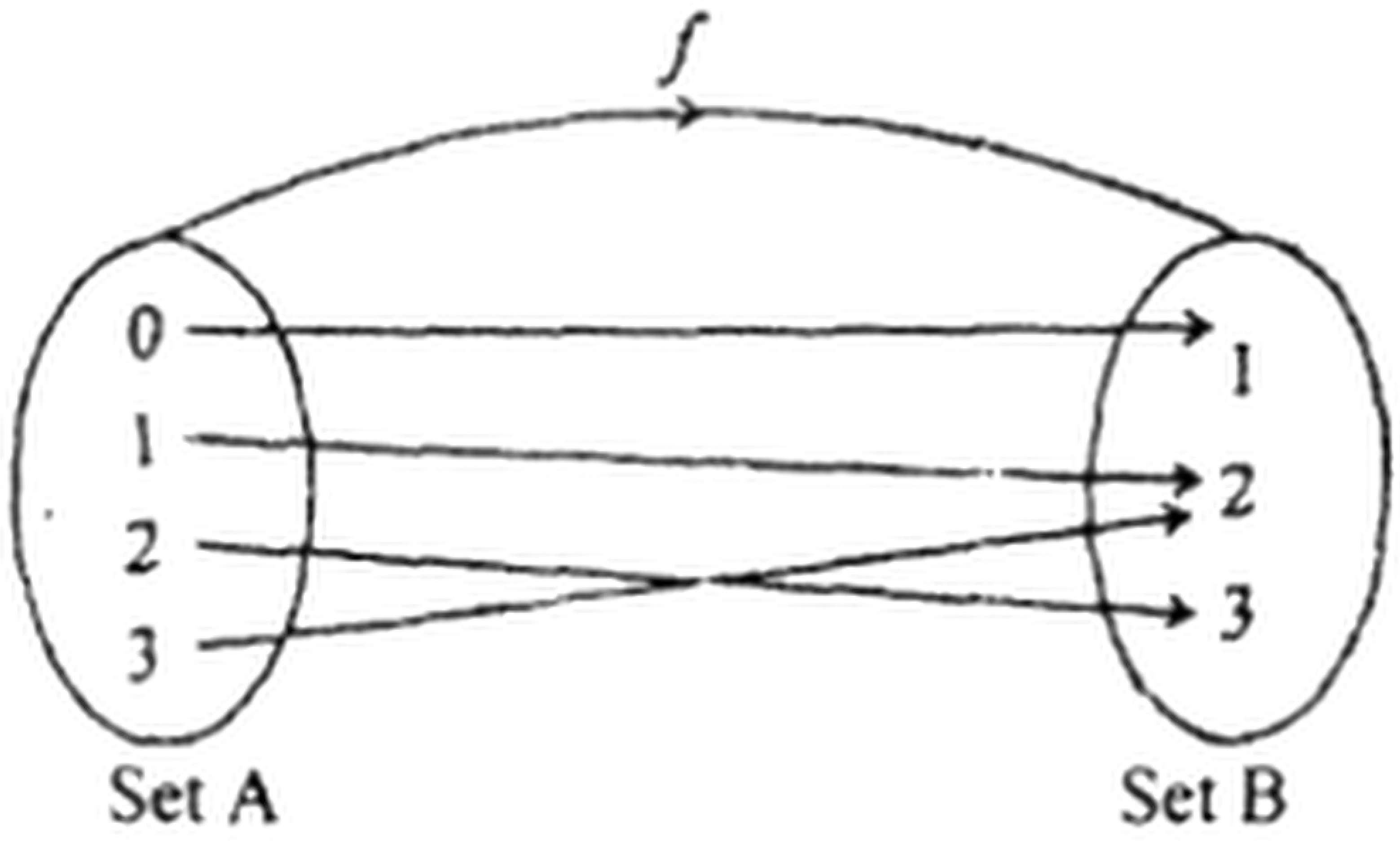
= {(0,1,), (1,2), (2, 3), (3,4)} f is one-one function.



(c) Into and one-one function: (injective function)

The function f discussed in (b) is also an into function. Thus f is an into and one-one function.

(d) An onto or surjective function:



A function f: A → B is called an onto function, if every element of set B is an image

of at least one element of set A i.e. Range of f = B.

