Exercise 5.2

1. If X = {1, 3, 5, 7, ...,19}, Y = (0, 2, 4, 6, 8,...,20)

Z = (2, 3, 5, 7, 11, 13, 17, 19, 23), then find the following.

(i) X ∪ (Y ∪ Z)

Solution:

Y ∪ Z = {0, 2, 4, 6, 8,....,20} ∪ {2, 3, 5, 7, 11, 13, 17, 19, 23}

= {0, 2, 3, 4, ..., 17, 19, 20, 23)

X ∪ (Y ∪ Z) = {1, 3, 5, 7, ... 19) ∪ { 0, 2, 3, 4, ..., 17, 19, 20, 23)

= {0, 1, 2, 3, ..., 30, 33}

(ii) (X ∪ Y) ∪ Z

Solution:

X ∪ Y = {1, 3, 5, 7, ... 19) ∪ {0, 2, 4, 6, 8, ..., 20}

= {0, 1, 2, 3, ..., 19, 20)

(X ∪ Y) ∪ Z = {0, 1, 2, 3, ... 19, 20) ∪ { 2, 3, 5, 7, 11, 13, 17, 19, 23)

= {0, 1, 2, 3,....., 20, 23)

(iii) X ∩ (Y ∩ Z)

Solution:

Y ∩ Z = {0, 2, 4, 6, 8, ..., 20) ∩ {2, 3, 5, 7, 11, 13, 17, 19, 23}

$= \emptyset$

$X \cap (Y \cap Z)$

$X \cap Y = \{1, 3, 5, 7,...., 19\} \cap \emptyset$

$= \emptyset$

(iv) (X ∩ Y) ∩ Z

Solution:

$X \cap Y = \{1, 3, 5, 7, ..., 19) \cap \{0, 2, 4, 6, 8, ..,20\}$

$= \emptyset$

$(X \cap Y) \cap Z = \emptyset \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23)$

$= \emptyset$

(v) X ∪ (Y ∩ Z)

Solution:

$Y \cap Z = \{0, 2, 4, 6, 8, ..., 20) \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$

$= \{2)$

$X \cup (Y \cap Z) = \{1, 3, 5, 7,....,19\} \cup \{2\}$

$= \{1, 2, 3, 5, 7,....,19\}$

(vi) (X ∪ Y) ∩ (X ∪ Z)

Solution:

$X \cup Y = \{1, 3, 5, 7,....,19\} \cup \{0, 2, 4, 6, 8,...,20\}$

$= \{0, 1, 2, 3, ..., 19, 20\}$

$X \cup Z = \{1, 3, 5, 7, ..., 19\} \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$

$= \{1, 2, 3, 5, 7, ..., 17, 19, 23\}$

$(X \cup Y) \cap (X \cup Z) = \{0, 1, 2, 3, ..., 19, 20\} \cap \{1, 2, 3, 5, 7, ..., 17, 19, 23\}$

$= \{1, 2, 3, 5, 7, ..., 19\}$

(vii) X ∩ (Y ∪ Z)

Solution:

$Y \cup Z = \{0, 2, 4, 6, 8, ..., 20\} \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$

$= \{0, 2, 3, 4, 5, 6, ..., 19, 20, 23\}$

$X \cap (Y \cup Z) = \{1, 3, 5, 7, ..., 19\} \cap \{0, 2, 3, 4, 5, 6, ..., 19, 20\}$

$= \{3, 5, 7, ..., 19\} \cap \{0, 2, 4, 6, 8, ..., 20\}$

$= \emptyset$

$X \cap Z = \{1, 3, 5, 7, ..., 19\} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$

$= \{3, 5, 7, 11, 13, 17, 19\}$

(viii) (X ∩ Y) ∪ (X ∩ Z)

Solution:

$X \cap Y = \{1, 3, 5, 7, ..., 19\} \cap \{0, 2, 4, 6, 8, ..., 20\}$

$= \emptyset$

$X \cap Z = \{1, 3, 5, 7, ..., 19\} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$

$= \{3, 5, 7, 11, 13, 17, 19\}$

2. If 4 = (1, 2, 3, 4, 5, 6), B = (2, 4, 6, 8), C= {1, 4, 8).

Prove the following identities:

(ii) A ∩ B = B ∩ A

Solution:

L.H.S. = A ∩ B

= {1, 2, 3, 4, 5, 6) ∩ (2, 4, 6, 8)

= (2, 4, 6) .....(i)

R.HS. = B ∩ A

= (2, 4, 6, 8) ∩ (1, 2, 3, 4, 5, 6)

= (2, 4, 6} .....(ii)

From (i) and (ii), we have

L.H.S = R.H.S.

Hence proved.

(ii) A ∪ B = B ∪ A

Solution:

L.H.S. = A ∪ B

= (1, 2, 3, 4, 5, 6) ∪ {2, 4, 6, 8)

={1, 2, 3, 4, 5, 6, 8) ....(i)

R.HS. = B ∪ A

= {2, 4, 6, 8) ∪ (1, 2, 3, 4, 5, 6)

= {1, 2, 3, 4, 5, 6, 8) ......(ii)

From (i) and (ii), we have

L.H.S = R.H.S.

Hence proved.

(iii) A ∪ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)

Solution:

L.H.S. = A ∩ (B ∪ C)

= {1, 2, 3, 4, 5, 6} ∩ ({2, 4, 6, 8) ∩ {1, 4. 8))

={1, 2, 3, 4, 5, 6} ∩ {1, 2, 4, 6, 8}

={1, 2, 3, 4, 5, 6} ....(i)

R.H.S. = (A ∩ B) ∪ (A ∪ C)

= ({1, 2, 3, 4, 5, 6} ∩ {2, 4, 6, 8}) ∪ ({1, 2, 3, 4, 5, 6} ∩ {1, 4, 8))

= {2, 4, 6} ∪ {1, 4}

= {1, 2, 3, 4, 5, 6) ....(ii)

From (i) and (ii), we have

L.H.S = R.H.S.

Hence Proved.

(iv) A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)

Solution:

L.H.S.= A ∪ (B ∩ C)

= {1, 2, 3, 4, 5, 6} ∪ ({2, 4, 6, 8) ∩ {1, 4, 8})

= {1, 2, 3, 4, 5, 6.} ∪ {4, 8)

= {1, 2, 3, 4, 5, 6, 8} ....(i)

R.HS. = $(A \cup B) \cap (A \cup C)$

R.HS. = $(A \cup B) \cap (A \cup C)$

= ({1, 2, 3, 4, 5, 6} ∪ {2, 4, 6, 8}) ∩ ({1,2, 3, 4, 5, 6} ∩ {1, 4, 8})

= {1,2, 3, 4, 6, 8} ∩ {1.2, 3. 4. 5, 6, 8}

= {1, 2, 3, 4, 5, 6,8} ....(ii)

From (i) and (ii), we have L.H.S = R.H.S.

Hence Proved.

