

Exercise 4.1

Resolve into partial fractions.

$$1) \quad \frac{7x-9}{(x+1)(x-3)}$$

Solution:

$$\frac{7x-9}{(x+1)(x-3)} = \frac{A}{x+1} + \frac{B}{x-3}$$

Multiplying both sides by $(x+1)(x-3)$, we get

$$7x-9 = A(x-3) + B(x+1) \quad \dots\dots\dots(1)$$

To find A, we put $x+1=0 \Rightarrow x=-1$ in eq.(1), we get

$$7(-1)-9 = A(-1-3) + B(-1+1)$$

$$7-9 = A(-4) + B(0)$$

$$-16 = -4A$$

$$-4A = -16$$

Dividing both sides by '-4', we get

$$A = 4$$

To find B, we put $x-3=0 \Rightarrow x=3$ in eq.(1), we get

$$7(3)-9 = A(3-3) + B(3+1)$$

$$21-9 = A(0) + B(4)$$

$$12 = 4B$$

$$\text{or } 4B = 12$$

Dividing both sides by '4', we get

$$B = 3$$

Thus required partial fraction are $\frac{4}{x+1} + \frac{3}{x-3}$

Hence, $\frac{7x-9}{(x+1)(x-3)} = \frac{4}{x+1} + \frac{3}{x-3}$

2) $\frac{x-11}{(x-4)(x+3)}$

Solution:

$$\frac{x-11}{(x-4)(x+3)} = \frac{A}{x-4} + \frac{B}{x+3}$$

Multiplying both sides by $(x-4)(x+3)$, we get

$$x-11 = A(x+3) + B(x-4) \quad \dots\dots\dots(1)$$

To find A, we put $x-4=0 \Rightarrow x=4$ in eq.(1), we get

$$4-11 = A(4+3) + B(4-4)$$

$$-7 = A(7) + B(0)$$

$$-7 = 7A$$

$$\text{or } 7A = -7$$

Dividing both sides by '7', we get

$$A = -1$$

To find B, we put $x+3=0 \Rightarrow x=-3$ in eq.(1), we get

$$-3-11 = A(-3+3) + B(-3-4)$$

$$-14 = A(0) + B(-7)$$

$$-14 = -7B$$

$$\text{or } -7B = -14$$

Dividing both sides by '-7', we get

$$B = 2$$

Thus required partial fraction are $\frac{-1}{x-4} + \frac{2}{x+3}$

$$\text{Hence, } \frac{x-11}{(x-4)(x+3)} = -\frac{1}{x-4} + \frac{2}{x+3}$$

$$3) \quad \frac{3x-1}{x^2-1}$$

Solution:

$$\frac{3x-1}{x^2-1} = \frac{3x-1}{(x-1)(x+1)}$$

$$\text{Let } \frac{3x-1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$$

Multiplying both sides by $(x-1)(x+1)$, we get

$$3x-1 = A(x+1) + B(x-1) \quad \dots\dots\dots(1)$$

To find A, we put $x-1=0 \Rightarrow x=1$ in eq.(1), we get

$$3(1)-1 = A(1+1) + B(1-1)$$

$$3-1 = A(2) + B(0)$$

$$2 = 2A$$

$$\text{or } 2A = 2$$

Dividing both sides by '2', we get

$$A = 1$$

To find B, we put $x+1=0 \Rightarrow x=-1$ in eq.(1), we get

$$3(-1)-1 = A(-1+1) + B(-1-1)$$

$$-3-1 = A(0) + B(-2)$$

$$-4 = -2B$$

$$\text{or } -2B = -4$$

Dividing both sides by '-2', we get

$$B = 2$$

Thus required partial fraction are $\frac{1}{x-1} + \frac{2}{x+1}$

$$\text{Hence, } \frac{3x-1}{x^2-1} = \frac{1}{x-1} + \frac{2}{x+1}$$

$$4) \quad \frac{x-5}{x^2+2x-3}$$

Solution:

$$\begin{aligned} \frac{x-5}{x^2+2x-3} &= \frac{x-5}{x^2+3x-x-3} \\ &= \frac{x-5}{x(x+3)-1(x+3)} = \frac{x-5}{(x-1)(x+3)} \end{aligned}$$

$$\text{Let } \frac{x-5}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$$

Multiplying both sides by $(x-1)(x+3)$, we get

$$x-5 = A(x+3) + B(x-1) \quad \dots\dots\dots(1)$$

To find A, we put $x-1=0 \Rightarrow x=1$ in eq.(1), we get

$$1-5 = A(1+3) + B(1-1)$$

$$-4 = A(1+3) + B(0)$$

$$-4 = 4A$$

$$\text{or } 4A = -4$$