For example, if A = {0, 1, 2, 3}

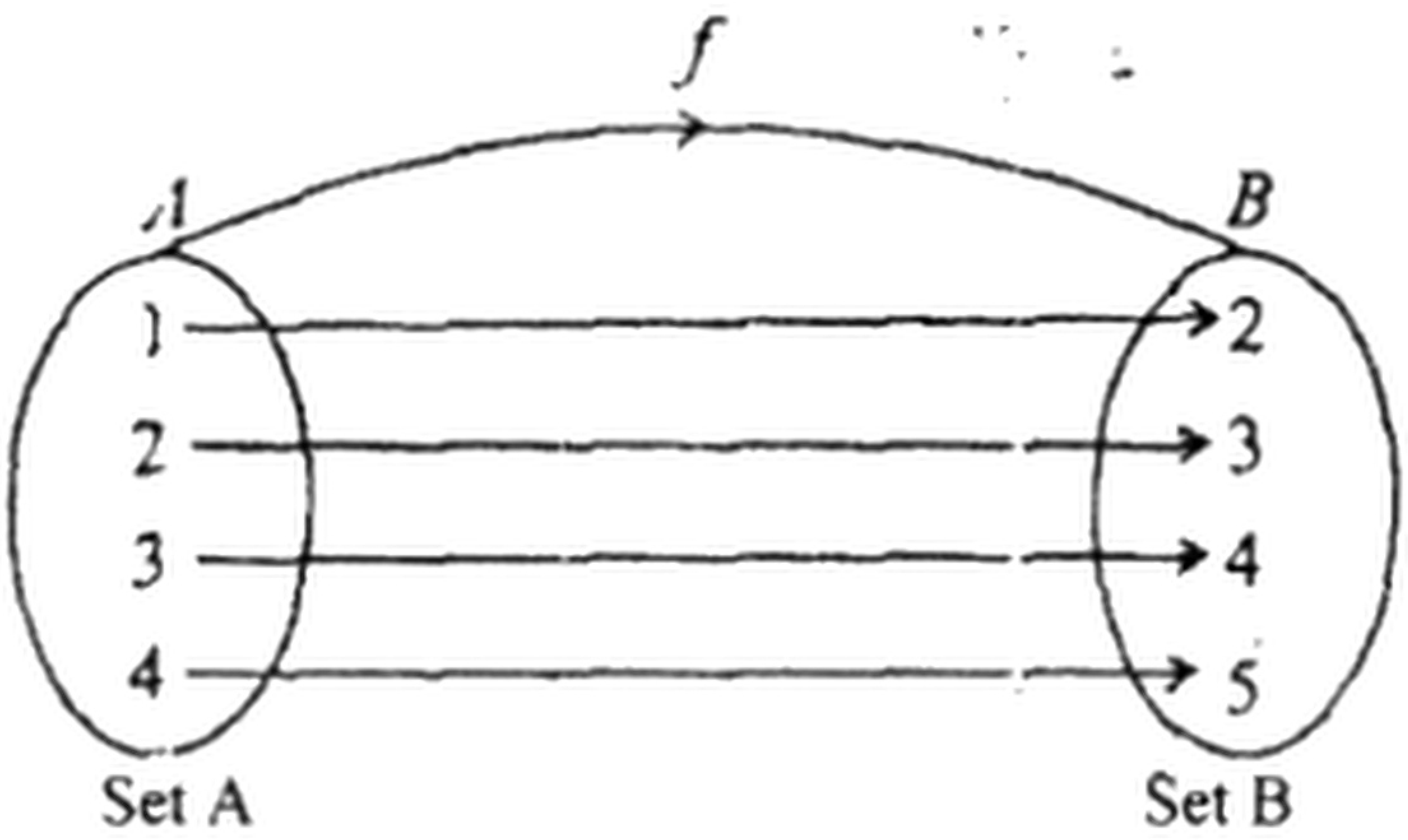
and B = {1, 2, 3}, then f: A → B such

that f = {(0, 1), (1, 2), (2, 3), (3, 2)}.

Here Rang f = {1, 2, 3} = B.

Thus, f so defined is an onto function.

(c) Bijective function or one to one correspondence:



A function f: A → B is called bijective function if f function f is one-one and onto.
For example, if A = {1, 2, 3, 4) and B = {2, 3, 4, 5)

We define a function f: A → B such that f = {(x, y) | y = x +1,∀ x ∈ A,y ∈ B)
Then f = {(1, 2), (2, 3), (3, 4), (4, 5))

Evidently this function is one-one because distinct elements of A have distinct images in B. This is an onto function also because every element of 0 is the image of at least one element of A.

Note: (1) Even function is a relation but converse may not be true.

(2) Even function may not be one-one

(3) Every function may not be onto.

