Exercise 5.3

1. If U = {1, 2, 3, 4, ..., 10)

A = {1, 3, 5, 7, 9)

B = (1, 4, 7, 10) then verify the following questions,

(i) A – B = A ∩ B'

L.H.S. = A - B

= {1, 3, 5, 7, 9) — {1, 4, 7, 10}

= {3, 5, 9} .....(i)

R.H.S. = A ∩ B'

=A ∩ (U - B)

= {1, 3, 5, 7, 9} ∩ ({1, 2, 3, 4,...., 10} – {1, 4, 7, 10})

= (1, 3, 5, 7, 9) ∩ (2, 3, 5, 6, 8, 9)

= (3, 5, 9) .....(ii)

From (1) and (ii), we have L.H.S. = R.H.S.

Hence Proved

(ii) B – A = B ∩ A'

L.H.S. = B - A

= (1, 4, 7, 10) - (1, 3, 5, 7, 9)

= (4, 10) ....(i)

R.H.S. = B ∩ A'

$=B \cap (U - A)$

$= (1, 4, 7, 10) \cap \{1, 2, 3, 4,...., 10\} - (1, 3, 4, 5, 7, 9))$

$= (1, 4, 7, 10) \cap (2, 4, 6, 8, 10)$

$= (4, 10)$ ....(ii)

From (i) and (ii), we have L.H.S. = R.H.S.

Hence Proved

(iii) (A ∪ B)' = A' ∩ B'

L.H.S. = $(A \cup B)'$

$= U - (A \cup B)$

$= \{1, 2, 3, 4, ..., 10\} - (\{1, 3, 5, 7, 9\} \cup \{1, 4, 7, 10\})$

$= (1, 2, 3, 4, ...,10) - (1, 3, 4, 5, 7, 9, 10)$

$= (2, 6, 8)$ .....(i)

R.H.S. = $A' \cap B'$

$= (U - A) \cap (U - B)$

$= (\{1, 2, 3, 4, ..., 10\} - \{1, 3, 5, 7, 9\}) \cap (1, 2, 3, 4,...., 10) - \{1, 4, 7, 10\})$

$= (2, 4, 6, 8, 10\} \cap (2, 3, 5, 6, 8, 9)$

$= (2, 6, 8)$ ......ii)

From (i) and (ii), we have L.H.S. = R.H.S.

Hence Proved

(iv) (A ∩ B)' = A' ∪ B'

L.H.S. = $(A \cup B)'$

=∪ -(A ∩ B)

= (1, 2, 3, 4, ….. 10) - ({1, 3, 5, 7, 9} ∩ {1, 4, 7, 10})

= (1, 2, 3, 4,….10) - {1, 7}

= (2, 3, 4, 5, 6, 8, 9,10) ……..(i)

R.H.S. = A' ∪ B'

= (U - A) ∪ (U - B)

= ({1, 2, 3, 4,……, 10} - {1, 3, 5, 7, 9}) ∪ ({1, 2, 3, 4,…., 10) - {1, 4, 7, 10))

= {2, 4, 6, 8, 10) ∪ (2,3, 5, 6, 8, 9)

= {2, 3, 4, 5, 6, 8, 9, 10) …..(ii)

From (i) and (ii), we have

L.H.S. = R.H.S. Hence Proved

(v) (A - B)' = A' ∪ B

L.H.S. = (A - B)'

= ∪ - (A - B)

= {1, 2, 3, 4, ….., 10) - ({l, 3, 5, 7, 9) - {1, 4, 7, 10})

= {1, 2, 3, 4, …. 10} - {3, 5, 9)

= {1, 2, 3, 6, 7, 8, 10) …..(i)

R.H.S. = A' ∪ B'

=(U - A) ∪ B

=({1, 2, 3, 4, ….., 10) - {1, 3, 5, 7, 9}) ∪ {1, 4, 7, 10)

= {2, 4, 6, 8, 10) ∪ {1, 4, 7, 10)

= {1, 2, 4, 6, 7, 8, 10) ....(ii)

From (i) and (ii), we have

L.H.S. = R.H.S. Hence Proved

(VI) (B - A)' = B' ∪ A

L.H.S. = (B ∪ A)'

= U - (B - A)

= {I, 2, 3, 4, ..., 10} - {1, 4, 7, 10} - {1, 3, 5, 7, 9})

= {1, 2, 3, 4, ..., 10) - {4, 10)

= {I, 2, 3, 5, 6, 7, 8, 9} .....(i)

R.H.S. = B' ∪ A

=(U - B)∪ A

=({1, 2, 3, 4, ..., 10} - {1, 3, 5, 7, 9}) ∪ {1, 3, 5, 7, 9)

= {2, 3, 5, 6, 8, 9} ∪ {1,3, 5, 7, 9)

= {1, 2, 3, 5, 6, 7, 8, 9} .....(ii)

From (i) and (ii), we have

L.H.S. = R.H.S.

Hence Proved.

2. If U = (1,2, 3,4,-,10); A = (1, 3, 5, 7, 9); B = (1, 4, 7,10); C = {1, 5, 8, 10) then verify the following:

Solution:

(i) (A ∪ B)∪ C = A ∪ (B ∪ C)

L.H.S.= $(A \cup B) \cup C$

$= (\{1, 3, 5, 7, 9\} \cup \{1, 4, 7, 10)) \cup \{1, 5, 8, 10\}$

$= \{1, 3, 4, 5, 7, 9, 10\} \cup \{1, 5, 8, 10\}$

$= \{1, 3, 4, 5, 7, 8, 9, 10\}$ ....(i)

R.H.S. = $A \cup (B \cup C)$

$= \{1, 3, 5, 7, 9) \cup (\{1, 4, 7, 10) \cup \{1, 5, 8, 10))$

$=\{1, 3, 5, 7, 9\} \cup \{1, 4, 5, 7, 8, 10)$

$= \{1, 3, 4, 5, 7, 8, 9, 10)$ ....(ii)

From (i) and (ii), we have

L.H.S. = R.H.S.