3. If U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} A = {1, 3, 5, 7, 9}, B = {2, 3, 5, 7}, then verify the De-Morgan's Laws.

i.e., $(A \cap B) = A' \cup B'$ and $(A \cup B)' = A' \cap B'$

Solution:

L.H.S. = $A' \cup B'$

= U - $(A \cap B)$

= {1,2, 3, 4, 5, 6, 7, 8, 9, 10} – ({1.3, 5, 7, 9} ∩ {2, 3, 5, 7})

= {1, 2, 3, 4, 5, 6, 7, 8. 9, 10} - {3, 5, 7}

= {l, 2, 4, 6, 8, 9, 10} ....(i)

R.H.S. = $A' \cup B'$

= [U - A] ∪ [U - B]

= ({1, 2, 3, 4, 5, 6, 7, 8, 9, 10} - {1, 3, 5, 7, 9})

∪ ({1,2, 3, 4, 5, 6, 7, 8, 9, 10} - {2, 3, 5, 7})

= {2, 4, 6, 8, 10} ∪ {1,4, 6, 8, 9, 10}

$= \{1, 2, 4, 6, 8, 9, 10\}$ ....(ii)

From (j) and (ii), we have L.H.S = R.H.S.

(ii) $(A \cup B)' = A' \cap B'$

L.H.S. = $A' \cup B'$

$= U - (A \cup B)$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - (\{1, 3, 5, 7, 9\} \cap \{2, 3, 5, 7\})$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{1, 2, 3, 5, 7, 9\}$

- $= \{4, 6, 8, 9, 10\}$ ....(i)

R.H.S. = $A' \cap B'$

$= [U - A] \cap [U - B]$

$= (\{1, 2, 3, ..., 10\} - \{1, 3, 5, 7, 9\}) \cap (\{1, 2, 3, ..., 10\} - \{2, 3, 5, 7\})$

$= \{2, 4, 6, 8, 10\} \cap \{1, 4, 6, 8, 9, 10\}$

$= \{4, 6, 8, 9, 10\}$ .....(ii)

From (i) and (ii), we have

L.H.S = R.H.S.

4. If U = {1, 2, 3, ...,20), X = {1, 3, 7, 9, 15, 18, 20} and

Y = (1, 3, 5,..., 17}, then show that

(i) $X - Y = X \cap Y'$

Solution:

L.H.S. = $X \cap Y'$.

= {1, 3, 5, 7, 9, 15, 18, 20} ∩ (U - Y)

= {1, 2, 5, 7, 9, 15, 18, 20} ∩ ({ 1, 2, 3,...,20} - {1, 3, 5, 17})

= {1, 3, 5, 7, 9, 15, 18, 20} ∩ {2, 4, 6, 8, 10, 12, 14, 16, 18, 20}

= {18, 20} ....(ii)

From (i) and (ii), we have

L.H.S = R.H.S.

Hence Proved.

(ii) Y - X = Y ∩ X'

Solution:

L.H.S = Y - X

= {1, 3, 5,.....,17} - {1, 2, 5, 7, 9, 15, 18, 20}

= {5, 11, 13, 17} ....(i)

R.H.S. = Y ∩ X'

= Y ∩ (U - X)

= {l, 3, 5,...., 17} ∩ ({1, 3,....., 20} - {1, 3, 5, 7, 9, 15, 18. 20}

= {1,3, 5,....., 17} ∩ {2, 4, 5, 6, 8, 10, 11, 12, 13, 14, 16, 19}

= {5, 11,13, 17} .....(ii)

From (i) and (ii). we have

L.H.S = R.H.S.

Hence Proved.

Verify the fundamental properties for given sets:

(a) A and B are any two subsets of U, then A ∪ B = B ∪ A (commutative law).

For example

A = {1, 3, 5, 7} and B = {2, 3, 5, 7}

then A ∪ B = {1, 3, 5, 7} ∪ {2, 3, 5, 7} = {1, 2, 3, 5, 7}

and B ∪ A = {2, 3, 5, 7} ∪ {1,3, 5, 7} = {1, 2, 3, 5, 7}

Hence, verified that A ∪ B = B ∪ A.

(b) Commutative property of intersection

For example A = {1, 3, 5, 7} and B = {2, 3, 5, 7}

Then A ∩ B = {1, 3, 5, 7} ∩ {2, 3, 5, 7} = {3, 5, 7}

and B ∩ A = {2, 3, 5, 7} ∩ {1,3, 5, 7} = {3, 5, 7}

Hence, verified that A ∩ B = B ∩ A.

(c) If A, B and C are the subsets of U, then (A ∪ B) ∪ C = A ∪ (B ∪ C).

(Associative law)

Suppose A = {1, 2, 4, 8}; B = {2, 4, 6}

And C = {3, 4, 5, 6}

Then L.H.S. = (A ∪ B) ∪ C

= ({1, 2, 4, 8} ∪ {2, 4, 6}) ∪ {3, 4, 5, 6}

= {1, 2, 4, 6, 8} ∪ {3, 4, 5, 6}

= {1, 2, 3, 4, 5, 6, 8}

and R.H.S. =A ∪ (B ∪ C)

= {42, 4, 8} ∪ ({2, 4, 6} ∪ {3, 4, 5, 6})

= {1, 2, 4, 8} ∪ {2, 3, 4, 5, 6}

= {42, 3, 4, 5, 6, 8}

L.H.S = R.H.S

Hence, union of Sets is associative.

(d) If A, B and C are the subsets of U, then (A ∩ B) ∩ C = A ∩ (B ∩ C)

(Associative Law).

Suppose., A = {1, 2, 4, 8}; 5 = {2, 4, 6} and C = {3, 4, 5, 6}

then L.H. S, = (A ∩ B) ∩ C

= ({1, 2, 4, 8} ∩ {2, 4, 6)) ∩ {3, 4, 5, 6}

= {2, 4} ∩ {3, 4, 5, 6} = {4}

and R.H.S. = A ∩ (B ∩ C)

= {1, 2, 4, 8} ∩ ({2, 4, 6} ∩ {3, 4, 5, 6})

= (1, 2, 4, 8}; ={4, 6} - {4}

L.H.S. = R.H.S.

Hence, intersection of sets is associative.

