Exercise 3.7

1. The surface area A of a cube varies directly as the square of the length l of an edge and A = 27 square units when l = 3 units.

Find

(i) A when l = 4 units

(ii) l when A = 12 sq. units.

Solution:

Given that $A \propto l^2$

$\Rightarrow$ $A = kl^2 \text{.......}(1)$

Put $A = 27$ and $l = 3$ in eq.(1), we get

$$27 = k(3)^2$$
$$27 = 9k$$
$$9\text{k} = 27$$
$$\text{k} = \frac{27}{9} = 3$$

Put k = 3 in eq.(1), we get

$$A = 3l^2 \text{........}(2)$$

(i) Put $l = 4$ in eq(2), we get

$$A = 3(4)^2$$
$$= 3(16) = 48\,\text{sq.units}$$

(ii) Put $A = 12$ in eq.(2), we get

$$12 = 3l^2$$

or $3l^2 = 12$

$\Rightarrow$ $l^2 = 4$

$$l = 2$$

2. The surface area S of the sphere varies directly as the square of radius r, and S = 16π when r = 2. Find r when S = 36π.

Solution:

Given that $S \propto r^2$

$\Rightarrow$ $S = kr^2$ ....(i)

Put $S = 16\pi$ and r = 2 in eq.(i), we get

$$16\pi = k(2)^2$$
$$16\pi = 4k$$
$$4k = 16\pi$$
$$k = 4\pi$$

Put $k = 4\pi$ in eq.(i), we get

$$S = 4\pi r^2 \quad ....(ii)$$

Put $S = 36\pi$ in eq(ii), we get

$$36\pi = 4\pi r^2$$

or $4\pi r^2 = 36\pi$

$$r^2 = \frac{36\pi}{4\pi}$$
$$r^2 = 9$$
$$r = 3$$

3. In Hook's law the force F applied to stretch a spring varies directly as the amount of elongation S and F = 32 lb when s = 1.6 in.

Find

(i) S when F = 50 lb

(ii) F when S = 0.8 in.

Solution:

$\Rightarrow \qquad F = kS \qquad ....(1)$

Put $F = 32$ and $S = 1.6$ in eq.(1), we get

$$32 = k(1.6)$$

or $\qquad 1.6k = 32$

$$k = \frac{32}{1.6} = 20$$

Put $k = 20$ in eq.(1), we get

$$F = 20S \qquad ....(2)$$

(i) Put $F = 50$ in eq(2), we get

$$50 = 20S$$

$\Rightarrow \qquad S = 2.5 in$

(ii) Put $S = 0.8$ in eq(2), we get

$$F = 20(0.8) = 16 lb$$

4. The intensity I of light from a given source varies inversely as the square of the distance d from it. If the intensity is 20 candlepower at a distance of 12ft. from the source, find the intensity at a point 8ft. from the source.

Solution:

Given that $\qquad I \propto \frac{1}{d^2}$

$\Rightarrow \qquad I = \frac{k}{d^2} \qquad ....(i)$

Put $I = 20$ and $d = 12$ in eq.(i), we get

$$20 = \frac{k}{(12)^2}$$

or $\qquad 20 = \frac{k}{144}$

$$k = 20 \times 144 = 2880$$

$$I = \frac{2880}{d^2} \qquad \text{........}(ii)$$

Put $d = 8$ in eq(ii), we get

$$I = \frac{2880}{(8)^2}$$

$$= \frac{2880}{64} = 45$$

5. The pressure P in a body of fluid varies directly as the depth d. If the pressure exerted on the bottom of a tank by a column of fluid 5ft. high is 2.25 lb/sq. in, how deep must the fluid be to exert a pressure of 9 lb/sq. in?

Solution:

Given that $\quad P \propto d$

$\Rightarrow \quad P = kd \qquad \text{.......}(i)$

Put $P = 2.25$ and d $= 5$ in eq.(i), we get

$$2.25 = k(5)$$

or $\quad 5k = 2.25$

$\Rightarrow \quad \text{k} = \frac{2.25}{5}$

$$= \frac{225}{500} = \frac{9}{20}$$

Put $\text{k} = \frac{9}{20}$ in eq.(i), we get

$$P = \frac{9}{20}d \qquad \text{........}(ii)$$

Put $\text{P} = 9$ in eq(ii), we get

$$9 = \frac{9}{20}d$$

$$d = 9 \times \frac{20}{9}$$

$$d = 20\ ft.$$

6. Labor costs c varies jointly as the number of workers n and the average number of days d. if the cost of 800 workers for 13 days is Rs. 286000, then find the labor cost of 600 workers for 18 days.