Hence Proved.

(ii) $(A \cap B) \cap C = A \cap (B \cap C)$

L.H.S. = $(A \cap B) \cap C$

$=(\{1, 3, 5, 7, 9\} \cap \{1, 4, 7, 10\}) \cap \{1, 5, 8, 10\}$

$= \{1, 7\} \cap \{1, 5, 8, 10)$

$= \{1\}$ ......(i)

R.H.S. = $A \cap (B \cap C)$

$= \{1, 3, 5, 7, 9\} \cap (\{1, 4, 7, 10\} \cap \{1, 5, 8, 10\})$

$= (1, 3, 5, 7, 9) \cap \{1, 10\}$

$= \{1)$ ......(ii)

From (i) and (ii), we have

L.H.S. = R.H.S.

Hence Proved.

(iii) A ∪ (B ∪ C) = (A ∪ B) ∩ (A ∪ C)

L.H.S. = A ∪ (B ∩ C)

= {1, 3, 5, 7, 9) ∪ {1, 4, 7, 10)) ∩ {1, 5, 8, 10)

= {1, 3, 5, 7, 9) ∪ {1, 10)

= {1, 3, 5, 7, 9, 10) ......(i)

R.H.S. = (A ∪ B ∩ (A ∪ C)

=({1, 3, 5, 7, 9} ∪ ({1, 4, 7, 10}) ∩ ({1, 3, 5, 7} ∪ {1, 5, 8, 10))

= {1, 3, 4, 5, 7, 9, 10} ∩ {1, 3, 5, 7, 8, 9, 10}

= {1, 3, 5, 7, 9, 10) ....(ii)

From (i) and (ii), we have

L.H.S. = R.H.S.

Hence Proved.

(iv) A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)

L.H.S. = A ∩ (B ∪ C)

= {1, 3, 5, 7, 9} ∩ ({1, 4, 7, 10} ∪ {1, 5, 8, 10})

={1, 3, 5, 7, 9} ∩ {1,4, 5, 7, 8, 10)

= {1, 5, 7) ....(i)

R.H.S. = (A ∩ B ∪ (A ∩ C)

=({1, 3, 5, 7, 9} ∩ ({1, 4, 7, 10}) ∪ ({1, 3, 5, 7 ,9} ∩ {1, 5, 8, 10})

= {1, 7} ∪ {1, 5}

= {1, 5, 7} ....(ii)

From (i) and (ii), we have

L.H.S. = R.H.S.

Hence Proved.

3. If U = N; then verify De-Morgan's laws by using A = ∅ and B = P. Solution:

Solution:

U = N , A = ∅, B = P

(i) (A ∩ B)' = A' ∪ B'

L.H.S. = (A ∩ B)'

=U - (A ∩ B)

=N – (∅ ∩ P)

=N - ∅

= N ....(i)

R.H.S. = A' = B'

= (U - A) ∪ (U - B)

=(N - ∅) ∪ (N - P)

= N ∪ (N - P)

= N ....(ii)

From (i) and (ii), we have

L. H.S. = R.H.S

Hence Proved.

(ii) (A ∪ B)' = A' ∩ B'

L.H.S. = (A ∪ B)'

= U - (A ∪ B)

= N - (∅ ∪ P)

=N - ∅

= N .....(i)

R.H.S. = A' ∩ B'

= (U - A) ∩ (U - B)

=(N - ∅) ∩ (N - P)

=N ∩ (N - P)

= N – P ......(ii)

From (i) and (ii), we have

L.H.S. = R.H.S

Hence Proved.

4. If U = {1, 2, 3, 4, ... 10), A - {1, 3, 5, 7, 9) and B = {2, 3, 4, 5, 8) then prove the following questions by Venn diagram:

(i) A – B = A ∩ B'

Solution:

U = {1, 2, 3, 4,.....,10} A = {1, 3, 5, 7, 9}

B = {2, 3, 4, 5, 8}

(i) A - B = A ∩ B'

L.H.S. = A - B

= {1, 3, 5, 7, 9} - {2, 3, 4, 5, 8}

= {1,7, 9} ......(i)

R.H.S. = A ∩ B'

= A ∩ (U - B)

= {1,3, 5, 7, 9} ∩ ({l, 2, 3,4,...., 10} - {2, 3, 4, 5, 8})

