Exercise 5.1

1. If X = {1, 4, 7, 9} and Y = {2, 4, 5, 9)

Then find:

(i) $X \cup Y$

(ii) $X \cap Y$

(iii) $Y \cup X$

(iv) $Y \cap X$

Solution:

(i) $X \cup Y = \{1, 4, 7, 9\} \cup \{2, 4, 5, 9\}$

$= \{1, 2, 4, 7, 9\}$

(ii) $X \cap Y = \{l,4,7,9\} \cap \{2,4,5,9\}$

$= \{4, 9\}$

(iii) $Y \cup X = \{2, 4, 5, 9\} \cup \{l, 4, 7, 9\}$

$= \{1, 2, 4, 5, 7, 9\}$

(iv) $Y \cup X = \{2, 4, 5, 9\} \cap \{l, 4, 7, 9\}$

$= \{4, 9\}$

2. If X Set of prime numbers less than or equal to 17 and Set of first 12 natural numbers, then find the following.

(i) X ∪ Y **(ii) Y ∪ X** **(iii) Z ∩ Y** **(iv) Y ∩ X**

Solution:

X = {2, 3, 5, 7, 11, 13, 17}, Y = {l, 2, 3, 4, ,12}

(i) **X ∪ Y** = {2, 3, 5, 7, 11,13, 17} ∪ {l, 2, 3, 4,.....,12}

={1, 2, 3, 4,.....,12, 13, 17}

={1, 2, 3, 4,......, 12} ∪ {13, 17}

= Y ∪ {13,17}

(ii) **Y ∪ X** = {l, 2, 3, 4,..,12} ∪ {2, 3, 5, 7,11, 13, 17}

= {1, 2, 3, 4,....12, 13, 17}

= {l, 2, 3, 4...,12} ∪ {l3, 17}

=Y ∪ {13,17}

(iii) **X ∩ Y** = {2, 3, 5, 7, 11, 13, 17} ∩ {1, 2, 3,.., 12}

= {2, 3, 5, 7, 11}

(iv) **Y ∩ X** = {l, 2, 3, 4,....,12} ∩ {2, 3, 5, 7, l l, 13, 17}

= {2, 3, 5, 7, 11}

3. If X = Ø, Y = Z^+, F = O^+, then

find: (i) X ∪ Y **(ii) X ∪ T** **(iii) Y ∪ T** **(iv) X ∩ Y**

(v) $X \cap T$ **(vi) $Y \cap T$**

Solution:

$X = \emptyset$, $Y = Z^+$, $T = O^+$

(i) $\mathbf{X \cup Y} = \emptyset \cup Z^+$

$= Z^+ = Y$

(ii) $\mathbf{X \cup T} = \emptyset \cup O^+$

$= O^+ = T$

(iii) $\mathbf{Y \cup T} = Z^+ \cup O^+$

$= Z^+ = Y$

(iv) $\mathbf{X \cap Y} = \emptyset \cap Z^+$

$= \emptyset$

(v) $\mathbf{X \cap T} = \emptyset \cap O^+$

$= \emptyset$

(vi) $\mathbf{Y \cap T} = Z^+ \cap O^+$

$= O^+ = T$

4. If U = {x | x ∈ N ^ 3 < x ≤ 25}, X = {x | x is prime ^8 < x < 25} and Y = {x | x ∈ W ^ 4 ≤ x ≤ 17). Find the value of:

i) (X ∪ Y)' (ii) X' ∩ Y' (iii) (X ∩ Y)' (iv) X' ∪ Y'

Solution:

U = {4, 5, 6, 7,.....,24, 25}

X = {11, 13, 17, 19, 23}

Y = {4, 5, 6, 7,......,16,17}

(i) (X ∪ Y)'= U - (X ∪ Y)

Now

X ∪ Y = {11, 13, 17, 19, 23} {4, 5, 6, 7,....,16, 17}

= {4, 5, 6, 7,....,16, 17, 19, 23}

(X ∪ Y)' = {4, 5, 6, 7,......,24, 25} - {4, 5, 6, 7,......,16, 17, 19, 23}

= {18, 20, 21, 22, 24, 25}

(ii) X' ∩ Y'

