K-Method:

(i) Use k - method to prove conditional equalities involving proportions.

If a : b :: c : d is a proportion, then putting each ratio equal to k

$$i.e., \frac{a}{b}=\frac{c}{d}=k$$

$$\frac{a}{b}=k \text{ and } \frac{c}{d}=k$$

$$a=bk \text{ and } c=dk$$

Using the above equations, we can solve certain problems relating to proportions more easily. This method is known as k-method.

Exercise 3.6

1. If a: b = c : d, (a, b, c, d ≠ 0), then show that

(i) $$\frac{4a-9b}{4a+9b}=\frac{4c-9d}{4c+9d}$$

Solution:

$$Given \quad a:b=c:d$$

$$Let \quad \frac{a}{b}=\frac{c}{d}=k$$

$$\Rightarrow \quad \frac{a}{b}=k \quad \text{and} \quad \frac{c}{d}=k$$

$$a=bk \text{ and } c=dk$$

$$L.H.S=\frac{4a-9b}{4a+9b} \qquad R.H.S=\frac{4c-9d}{4c+9d}$$

$$=\frac{4bk-9b}{4bk+9b} \qquad =\frac{4dk-9d}{4dk+9d}$$

$$=\frac{b(4k-9)}{b(4k+9)} \qquad =\frac{d(4k-9)}{d(4k+9)}$$

From (i) and (ii) we have
L.H.S = R.H.S

Hence $\frac{4a-9b}{4a+9b}=\frac{4c-9d}{4c+9d}$

(ii) $\frac{6a-5b}{6a+5b}=\frac{6c-5d}{6c+5d}$

Solution:

Given $a:b=c:d$

Let $\frac{a}{b}=\frac{c}{d}=k$

$\Rightarrow \quad \frac{a}{b}=k$ and $\frac{c}{d}=k$

$a=bk$ and $c=dk$

$L.H.S=\frac{6a-5b}{6a+5b} \qquad R.H.S=\frac{6c-5d}{6c+5d}$

$=\frac{6bk-5b}{6bk+5b} \qquad =\frac{6dk-5d}{6dk+5d}$

$=\frac{b(6k-5)}{b(6k+5)} \qquad =\frac{d(6k-5)}{d(6k+5)}$

$=\frac{6k-5}{6k+5} \quad .....(i) \qquad =\frac{6k-5}{6k+5} \quad ....(ii)$

From (i) and (ii) we have
L.H.S = R.H.S

Hence $\frac{6a-5b}{6a+5b}=\frac{4c-5d}{4c+5d}$

(iii) $\frac{a}{b}=\sqrt{\frac{a^2+c^2}{b^2+d^2}}$

Given $a : b = c : d$

Let $\frac{a}{b} = \frac{c}{d} = k$

$\Rightarrow \quad \frac{a}{b} = k$ and $\frac{c}{d} = k$

$a = bk$ and $c = dk$

$L.H.S = \frac{a}{b}$ $\qquad$ $R.H.S = \sqrt{\frac{a^2 + c^2}{b^2 + d^2}}$

$= \frac{bk}{b}$ $\qquad$ $= \sqrt{\frac{b^2k^2 + d^2k^2}{b^2 + d^2}}$

$= k \quad .....(i)$ $\qquad$ $= \sqrt{\frac{k^2\left(b^2 + d^2\right)}{b^2 + d^2}}$

$= \sqrt{k^2}$

$= k \quad ....(ii)$

From (i) and (ii) we have

L.H.S = R.H.S

Hence $\frac{a}{b} = \sqrt{\frac{a^2 + c^2}{b^2 + d^2}}$

(iv) $a^6 + c^6 : b^6 + d^6 = a^3c^3 : b^3d^3$

Solution:

Given $a : b = c : d$

Let $\frac{a}{b} = \frac{c}{d} = k$

$\Rightarrow \quad \frac{a}{b} = k$ and $\frac{c}{d} = k$

$a = bk$ and $c = dk$

$L.H.S = a^6 + c^6 : b^6 + d^6$ $\qquad$ $R.H.S = a^3c^3 : b^3d^3$

$= \frac{a^6 + c^6}{\ldots}$ $\qquad$ $= \frac{a^3c^3}{\ldots}$

$$= \frac{b^6k^6 + d^6k^6}{b^6 + d^6} \qquad = \frac{(b^3k^3)(d^3k^3)}{b^3d^3}$$

$$= \frac{k^6(b^6 + d^6)}{b^6 + d^6} \qquad = \frac{(b^3d^3)(k^6)}{b^3d^3}$$

$$= k^6 \quad .....(i) \qquad = k^6 \quad ....(ii)$$

From (i) and (ii) we have

L.H.S = R.H.S

Hence $a^6 + c^6 : b^6 + d^6 = a^3c^3 : b^3d^3$

(v) $p(a+b)+qb : p(c+d)+qd = a:c$

Solution:

Given $a:b = c:d$

Let $\frac{a}{b} = \frac{c}{d} = k$

$\Rightarrow \quad \frac{a}{b} = k$ and $\frac{c}{d} = k$

$a = bk$ and $c = dk$

$L.H.S = p(a+b)+qb : p(c+d)+qd \qquad R.H.S = a:c$

$$= \frac{p(a+b)+qb}{p(c+d)+qd} \qquad = \frac{a}{c}$$

$$= \frac{p(bk+b)+qb}{p(dk+d)+qd} \qquad = \frac{bk}{dk}$$

$$= \frac{pbk+pb+qb}{pdk+pd+qd} \qquad = \frac{b}{d} \quad ....(ii)$$

$$= \frac{b(pk+p+q)}{d(pk+p+q)}$$

$$= \frac{b}{d} \quad ....(i)$$

From (i) and (ii) we have

L.H.S = R.H.S

(vi) $a^2+b^2 : \frac{a^3}{a+b} = c^2+d^2 : \frac{c^3}{c+d}$

Solution:

Given $a:b=c:d$

Let $\frac{a}{b}=\frac{c}{d}=k$

$\Rightarrow \quad \frac{a}{b}=k$ and $\frac{c}{d}=k$

$a=bk$ and $c=dk$

$L.H.S = a^2+b^2 : \frac{a^3}{a+b}$

$= a^2+b^2 \div \frac{a^3}{a+b}$

$= a^2+b^2 \times \frac{a+b}{a^3}$

$= \left(b^2k^2+b^2\right) \times \frac{bk+b}{b^3k^3}$

$= \left(b^2\right)\left(k^2+1\right) \times \frac{b(k+1)}{b^3k^3}$

$= \frac{b^3}{b^3k^3}\left(k^2+1\right)(k+1)$

$= \frac{1}{k^3}\left(k^2+1\right)(k+1) \quad ....(i)$

$R.H.S = c^2+d^2 : \frac{c^3}{c+d}$

$= c^2+d^2 \div \frac{c^3}{c+d}$

$= c^2+d^2 \times \frac{c+d}{c^3}$

$= \left(d^2k^2+d^2\right) \times \frac{dk+d}{d^3k^3}$

$= \left(d^2\right)\left(k^2+1\right) \times \frac{d(k+1)}{d^3k^3}$

$= \frac{d^3}{d^3k^3}\left(k^2+1\right)(k+1)$

$= \frac{1}{k^3}\left(k^2+1\right)(k+1) \quad ....(ii)$

From (i) and (ii) we have

L.H.S = R.H.S

Hence $a^2+b^2 : \frac{a^2}{a+b} = c^2+d^2 : \frac{c^2}{c+d}$

$\frac{a}{\cdot} : \frac{a+b}{\cdot} = \frac{c}{\cdot} : \frac{c+d}{\cdot}$

Given $a:b=c:d$

Let $\frac{a}{b}=\frac{c}{d}=k$

$\Rightarrow \quad \frac{a}{b}=k$ and $\frac{c}{d}=k$

$a=bk$ and $c=dk$

$$L.H.S=\frac{a}{a-b}:\frac{a+b}{b}$$
$$=\frac{a}{a-b}\div\frac{a+b}{b}$$
$$=\frac{a}{a-b}\times\frac{b}{a+b}$$
$$=\frac{bk}{bk-b}\times\frac{b}{bk+b}$$
$$=\frac{bk}{b(k-1)}\times\frac{b}{b(k+1)}$$
$$=\frac{k}{(k-1)(k+1)}$$
$$=\frac{k}{k^2-1}\quad ....(i)$$

$$R.H.S=\frac{c}{c-d}:\frac{c+d}{d}$$
$$=\frac{c}{c-d}\div\frac{c+d}{d}$$
$$=\frac{c}{c-d}\times\frac{d}{c+d}$$
$$=\frac{dk}{dk-d}\times\frac{d}{dk+d}$$
$$=\frac{dk}{d(k-1)}\times\frac{d}{d(k+1)}$$
$$=\frac{k}{(k-1)(k+1)}$$
$$=\frac{k}{k^2-1}\quad ....(ii)$$