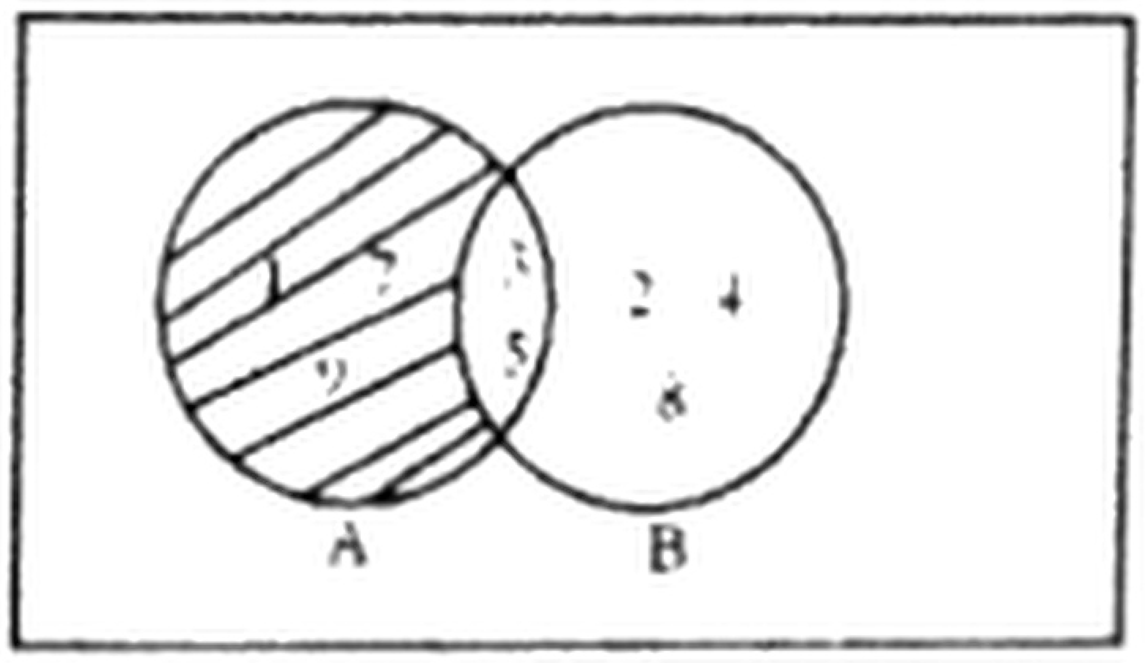
= {1, 3, 5, 7, 9} ∩ {1, 6, 7, 9, 10}

= {1, 7, 9} ......(ii)

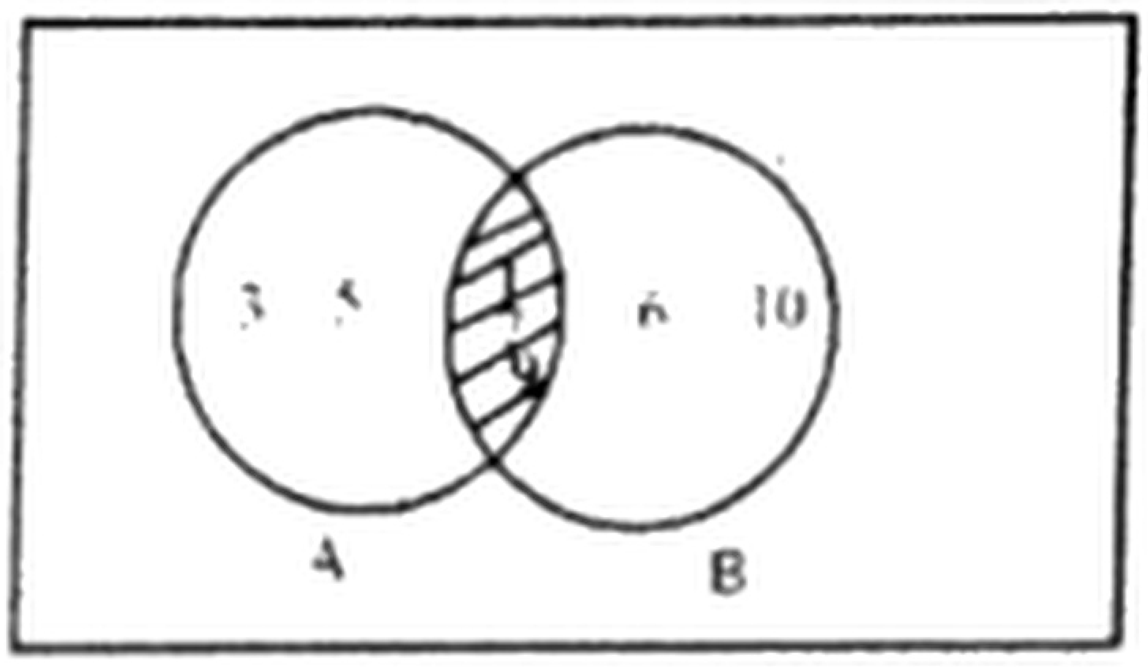
From (i) and (ii), we have

L.H.S. = R.H.S

Hence Proved.



A B



A ∩ B'

(ii) B - A = B ∩ A'

Solution:

L.H.S = B - A

= {2, 3, 4, 5, 8} - {l, 3, 5, 7, 9}

= {2, 4, 8} ....(i)

R.H.S. = $B \cap A'$

$= B \cap (U - A)$

$= \{2, 3, 4, 5, 8\} \cap (\{1, 2, 3, 4, \ldots\ldots, 10\} - \{1, 3, 5, 7, 9\})$