Distributive laws

(e) Union is distributive over intersection of sets

If A, B and C are the subsets of universal set U, then A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C). **Solution:** Suppose A = {1, 2, 4, 8}, B = {2, 4, 6} and C = {3, 4, 5, 6}

then L.H.S =A ∪ (B ∩ C)

$=\{1, 2, 4, 8\} \cup (\{2, 4, 6) \cap \{3, 4, 5, 6\})$

$= \{1, 2, 4, 8) \cup \{4, 6\} - \{1, 2, 4, 6, 8\}$

and R.H.S $= (A \cup B) \cap (A \cup C)$

$= (\{1, 2, 4, 8\} \cup \{2, 4, 6\}) \cap (\{1, 2, 4, 8\} \cup \{3, 4, 5, 6\})$

$= (1, 2, 4, 6, 8\} \cap \{1, 2, 3, 4, 5, 6, 8\}$

$= (1, 2, 4, 6, 8\}$

L.H.S = R.H.S

(f) Intersection is distributive over union of sets

To prove $\cap (B \cup C) = (A \cap B) \cup (A \cap B)$

Suppose $A = \{1, 2, 3, 4, 5,....,20\}$

$B = \{5, 10, 15, 20, 25, 30\}$

$C = \{3, 9, 15, 21, 27, 33\}$

L.H.S. $= A \cap (B \cup C)$

$= \{1, 2, 3, 4, 5, ..., 20\} \cap (\{5, 10, 15, 20, 25, 30\} \cup \{3, 9, 15, 21, 27, 33\})$

$= \{1, 2, 3, 4, 5, ... 20\} \cap \{3, 5, 9, 10, 15, 20, 21, 25, 27, 30, 33\}$

$= \{3, 5, 9, 10, 15, 20\}$

R.H.S. $= (A \cap B) \cup (A \cap C)$

$= (\{1, 2, 3, 4, ..., 20\} \cap \{5, 10, 15, 20, 25, 30\})$

$\cup (\{1, 2, 3, 4, 5, ..., 20\} \cap \{3, 9, 15, 21, 27, 33\})$

$= \{5, 10, 15, 20\} \cup \{3, 9, 15\} = \{3, 5, 9, 10, 15, 20\}$,

L.H.S. = R.H.S.

(g) De Morgan's Laws $(A \cap B)' = A' \cup B'$ and $(A \cup B)' = A' \cap B'$

Suppose $U = \{1, 2, 3, 4, ..., 10\}$

$A = \{2, 4, 6, 8, 10\}$ $\Rightarrow \{1, 3, 5, 7, 9\}$

$B = \{1, 2, 3, 4, 5, 6\}$ $\Rightarrow B' = \{7, 8, 9, 10\}$

Now consider $A \cap B = \{2, 4, 6, 8, 10\} \cap \{1, 2, 3, 4, 5, 6\}$

. $=\{2, 4, 6\}$

Then L.H.S. $= (A \cap B)' = U - (A \cap B)$

$= \{1, 2, 3, 4, ..., 10\} - \{2, 4, 6\}$

$= \{1, 3, 5, 7, 8, 9, 10\}$

and R.H.S. = $A \cup B'$

$= \{1, 3, 5, 7, 9\} \cup \{7, 8, 9, 10\}$

$= \{1, 3, 5, 7, 8, 9, 10\}$

L.H.S. = R.H.S.

$(A \cup B)' = A' \cap B'$.

Suppose $U = \{1, 2, 3, 4, ..., 10\}$

$A = \{2, 4, 6, 8, 10\} \Rightarrow A' = \{-1, 3, 5, 7, 9\}$

$B = \{1, 2, 3, 4, 5, 6\} \Rightarrow B' = \{7, 8, 9, 10\}$

Now consider $A \cup B = \{2, 4, 6, 8, 10\} \cup \{1, 2, 3, 4, 5, 6\}$

$= \{1, 2, 3, 4, 5, 6, 8, 10\}$

L.H.S. = $(A \cup B)' = U - (A \cup B)$

$= \{1, 2, 3, 4, ..., 10\} - \{1, 2, 3, 4, 5, 6, 8, 10\}$

$= \{7, 9\}$

and R.H.S $A' = B' = \{1, 3, 5, 7, 9\} \cap \{7, 8, 9, 10\}$

$= \{7, 9\}$

L.H.S. = R.H.S.

Venn Diagram:

British mathematician John Venn (1834 - 1923) introduced rectangle for a universal set U and its subsets A and B as closed figures inside this rectangle.

Use Venn diagrams to represent:

(a) Union and intersection of sets

Disjoint sets Overlapping sets $A \subseteq B$

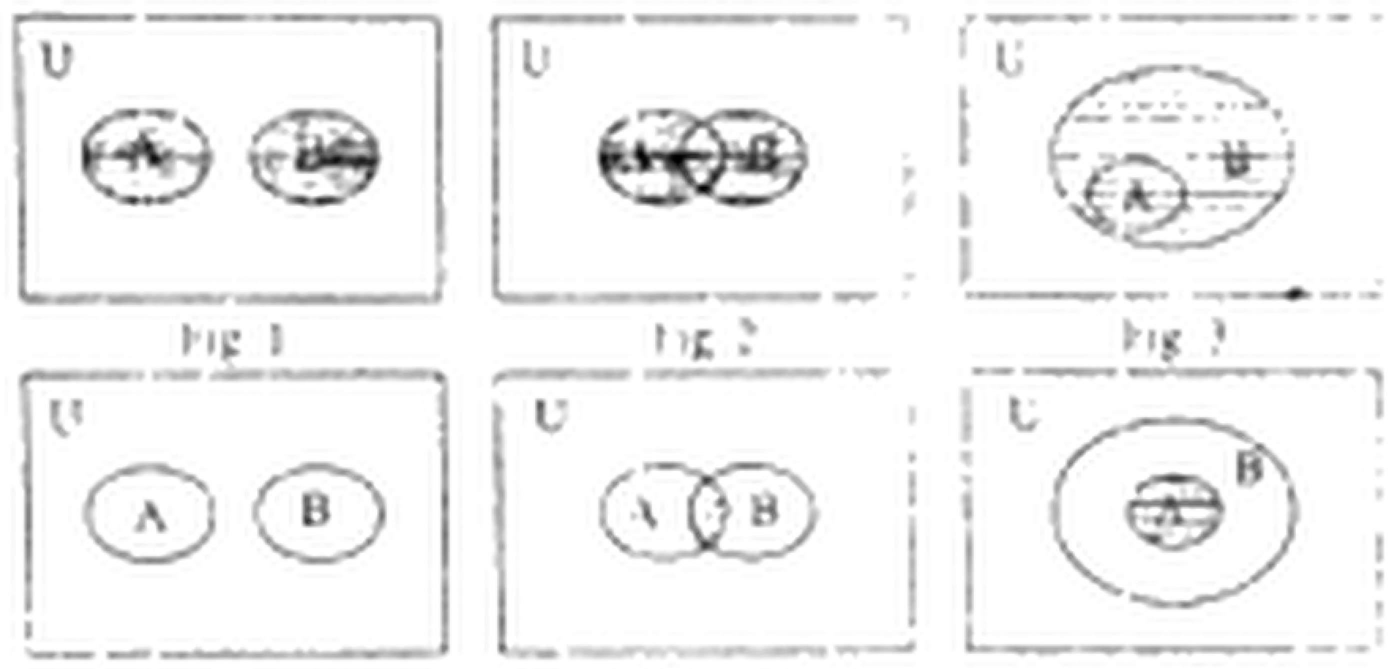
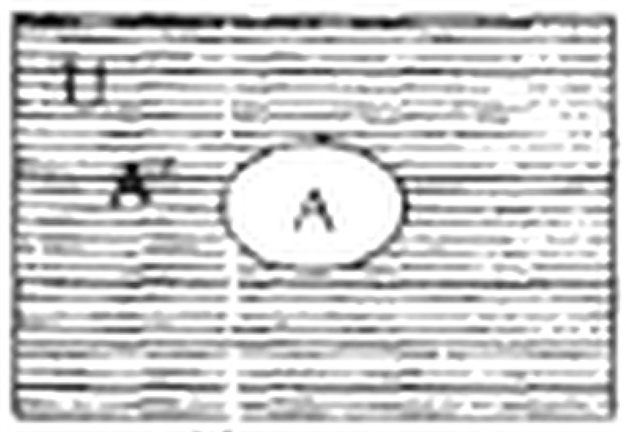


Fig 1 Fig 2 Fig 3

Fig. 4 Fig. 5 Fig. 6

(Regions shown by horizontal line segments in figures 1 to 6.)