Now X' = U - X

= {4, 5, 6, 7,....,24, 25} - {11, 13, 17, 19, 23}

= {4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25}

Y' = U - Y

= {4, 5, 6, 7,....,24, 25} - {4, 5, 6, 7,.....,16, 17}

= {18, 19, 20,........., 24, 25}

X' ∩ Y' = {4, 5, 6, 7, 8, 9, 10, 12, 14, 16, 18, 20, 21, 22, 24, 25}

$\cap$ {18, 19, 20,....,24, 25}

= {18, 20, 21, 22, 24, 25}

(iii) $(X \cap Y)' = U - (X \cap Y)$

Now

$X \cap Y$ = {11, 13, 17, 19, 23} $\cap$ {4, 5, 6, 7,.., 16, 17}

= {11,13, 17}

$(X \cap Y)'$ = {4, 5, 6, 7,...., 24, 25} - {11, 13, 17}

= {4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 18, 20, 21, 22, 24, 25}

Y' = U - Y

= {4, 5, 6, 7,....., 24, 25} - {4, 5, 6, 7,...., 16, 17}

{18, 19, 20,...,24, 25}

Y' = U - Y

= {4, 5, 6,..., 10, 12, 14, 15, 16, 18,....., 25} $\cup$ = { 18, 19, 20,...., 24, 25}

= {4, 5 .... ,10, 12, 14, 16, 18, ... 25}

5. If X = {2, 4, 6, ..., 20} and Y = {4, 8, 12,...., 24}, then find the following:

(i) X -Y **(ii) Y – X**

Solution:

X = {2, 4, 6,...., 20} , Y = {4, 8, 12,..., 24}

(i) X – Y = {2,4, 6,.., 20} - {4, 8, 12,.., 24}

= {2, 6,10, 14, 18}

(ii) Y - X $= \{4, 8, 12, \ldots, 24\} - \{2, 4, 6, \ldots\ldots, 20\}$

$= \{24\}$

6. If A = N and B = W, then find the value of

(i) A - B **(ii) B – A**

Solution:

A = N and B = W

(i) A - B = N – W

$= \emptyset$

(ii) B - A = W – N

$= \{0, 1, 2, \ldots\} - \{1, 2, \ldots\}$

$= \{0\}$

Properties of Union and Intersection:

(a) Commutative property of union.

For any two sets A and B, prove that $A \cup B = B \cup A$.

Proof:

Let $x \in A \cup B$

$\Rightarrow x \in A$ or $x \in B$ (by definition of union of sets)

$\Rightarrow x \in B$ or $x \in A$

⇒ $x \in B \cup A$

⇒ $A \cup B \subseteq B \cup A$ (i)

Now let $y \in B \cup A$

⇒ $y \in B$ or $y \in A$ (by definition of union of sets)

⇒ $y \in A$ or $y \in B$

⇒ $y \in A \cup B$

⇒ $B \cup A \subseteq A \cup B$ (ii)

From (i) and (ii), we have $A \cup B = B \cup A$. (by definition of equal sets)

(b) Commutative property of intersection

For any two sets A and B, prove that $A \cup B$

Proof: Let $x \in A \cap B$

⇒ $x \in A$ and $x \in B$ (by definition of intersection of sets)

⇒ $x \in B$ and $x \in A$

⇒ $x \in B \cap A$

⇒ $A \cap B \subseteq B \cap A$ (i)

Now let $y \in B \cap A$

⇒ $y \in B$ and $y \in A$ (by definition of intersection of sets)

⇒ $y \in A$ and $y \in B$

⇒ $y \in A \cap B$

Therefore, $B \cap A \subseteq A \cap B$ (ii)

From (i) and (ii), we have $A \cap B = B \cap A$ (by definition of equal sets)**(c)**

Associative property of union

For any three sets A, B and C, prove that $(A \cup B) \cup C = A \cup (B \cup C)$

Proof: Let $x \in (A \cup B) \cup C$

$\Rightarrow x \in (A \cup B)$ or $x \in C$

$\Rightarrow x \in A$ or $x \in B$ or $x \in C$

$\Rightarrow x \in A$ or $x \in B$ or $A \in C$

$\Rightarrow x \in A$ or $x \in B \cup C$

$\Rightarrow x \in A \cup (B \cap C)$

$(A \cup B) \cup C \subseteq A \cup (B \cup C)$ (i)

Similarly, $A \cup (B \cup C) \subseteq (A \cup B) \cup C$ (ii)

From (i) and (ii), we have

$(A \cup B) \cup C = A \cup (B \cup C)$

(d) Associative property of intersection

For any three sets A, B and C, prove that $(A \cap B) \cap C = A \cap (B\ C\}$

Proof: Let $\in (A \cap B) \cap C$.