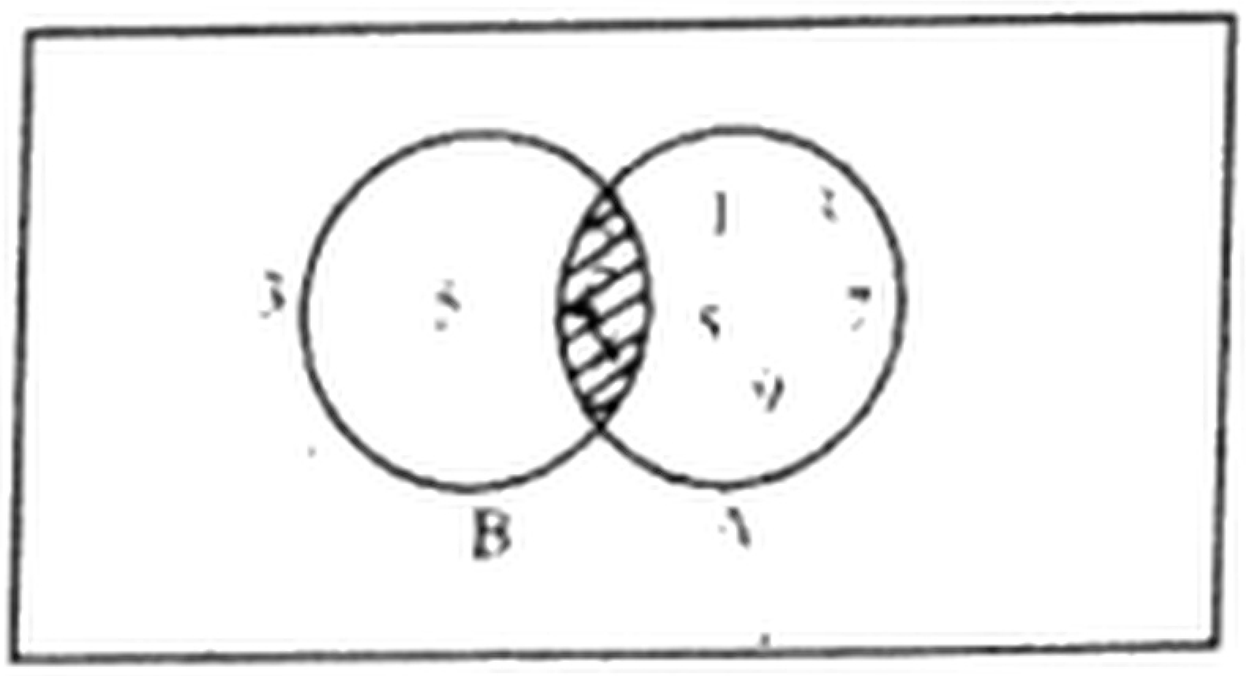
$= \{2, 3, 4, 5, 8\} \cap \{2, 4, 6, 8\}$

$= \{2, 4, 8\}$ ....(ii)

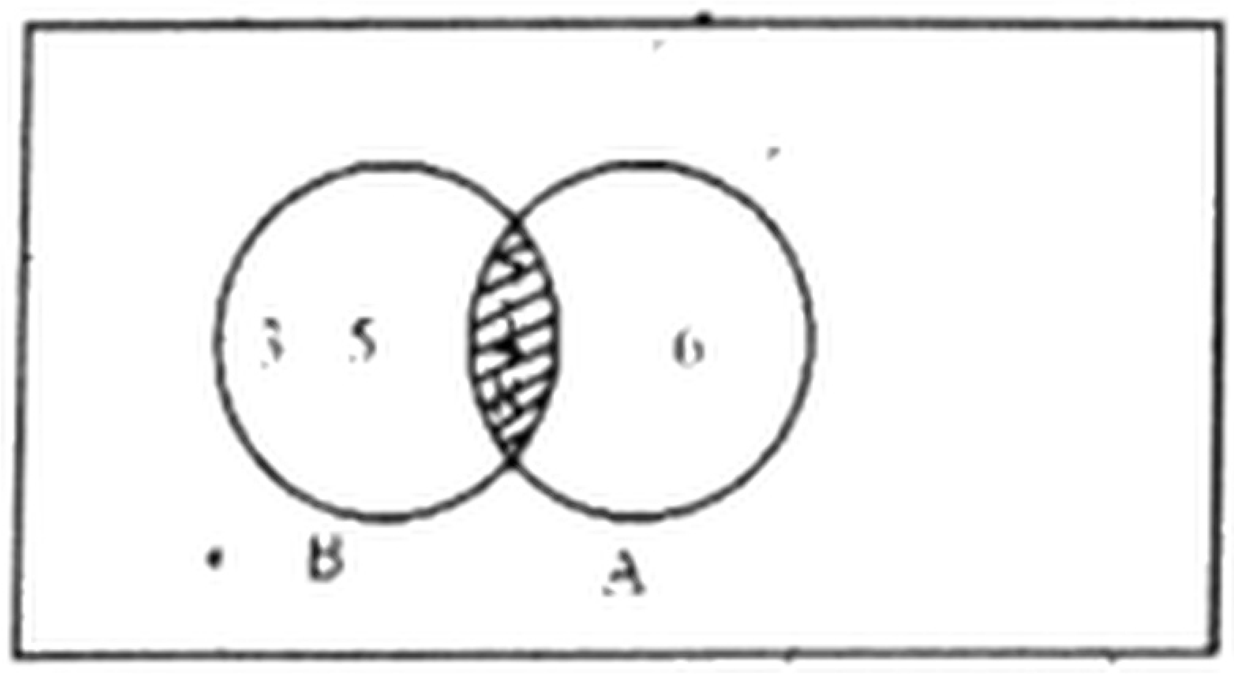
From (i) and (ii), we have

L.H.S. = R.H.S

Hence Proved.



$B - A$



$B \cap A'$

(iii) $(A \cup B)' = A' \cap B'$

Solution:

L.H.S. = $(A \cup B)'$

$= \{1, 2, 3, 4, \ldots\ldots, 10\} - (\{1, 3, 5, 7, 9\} \cup \{2, 3, 4, 5, 8\})$

$= \{1, 2, 3, 4, \ldots\ldots, 10\} - \{1, 2, 3, 4, 5, 7, 8, 9\}$

$= \{6, 10\}$ .....(i)

R.H.S. = $A' \cap B'$

$= (U - A) \cap (U - B)$

$= (\{1, 2, 3, 4, \ldots\ldots, 10\} - (\{1, 3, 5, 7, 9\}) \cap (\{1, 2, 3, 4, \ldots\ldots, 10\} - \{2, 3, 4, 5, 8\})$