(b) Complement of a set



U – A = A' is shown by horizontal line segments.

Use Venn diagram to verily:

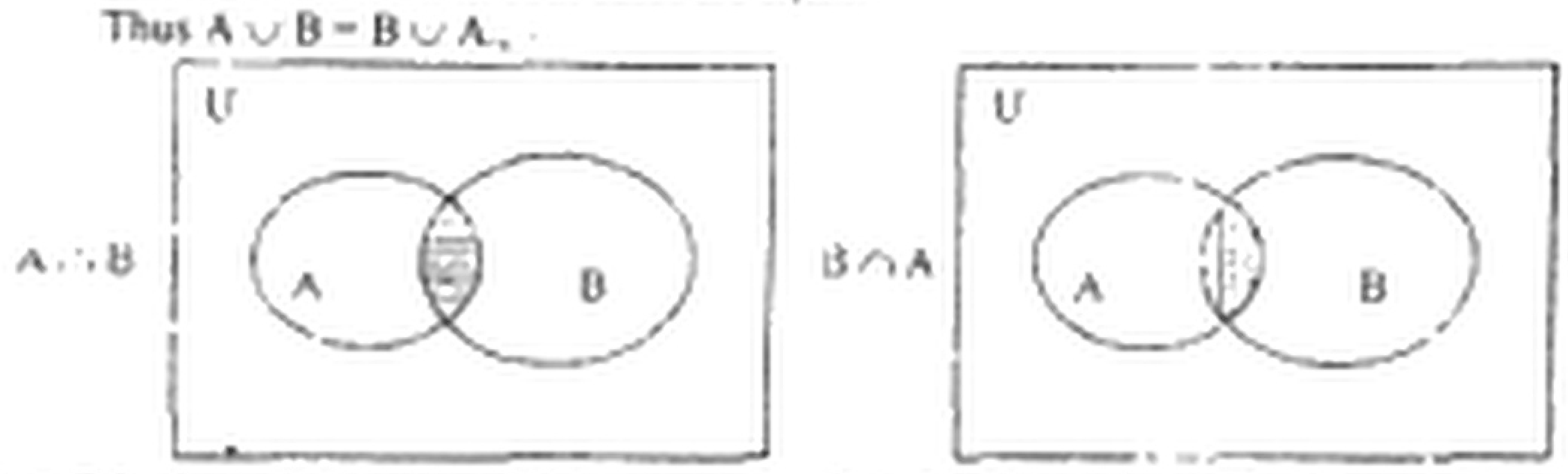
(a) Commutative law for union and intersection of sets.

A ∪ B is shown by horizontal line segments,

B ∪ A is shown by vertical line segments.

The regions shown in both cases are equal.

Thus $A \cup B = B \cup A$.



A ∩ B is shown by horizontal line segments. B ∩ A is shown by vertical line segments.

The regions shown in both cases are equal.

Thus $A \cap B = B \cap A$.