$\Rightarrow X \in (A \cap B)$ and $X \in C$

$\Rightarrow (X \in A$ and $x \in 5)$ and $x \in C$

$\Rightarrow x \in A$ and $(x \in B)$ and $\in C)$

$\Rightarrow x \in A$ and $x \in B \cap C$

$\Rightarrow X \in A \cap (B \cap C)$

∴ $(A \cap B) \cap C \subseteq A \cap (B \cap C)$ (i)

Similarly, $A \cap (B \cap C) \subseteq (A \cap B) \cap C$ (ii)

From (i) and (ii), we have

$(A \cap B) \cap C = A \cap (B \cap C)$

(e) Distributive property of union over intersection

For any three sets A, B and C, prove that $A \cup (B \cup C) = (A \cup B) \cap (A \cup C)$

Proof: Let $x \in A \cup (B \cup C)$

$\Rightarrow x \in A$ or $x \in B \cap C$

$\Rightarrow x \in A$ or ($x \in B$ and $X \in C$)

$\Rightarrow$ ($X \in A$ or $x \in B$) and ($x \in A$ or $X \in C$)

$\Rightarrow x \in A \cup B$ and $x \in A \cup C$

$\Rightarrow x \in (A \cup B) \cap (A \cup C)$

Therefore, $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$

Similarly, now let $y \in (A \cup B) \cap (A \cup C)$

$\Rightarrow y \in (A \cup B)$ and $y \in (A \cup C)$

$\Rightarrow$ ($y \in A$ or $y \in B$) and ($y \in A$ or $y \in C$)

$\Rightarrow y \in A$ or ($y \in B$ and $y \in C$)

$\Rightarrow y \in A$ or $y \in B \cap C$

$\Rightarrow y \in y \in \cup (B \cap C)$

$\Rightarrow (A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$ (ii)

From (i) and (ii), we have $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(f) Distributive property of intersection over union

For any three sets A, B and C, prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Proof: Let $x \in A \cap (B \cup C)$

$\Rightarrow x \in A$ and $x \in B \cap C$

$\Rightarrow x \in A$ and $[x \in B$ or $x \in C]$

$\Rightarrow [x \in A$ and $X \in B]$ or $[x \in A$ and $X \in C]$

$\Rightarrow [x \in A \cap B]$ or $[x \in A \cap C]$

$\Rightarrow x \in (A \cap B) \cup (A \cup C)$

Hence by def. of subsets

$A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$ (i)

Similarly $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$ (ii)

From (i) and (ii), we have, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(g) De-Morgan's laws

For any two sets A and B, prove that

(i) $(A \cup B)' = A' \cap B'$

Proof: Let $x \in (A \cup B)'$

$\Rightarrow x \notin A \cup B$ (by definition of complement of set)

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \in A'$ and $X \in B'$

$\Rightarrow x \in A' \cap B'$ (by definition of intersection of sets)

$\Rightarrow (A \cup B) \subseteq (A \cup B)'$ (i)

Similarly, $A' \cap B' \subseteq (A \cup B)$ (ii)

Using (i) and (ii), we have $(A \cup B)' = A' \cap B'$

(ii) Let $x \in (A \cap B)'$

$\Rightarrow x \notin A \cap B$

$\Rightarrow x \notin A$ or $x \notin B$

$\Rightarrow x \in A'$or $x \in B'$

$\Rightarrow x\ A' \cup B'$

$\Rightarrow (A \cap B)' \subseteq A' \cup B'$ (i)

Let $y \in A' \cap B'$

$\Rightarrow y \in A \cap B$

$\Rightarrow y \notin A$ or $x \notin B$

$\Rightarrow y \notin A \cap B$

$\Rightarrow y \in (A \cap B)'$

$\Rightarrow (A' \cap B)' \subseteq A' \cap B'$ (ii)

From (i) and (ii) we have proved that

$(A \cap B)' = A' \cup B'$