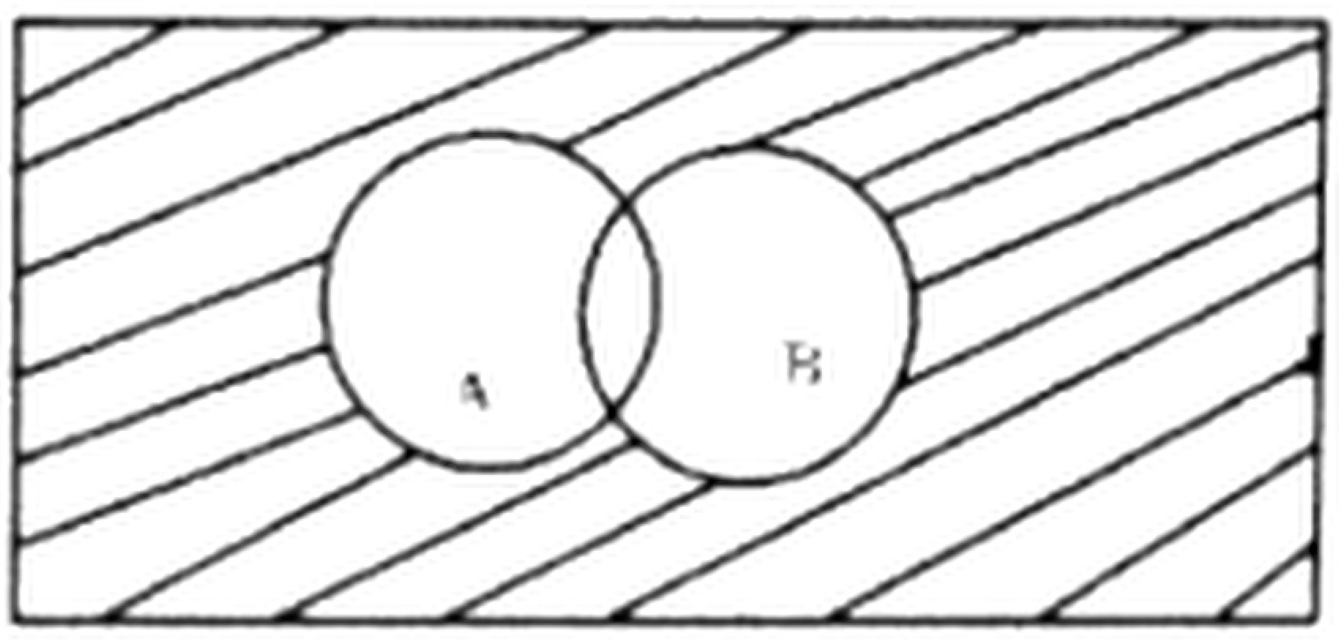
= {2, 4, 6, 8, 10} ∩ {1, 6, 10}

= {6, 10} .....(ii)

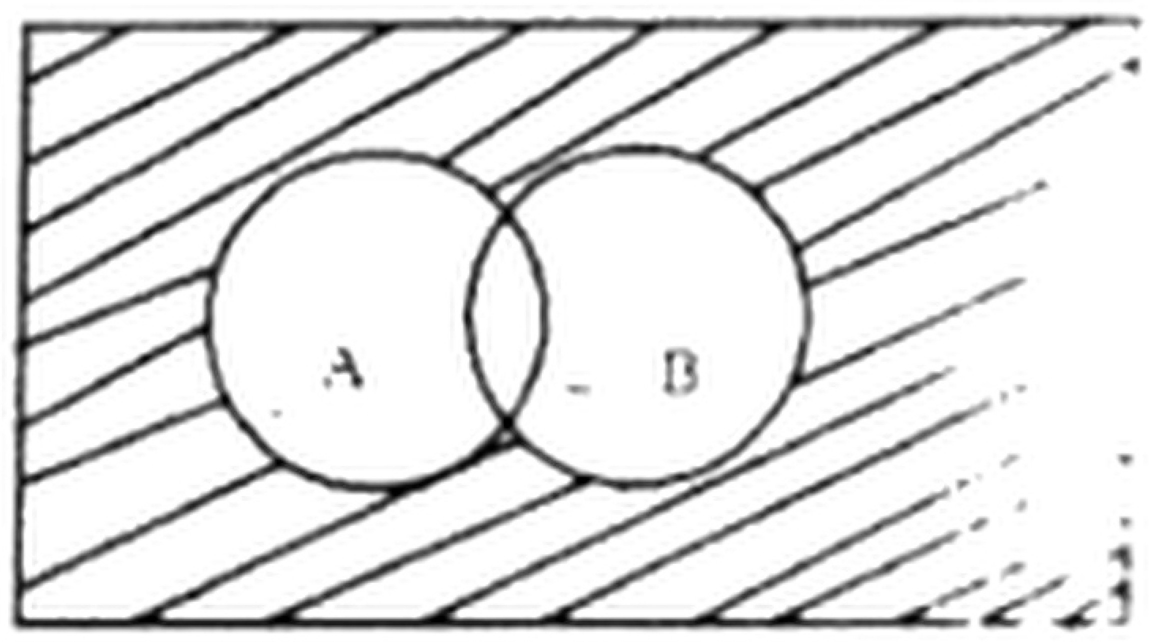
From (i) and (ii), we have

L.H.S. = R.H.S

Hence Proved.