(b) De Morgan's laws

(i) $(A \cup B)' = A' \cap B'$

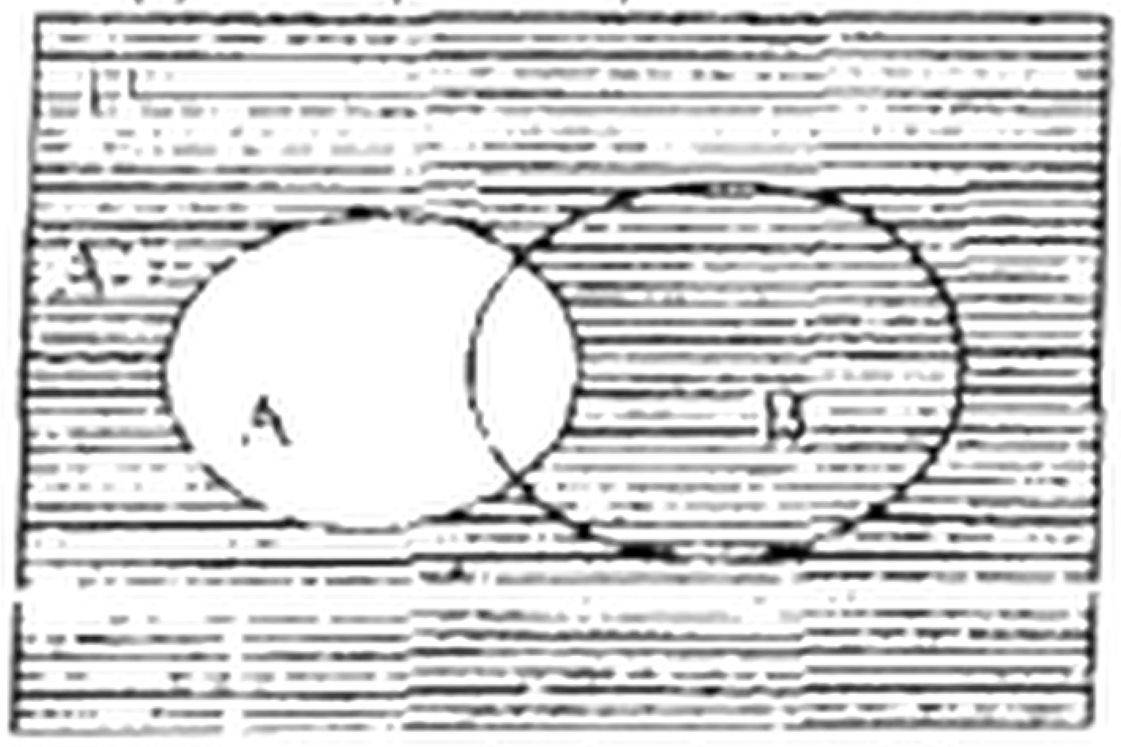


Fig. 1: A' is shown by horizontal line segments

(ii) $(A \cap B)' = A' \cup B'$

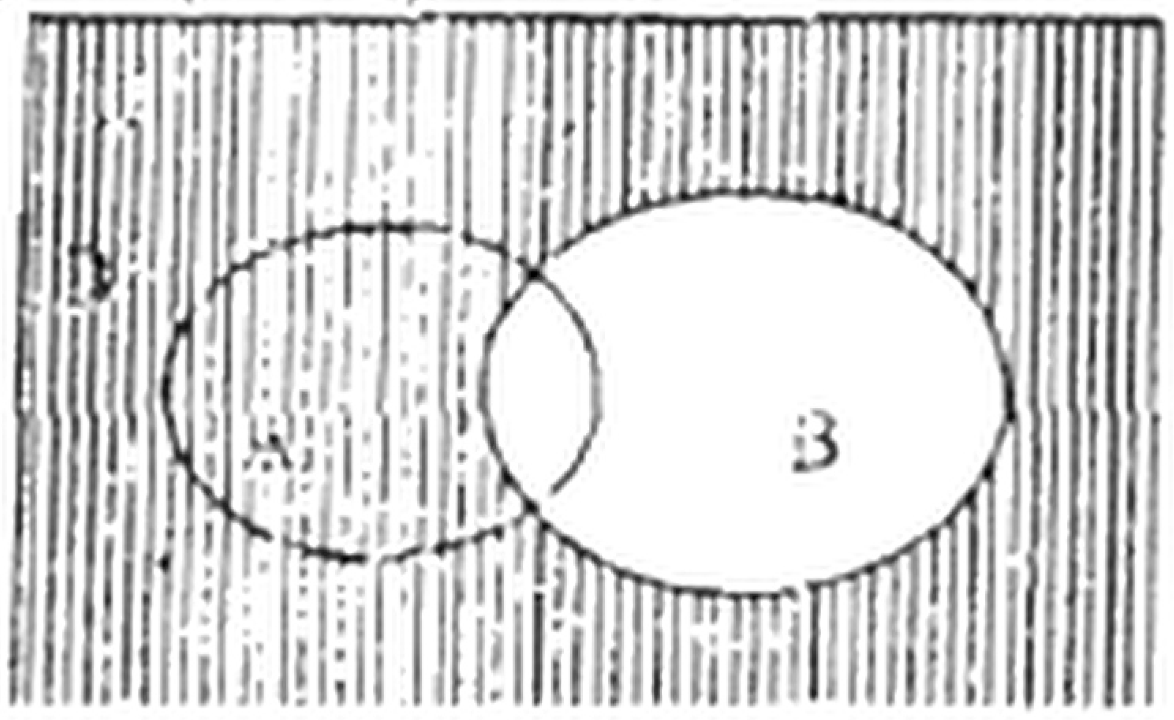


Fig. 2: B' is shown by vertical line segments

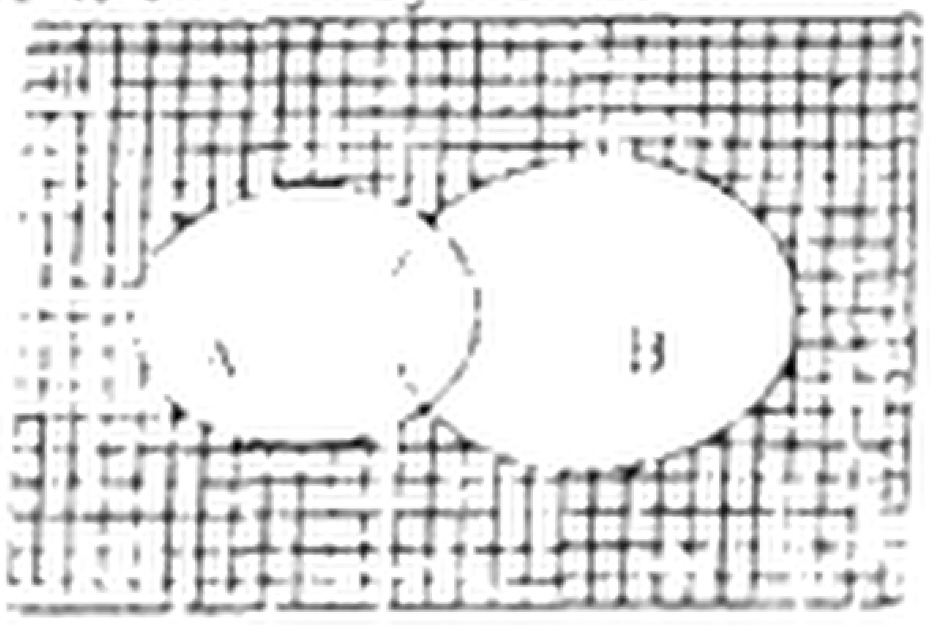


Fig. 3: $A' \cap B'$ is shown by squares

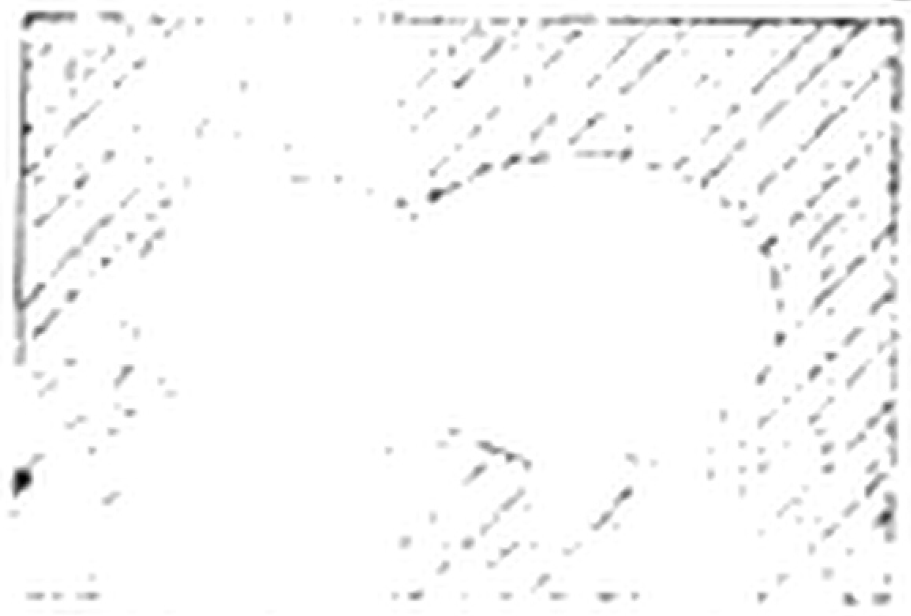


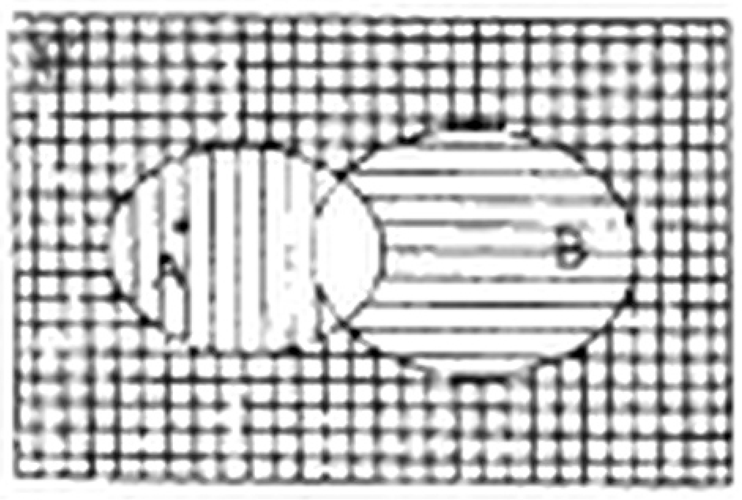
Fig. 4: $(A \cup B)'$ is shown by slanting line segments

Regions shown in Fig. 3 and Fig. 4 are equal.