From (i) and (ii) we have
L.H.S = R.H.S

Hence $\frac{a}{a-b}:\frac{a+b}{b}=\frac{c}{c-d}:\frac{c+d}{d}$

2. If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}\,(a,b,c,d,e,f\neq 0)$ then show that

(i) $\frac{a}{b}=\sqrt{\frac{a^2+b^2+e^2}{b^2+d^2+f^2}}$

Solution:

$Let \quad \frac{a}{b}=\frac{c}{d}=\frac{e}{f}=k$

$\Rightarrow \quad \frac{a}{b}=k$ and $\frac{c}{d}=k$ *and* $\frac{e}{f}=k$

$a=bk \qquad c=dk \qquad e=fk$

$$L.H.S=\frac{a}{b}$$
$$=\frac{bk}{b}$$
$$=k \quad ....(i)$$
$$R.H.S=\sqrt{\frac{a^2+b^2+e^2}{b^2+d^2+f^2}}$$
$$=\sqrt{\frac{b^2k^2+d^2k^2+f^2k^2}{b^2+d^2+f^2}}$$
$$=\sqrt{\frac{k^2\left(b^2+d^2+f^2\right)}{b^2+d^2+f^2}}$$
$$=\sqrt{k^2}$$
$$=k \quad ....(ii)$$

From (i) and (ii) we have

L.H.S = R.H.S

Hence $\frac{a}{b}=\sqrt{\frac{a^2+b^2+e^2}{b^2+d^2+f^2}}$

(ii) $\frac{ac+ce+ea}{bd+df+fb}=\left[\frac{ace}{bdf}\right]^{\frac{2}{3}}$

Solution:

$Let \quad \frac{a}{b}=\frac{c}{d}=\frac{e}{f}=k$

$\Rightarrow \quad \frac{a}{b}=k$ and $\frac{c}{d}=k$ *and* $\frac{e}{f}=k$

$a=bk \qquad c=dk \qquad e=fk$

$$L.H.S = \frac{ac+ce+ea}{bd+df+fb}$$

$$= \frac{(bk)(dk)+(dk)(fk)+(fk)(bk)}{bd+df+fb}$$

$$= \frac{bdk^2+dfk^2+fbk^2}{bd+df+fb}$$

$$= \frac{k^2(bd+df+fb)}{bd+df+fb}$$

$$= k^2 \quad ....(i)$$

$$R.H.S = \left[\frac{ace}{bdf}\right]^{\frac{2}{3}}$$

$$= \left[\frac{(bk)(dk)(fk)}{bdf}\right]^{\frac{2}{3}}$$

$$= \left[\frac{bdf(k^3)}{bdf}\right]^{\frac{2}{3}}$$

$$= \left[k^3\right]^{\frac{2}{3}}$$

$$= k^2 \quad ....(ii)$$

From (i) and (ii) we have

L.H.S = R.H.S

Hence $\frac{ac+ce+ea}{bd+df+fb} = \left[\frac{ace}{bdf}\right]^{\frac{2}{3}}$

(iii) $\frac{ac}{bd}+\frac{ce}{df}+\frac{ea}{fb} = \frac{a^2}{b^2}+\frac{c^2}{d^2}+\frac{e^2}{f^2}$

Solution:

Let $\quad \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

$\Rightarrow \quad \frac{a}{b} = k$ and $\frac{c}{d} = k$ *and* $\frac{e}{f} = k$

$$L.H.S = \frac{ac}{bd} + \frac{ce}{df} + \frac{ea}{fb}$$

$$= \frac{(bk)(dk)}{bd} + \frac{(dk)(fk)}{df} + \frac{(fk)(bk)}{fb}$$

$$= \frac{bd(k^2)}{bd} + \frac{df(k^2)}{df} + \frac{fb(k^2)}{fb}$$

$$= k^2 + k^2 + k^2$$

$$= 3k^2 \quad ....(i)$$

$$R.H.S = \frac{a^2}{b^2} + \frac{c^2}{d^2} + \frac{e^2}{f^2}$$

$$= \frac{b^2k^2}{b^2} + \frac{d^2k^2}{d^2} + \frac{f^2k^2}{f^2}$$

$$= k^2 + k^2 + k^2$$

$$= 3k^2 \quad ....(ii)$$

From (i) and (ii) we have

L.H.S = R.H.S

Hence $\frac{ac}{bd} + \frac{ce}{df} + \frac{ea}{fb} = \frac{a^2}{b^2} + \frac{c^2}{d^2} + \frac{e^2}{f^2}$