(A ∪ B)'



A' ∩ B'

(iv) (A ∩ B) ' = A ∪ B'

Solution:

L.H.S = (A ∩ B)'

= U - (A ∩ B)

= {1,2, 3, 4,......, 10} - ({1, 3, 5, 7, 9} ∩ {2, 3, 4, 5, 8})

= {1,2, 3, 4,....., 10} - {3, 5}

= {1,2, 4, 6, 7, 8, 9, 10} .....(i)

R.H.S. = A' ∪ B'

= (U - A) ∪ (U - B)

= ({l, 2, 3, 4, , 10} - ({1, 3, 5, 7, 9} ∪ ({1, 2, 3, 4,......, 10} - {2, 3, 4, 5, 8})

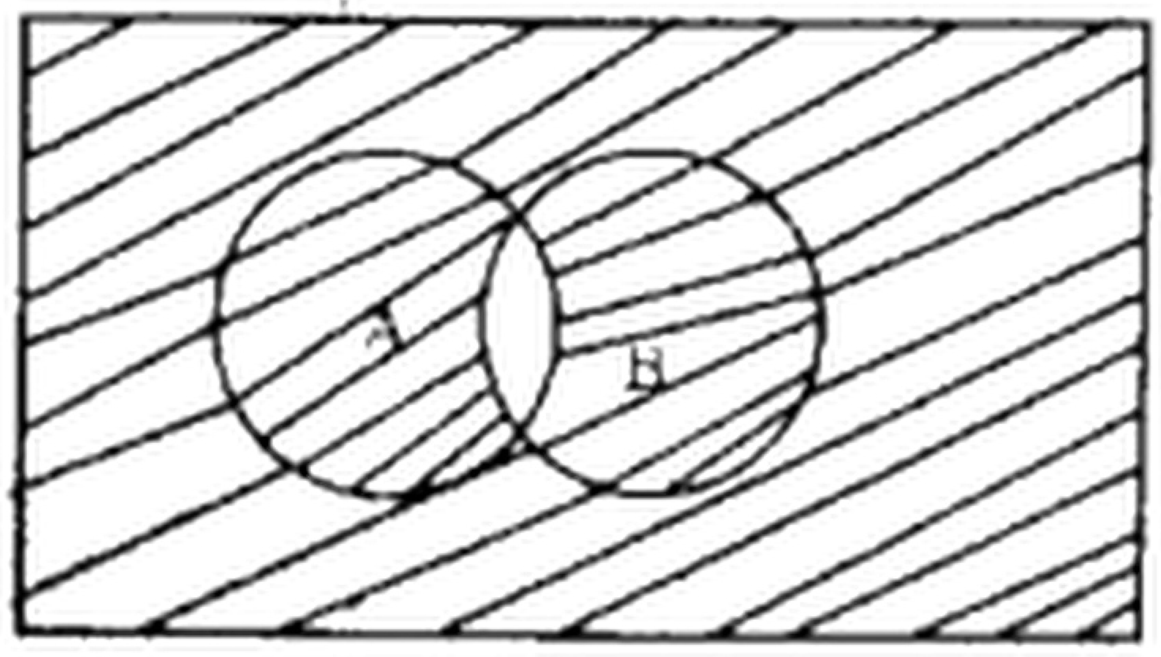
= {2, 4, 6, 8, 10} - {1, 6, 7, 9, 10}

= {1, 2, 4, 6, 7, 8, 9, 10} ....(i)

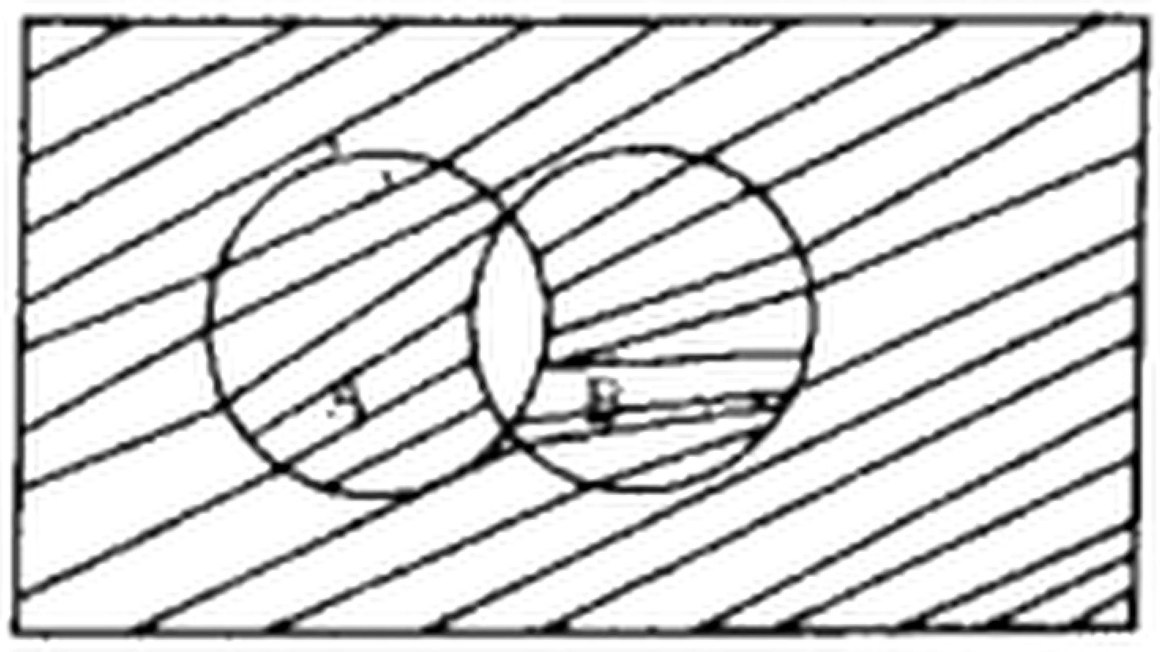
From (i) and (ii), we have

L. H.S. = R.H.S

Hence Proved.