Thus $(A \cup B)' = A' \cap B'$

$(A \cap B)' = A' \cup B'$

$A' \cup B'$



$(A \cap B)'$

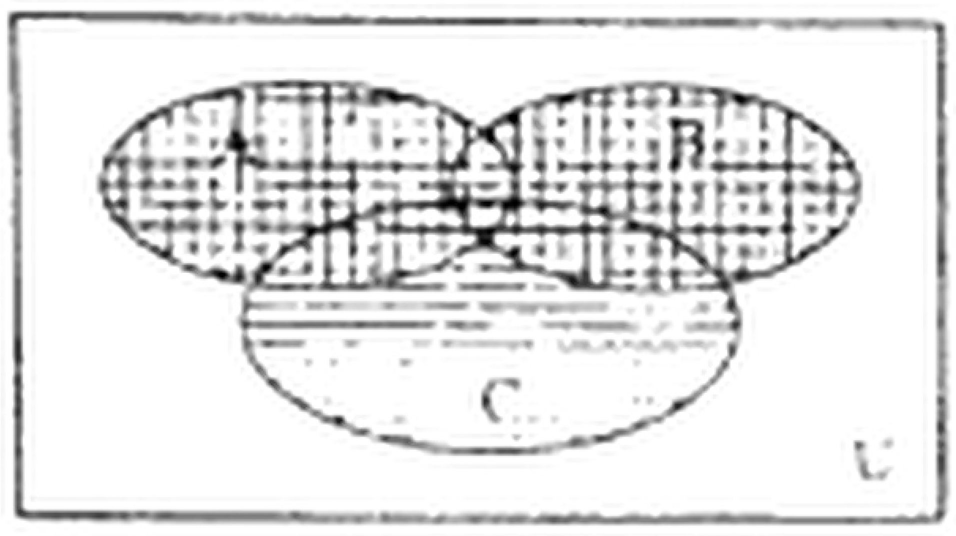
Fig. 5 $A' \cup B'$ is shown by squares, horizontal and vertical line segments.

Fig.6 $U - (A \cap B) = (A \cap B)$ is shown by squares, horizontal

Regions shown in Fig. 5 and Fig. 6 are equal.

Thus $(A \cap B)' = A' \cup B'$

(c) Associative law:



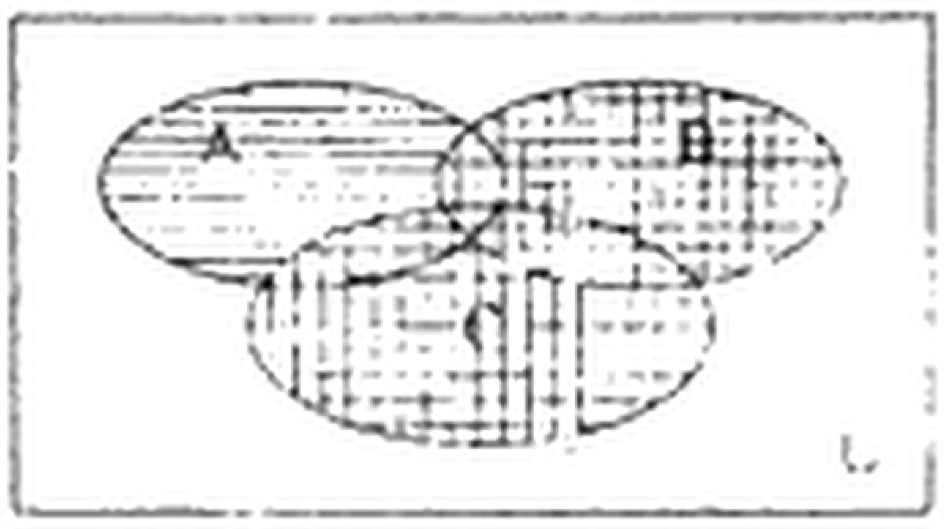
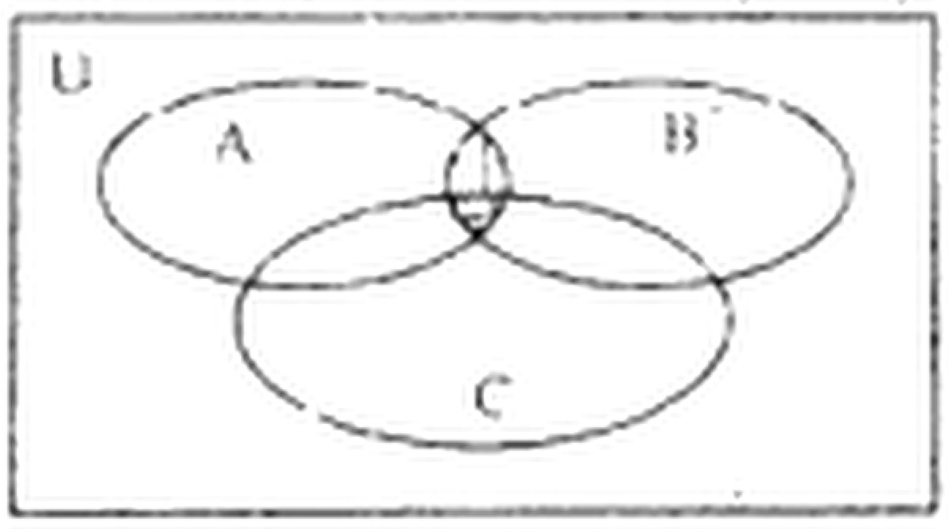


Fig. 1
$(A \cup B) \cup C$ is shown in the above figure.

Fig. 2
$A \cup (5 \cup C)$ is shown in the above figure.

Regions shown in fig. 1 and fig. 2 by different ways are equal.