Solution:

Given that $c \propto nd$

$\Rightarrow \quad c = knd \quad ....(i)$

$Put\ n = 800$ and $\mathrm{d} = 13$ and $\mathrm{c} = 286000$ in eq.(i), we get

$$286000 = k(800)(13)$$
$$10400k = 286000$$
$$\mathrm{k} = \frac{286000}{10400} = \frac{55}{2}$$

Put $\mathrm{k} = \frac{55}{2}$ in eq.(i), we get

$$c = \frac{55}{2}nd \quad ....(ii)$$

Put $n = 600$ and $\mathrm{d} = 18$ in eq(ii), we get

$$c = \frac{55}{2} \times 600 \times 18$$
$$c = 297000$$

7. The supporting load c of a pillar varies as the fourth power of its diameter d and inversely as the square of its length l. A pillar of diameter 6 inch and of height 30 feet will support a load of 63 tons. How high a 4-inch pillar must be to support a load of 28 tons?

Solution:

Given that $c \propto \frac{d^4}{l^2}$

$\Rightarrow \quad c = k\frac{d^4}{l^2} \quad ....(i)$

$$63 = k\frac{(6)^4}{(30)^2}$$

$$63 = \frac{1296}{900}k$$

or $$k = 63 \times \frac{900}{1296}$$

$$k = \frac{175}{4}$$

Put $k = \frac{175}{4}$ in eq.(i), we get

$$c = \frac{175d^4}{4l^2} \qquad ....(ii)$$

Put $d = 4$ and c = 28 in eq(ii), we get

$$28 = \frac{175(4)^4}{4l^2}$$

$$l^2 = \frac{175 \times 256}{4 \times 28} = 400$$

$$l = 20\,ft$$

8. The time T required for an elevator to lift a weight varies jointly as the weight w and the lifting depth d varies and inversely as the power p of the motor. If 25 sec. are required for a 4-hp motor to lift 500 lb through 40 ft, what power is required to lift 800 lb, through 120 ft in 40 sec.?

Solution:

Given that $T \propto wd$

Also given that $T \propto \frac{1}{p}$

In joint variation, we can write

$$T \propto \frac{wd}{p}$$

$\Rightarrow$ $$T = k\frac{wd}{p} \qquad ....(i)$$

$$25=\frac{k\times 500\times 40}{4}$$

$$k=\frac{25\times 4}{500\times 40}$$

or $$\text{k}=\frac{1}{200}$$

Put $k=\frac{1}{200}$ in eq.(i), we get

$$T=\frac{wd}{200p} \qquad ....(ii)$$

Put $\text{w}=800, d=120$ and $\text{T}=40$ in eq(ii), we get

$$40=\frac{800\times 120}{200p}$$

$$p=\frac{800\times 120}{200\times 40}$$

$$p=12hp$$

9. The kinetic energy (K.E.) of a body varies jointly as the mass "m" of the body and the square of its velocity "v". If the kinetic energy is 4320 ft/lb when the mass is 45 lb and the velocity is 24 ft/sec. Determine the kinetic energy of a 3000 lb automobile travelling 44 ft/sec.

Solution:

Given that $K.E \propto mv^2$

$\Rightarrow$ $K.E=kmv^2 \qquad ....(i)$

Put $K.E=4320, m=45, v=24$ in eq.(i), we get

$$4320=k(45)(24)^2$$

$$k=\frac{4320}{45\times 576}$$

or $$\text{k}=\frac{1}{6}$$

Put $k=\frac{1}{6}$ in eq.(i), we get

$$K.E = \frac{1}{6}mv^2 \qquad ....(ii)$$

Put $m = 3000$ and $v = 44$ in eq(ii), we get

$$K.E = \frac{1}{6}(3000)(44)^2$$

$$P = 968000$$