(A' ∪ B)'



A' ∪ B'

(v) (A - B)' = A' ∪ B

Solution:

L.H.S. = (A - B)'

= U - (A - B)

= {1,2, 3, 4,.......,10} - ({1, 3, 5, 7, 9} - {2, 3, 4, 5, 8})

= {1, 2, 3, 4,......, 10} - {1, 7, 9}

= {2, 3, 4, 5, 6, 8, 10} ......(i)

R.H.S. = A' ∪ B

= (U - A) ∪ B

= {1, 2, 3, 4, ...,10} - ({1, 3, 5, 7, 9}) ∪ {2, 3, 4, 5, 8}

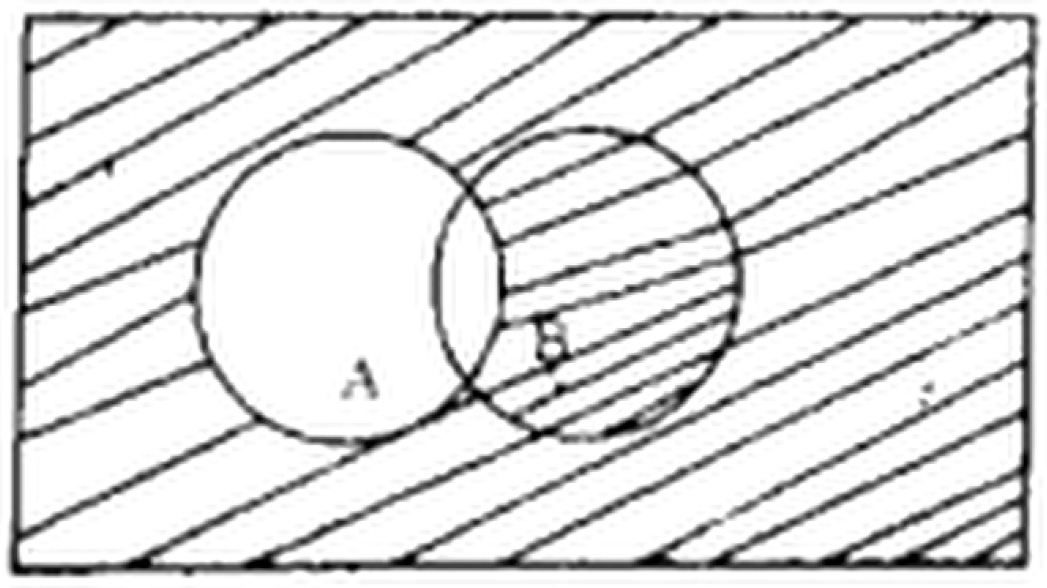
= {2, 4, 6, 8, 10} ∪ {2, 3, 4, 5, 8}

= {2, 3, 4, 5, 6, 8, 10} ....(ii)

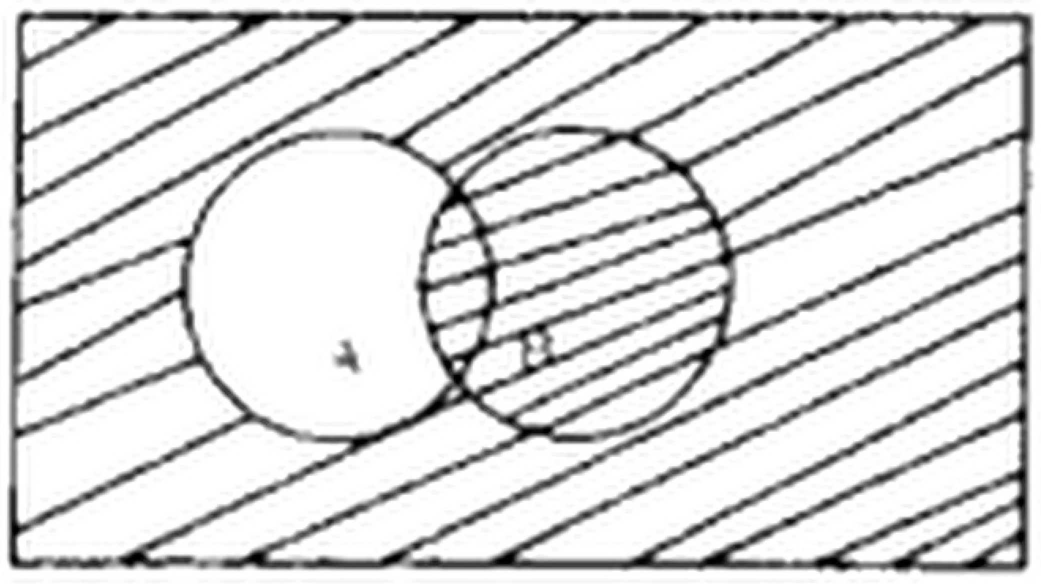
From (i) and (ii), we have

L. H.S. = R.H.S

Hence Proved.



(A ∪ B)'



A' ∪ B

(vi) (B - A)' = B' ∪ A

Solution:

L.H.S. = (B - A)'

= U - (B - A)

= {1, 2, 3, 4, ...,10} - ({2, 3, 4, 5, 8} - {I, 3, 5, 7, 9})

= {1,2, 3, 4, ..., 10} - {2, 4, 8}

= {1, 3, 4, 5, 6, 7, 9, 10} .....(i)