Thus $(A \cup B) \cup C = A \cup (B \cup C)$



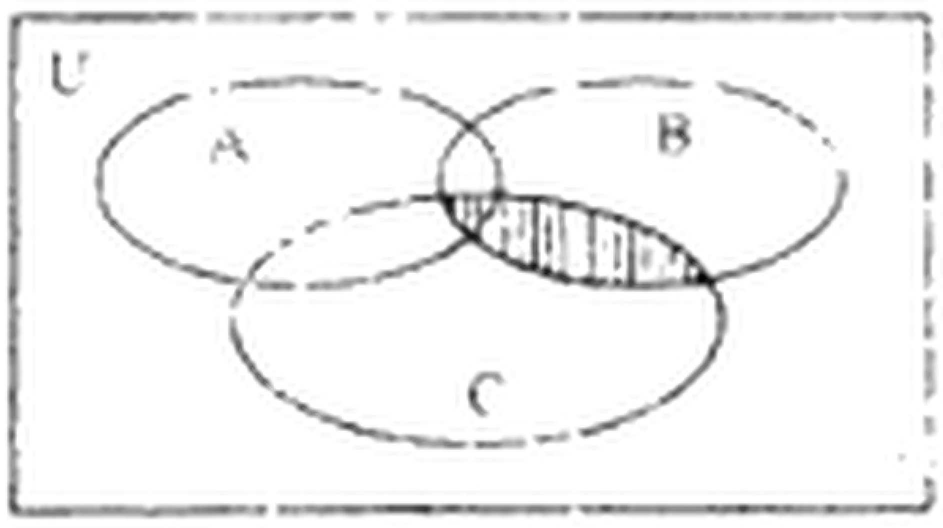


Fig. 3

Fig. 4

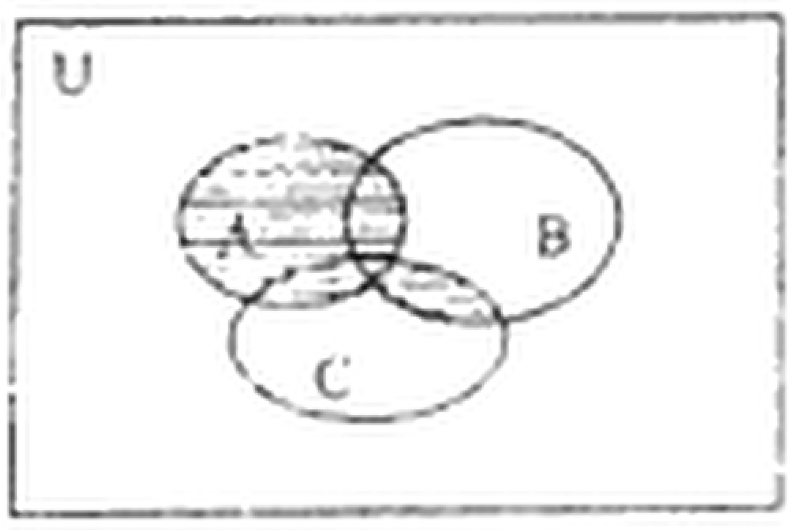
$(A \cap B) \cap C$ is shown in figure 3 by double

$A \cap (B \cap C)$ is shown in figure 4 by double crossing line segments

Regions shown in Fig. 3 and fig. 4 are equal.

Thus $(A \cap B)' \cap C' = A \cap (B \cap C)$

(d) Distributive law:



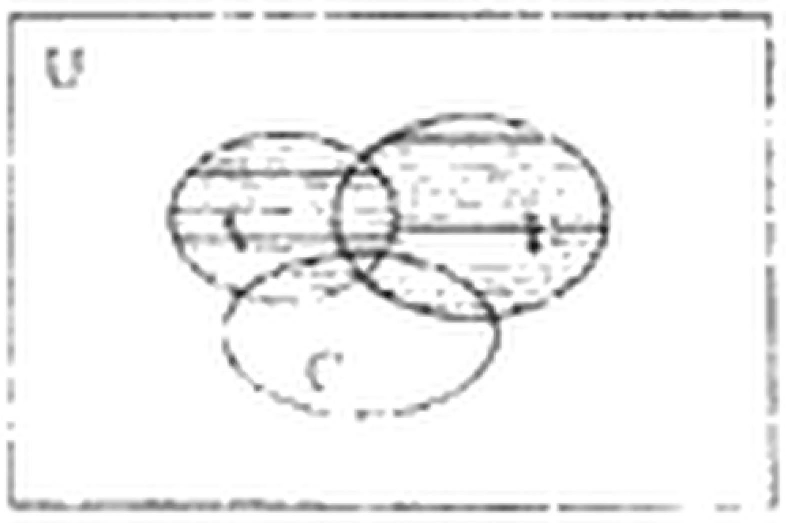
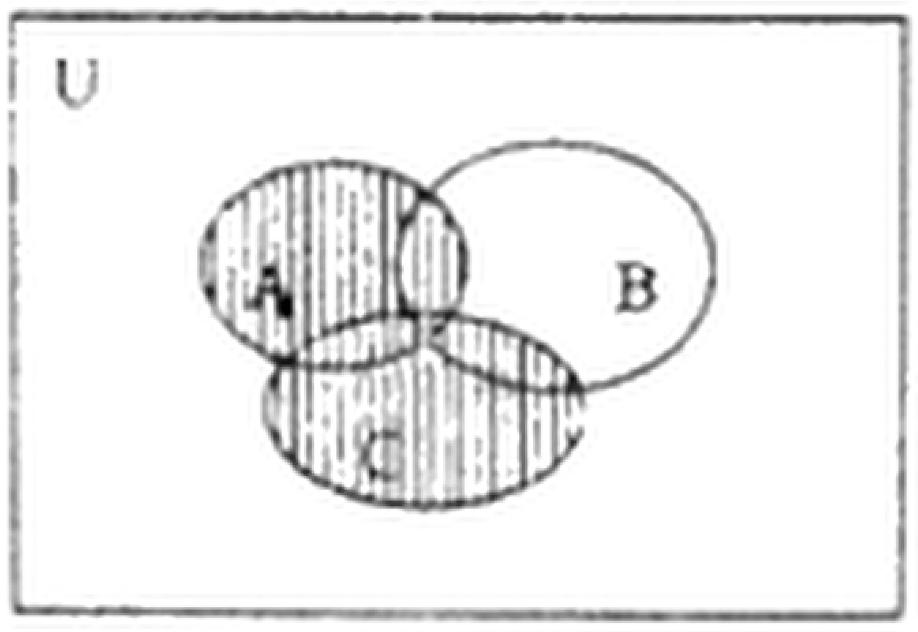


Fig. 1: $A \cup (B \cap C)$ is shown by horizontal line segments in the above figure.

Fig. 2: A u B is shown by horizontal line segments in the above figure.