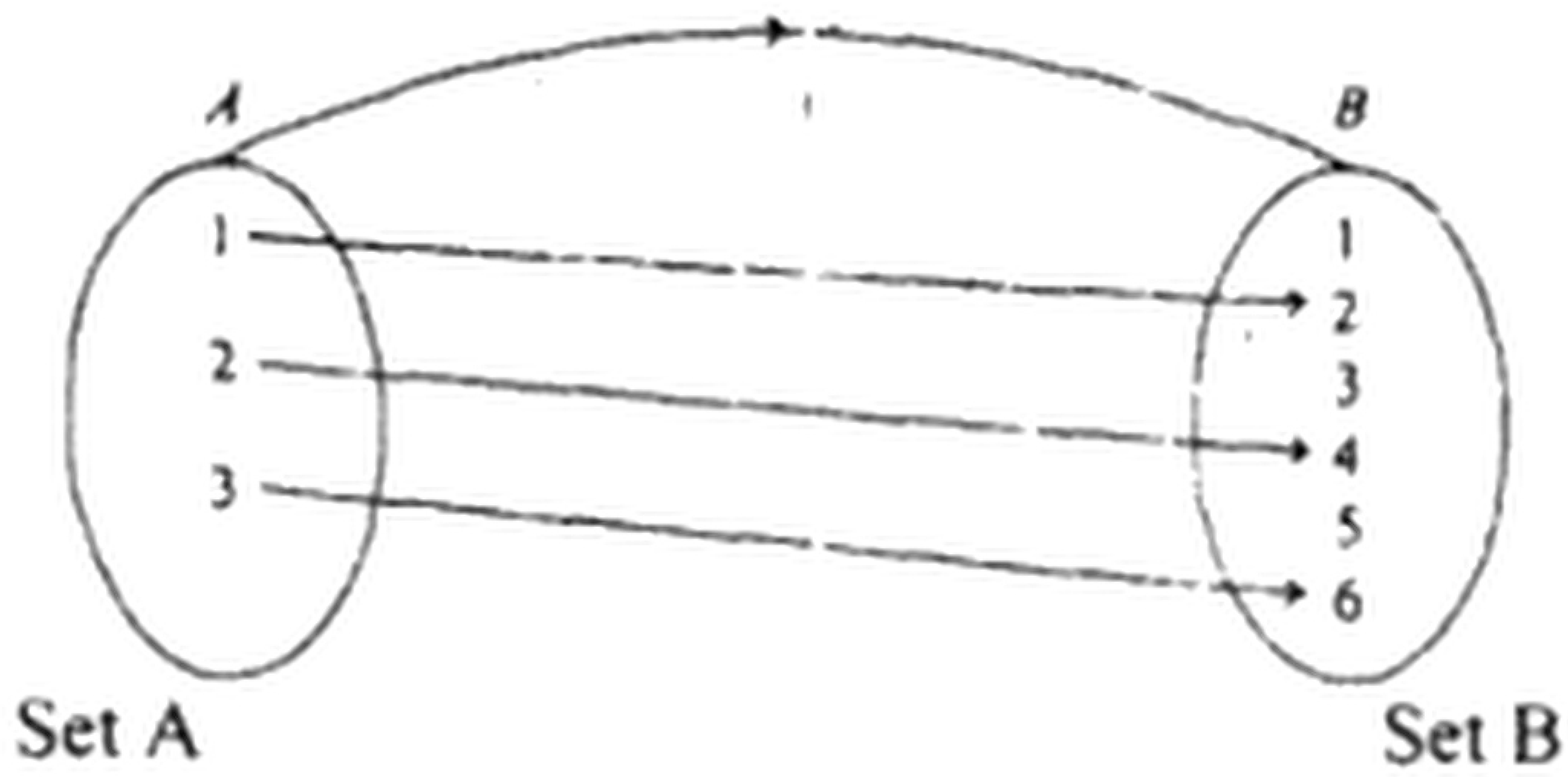
Example: Suppose A = {1, 2, 3)

B = (1, 2, 3, 4, 5, 6)

We define a function f: A → B = {(x, y) | y = 2x, ∀ x ∈ A ,y ∈ B}

Then f = {1, 2), (2, 4), (3, 6))

Evidently this function is one-one but riot an onto.



Examine whether a given relation is a function:

A relation in which each x e its domain, has a unique image in its range, is a function.

Differentiate between one-to-one correspondence and one-one function:

A function f from set A to set B is one-one if distinct elements of A has distinct images in B. The domain of/is A and its range is contained in B.

In one-to-one correspondence between two sets A and B, each element of either set is assigned with exactly one element of the other set. If the sets A and B are finite, then these sets have the same number of elements, that is, n (A) = n (B).