R.H.S. = B' ∩ A

= (U - B) ∪ A

= ({1, 2, 3, 4,...., 10} - {2, 3, 4, 5, 8}) ∪ {1, 3, 5, 7, 9}

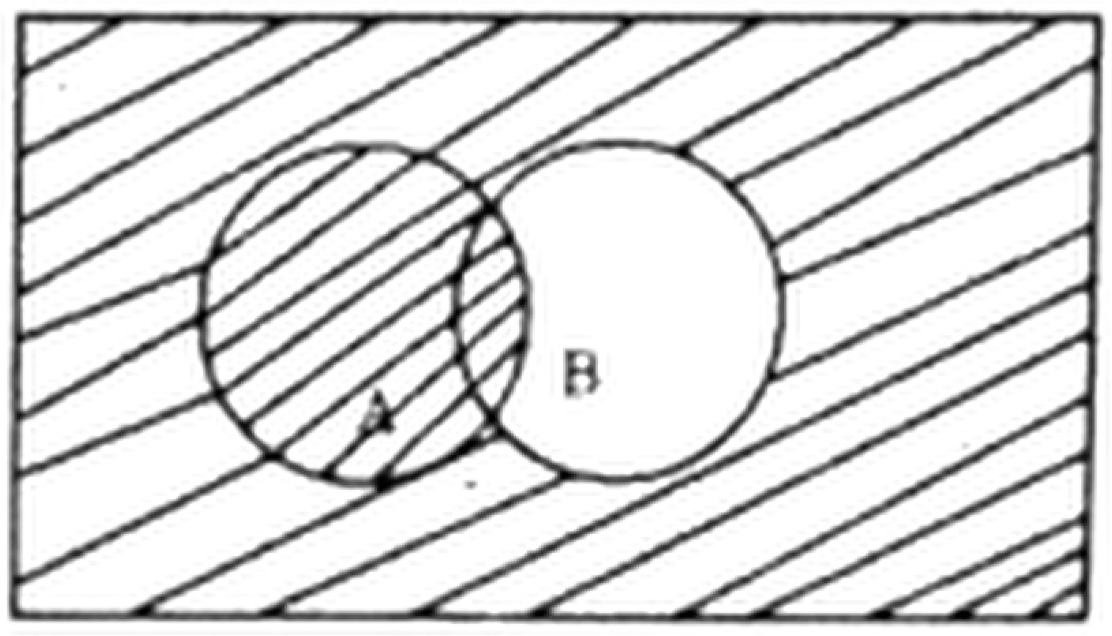
= {1, 3, 5, 7, 9, 10} ∪ {1, 3, 5, 7, 9}

= {1, 3, 5, 7, 9, 10} ....(ii)

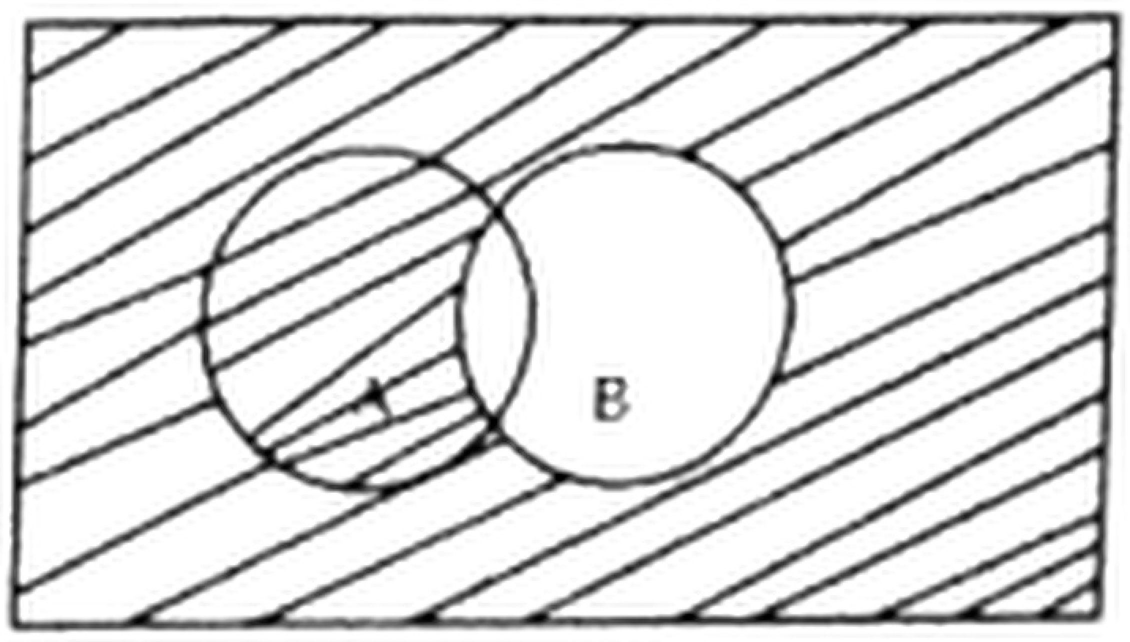
From (i) and (ii), we have

L.H.S. = R.H.S

Hence Proved.



$(A \cup B)'$



$B' \cup A$

5.1.4 (viii) Ordered pairs and Cartesian product:

5.1.4 (a) Ordered pairs:

Any two numbers x and y, written in the form (x, y] is called an ordered pair. In an ordered pair (x, y), the order of numbers is important, that is, x is the first co-ordinate and y is the second co-ordinate. For example, (3, 2) is different from (2, 3).

It is obvious that $(x, y) \neq (y, x)$ unless $x = y$.

Note that $(x, y) = (s, t)$, if $x = s$ and $y = t$.

5.1.4(b) Cartesian product:

Cartesian product of two non-empty sets A and B denoted by $A \times B$ consists of all ordered pairs (x, y) such that $x \in A$ and $y \in B$.

Example: If $A = \{1, 2, 3\}$ and $B = \{2, 5\}$, then find $A \times B$ and $B \times A$.

Solution: $A \times B = \{(1, 2), (1, 5), (2, 2), (2, 5), (3, 2), (3, 5)\}$

Since set A has 3 elements and set B has 2 elements.

Hence product set $A \times B$ has $3 \times 2 = 6$ ordered pairs.

We note that $B \times A$ - $\{(2, 1), (2, 2), (2, 3), (5, 1), (5, 2), (5, 3)\}$

Evidently $A \times B \neq B \times A$.