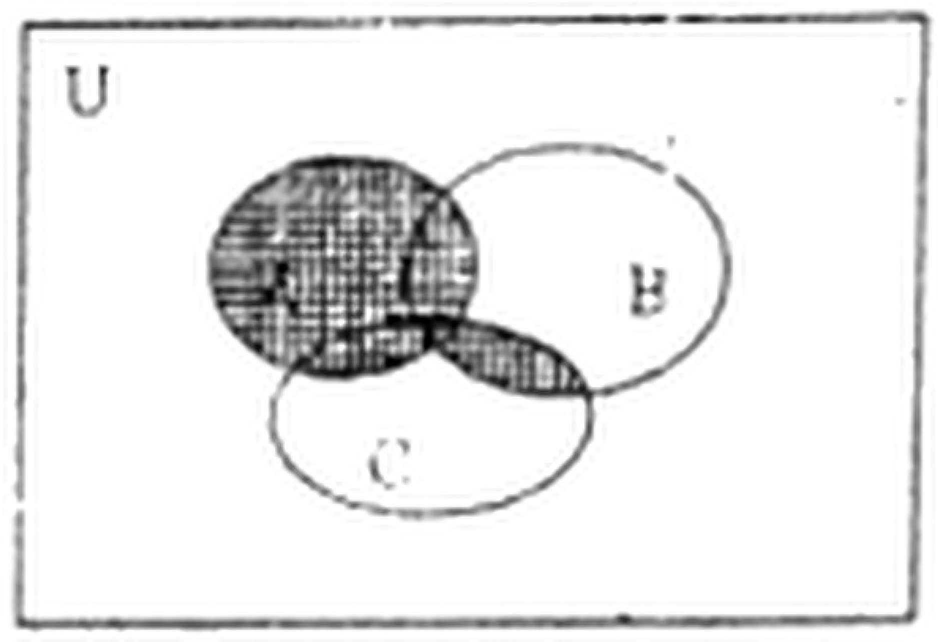
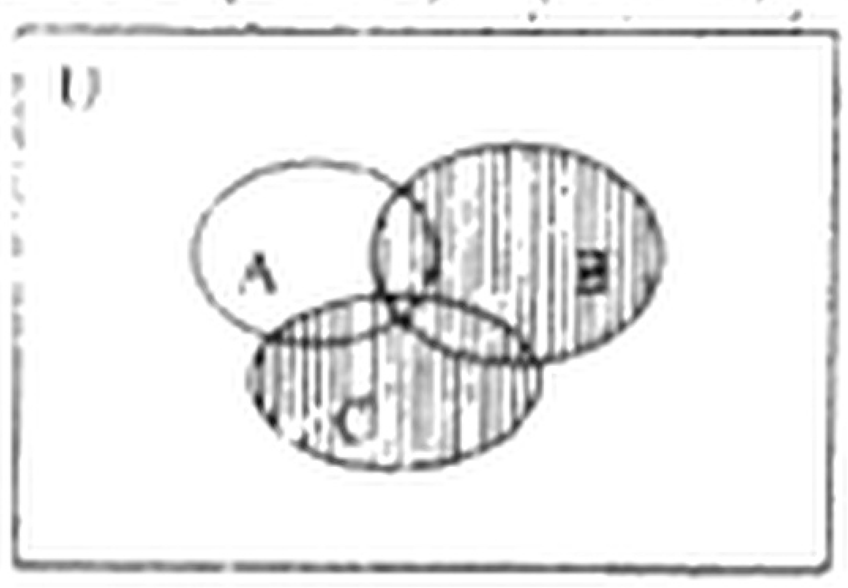


Fig. 3: $A \cup C$ is shown by vertical line segments in Fig. 3,

Fig. 4: $(A \cup B) \cap (A \cup C)$ is shown by double crossing line segments in Fig. 4.

Regions shown in Fig. 1 and Fig. 4 are equal.

Thus $A \cup (B \cap C) = (A \cup B') \cap (A \cup C)$



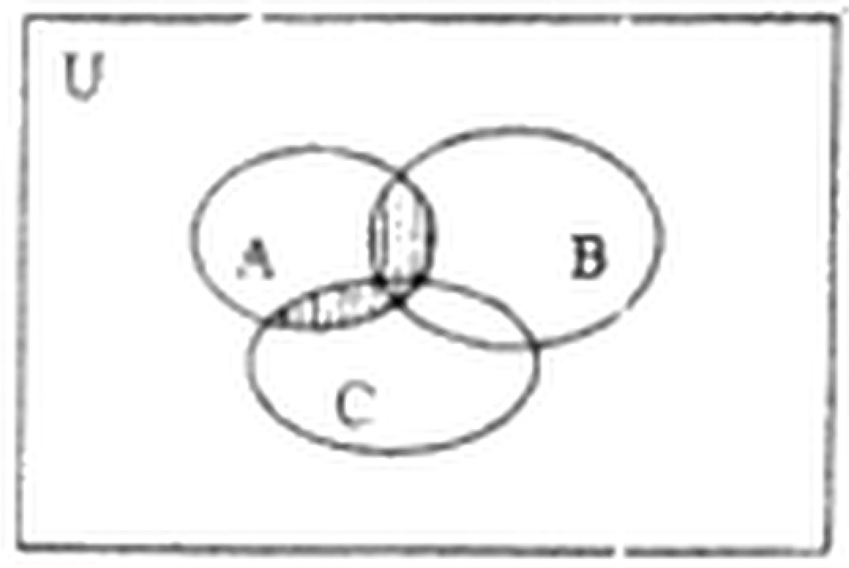


Fig. 5: $B \cup C$ is shown by vertical line segments in Fig. 5.

Fig. 6: $A \cap (B \cup C)$ is shown in Fig. 6 by vertical line segments.

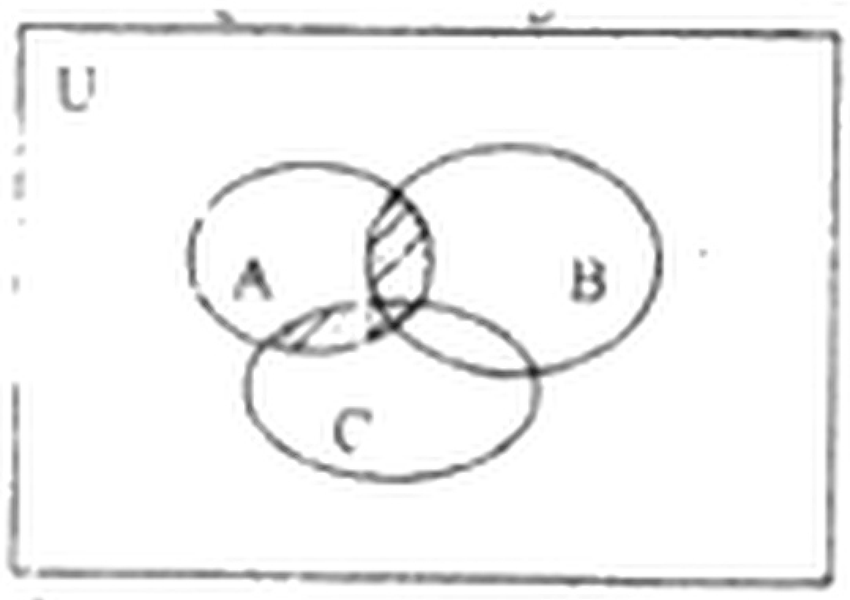


Fig. 7: $(A \cap B) \cup (A \cap C)$ is shown in Fig. 7 by slanting line segments.

Regions displayed in Fig. 6 and Fig, 7 are equal.

Thus $A \cap (A \cup C) = (A \cap B) \cup (A \cap C)$