(i) Joint variation

A combination of direct and inverse variations of one or more than one variable forms joint variation.

If a variable y varies directly as x and varies inversely as z.

Then $y \propto x$ and $y \propto \frac{1}{z}$

In joint variation, we write it as

$$y \propto \frac{x}{z}$$

$$i.e., \quad y = k\frac{x}{z}$$

Where $k \neq 0$ is the constant of variation.

Exercise 3.5

1. If s varies directly as u^2 and inversely as v and s = 7 when u = 3, v = 2. Find the value of s when u = 6 and v = 10.

Solution:

Given that s varies directly as u^2, so

$$s \propto u^2$$

Also given that S varies inversely as v, so

$$s \propto \frac{1}{v}$$

In joint variation, we can write

$$u^2$$

$\Rightarrow \quad s = k\dfrac{u^2}{v} \quad ....(i)$

Put s = 7, u = 3 and v = 2 in eq.(i), we get

$7 = k\dfrac{(3)^2}{2}$

$7 = \dfrac{9k}{2}$

$or\ \dfrac{9k}{2} = 7$

Multiplying both sides by $\dfrac{2}{9}$, we get

$k = 7 \times \dfrac{2}{9}$

$k = \dfrac{14}{9}$

Put k = $\dfrac{14}{9}$ in eq.(i), we get

$s = \dfrac{14u^2}{9v} \quad ....(ii)$

Put $u = 6$ and v = 10 in eq.(ii), we get

$S = \dfrac{14 \times (6)^2}{9(10)}$

$= \dfrac{14 \times 36}{9 \times 10} = \dfrac{504}{90} = \dfrac{28}{5}$

2. If w varies jointly as x, y² and 2 and w = 5 when x == 2, y = 3, z = 10. Find w When x = 4, y = 7 and z = 3.

Solution:

Given that s varies directly as x, y^2 and z.

Therefore $\quad w \propto xy^2z$

$\Rightarrow \quad w = kxy^2z \quad ....(i)$

Put w = 5, $x = 2$ and y = 3 and z = 10 in eq.(i), we get

$5 = k(2)(3)^2(10)$

$5 = k(2)(9)(10)$

$5 = 180k$

$180k = 5$

$k = \frac{5}{180} = \frac{1}{36}$

Put k = $\frac{1}{36}$ in eq.(i), we get

$w = \frac{1}{36}xy^2z \quad ....(ii)$

Put x = 4 and y = 7 and z = 3 in eq.(ii), we get

$w = \frac{1}{36}(4)(7)^2(3)$

$= \frac{1}{36}(4)(49)(3)$

$= \frac{588}{36} = \frac{49}{3}$

3. If y varies directly as x^3 and inversely as z^2 and t, and y = 16 when x = 4, z = 2, t = 3. Find the value of y when x = 2, z = 3 and t = 4.

Solution:

Given that s varies directly as x^3.

Therefore $y \propto x^3$

Also given that y varies inversely as z^2 and t.

Therefore $y \propto \frac{1}{z^2t}$

$$y \propto \frac{x^3}{z^2 t}$$

$$\Rightarrow \quad y = k\frac{x^3}{z^2 t} \quad ....(i)$$

Put y = 16, x = 4 and z = 2 and t = 3 in eq.(i), we get

$$16 = k\frac{(4)^3}{(2)^2(3)}$$

$$16 = k\frac{64}{4\times 3}$$

$$16 = k\frac{64}{12}$$

$$or \ \frac{64}{12}k = 16$$

$$k = 16\times\frac{12}{64}$$

$$k = 3$$

Put k = 3 in eq.(i), we get

$$y = \frac{3x^3}{z^2 t} \quad ....(ii)$$

Put x = 2 and z = 3 and t = 4 in eq.(ii), we get

$$y = \frac{3(2)^3}{(3)^2(4)}$$

$$= \frac{3\times 8}{9\times 4} = \frac{2}{3}$$

4. If u varies directly as x^2 and inversely as the product yz^3, and u = 2 when x = 8, y = 7, z = 2. Find the value of u when x = 6, y = 3, z = 2.

Solution:

Given that u varies directly as x^2.

Therefore $\quad u \propto x^2$

Therefore $u \propto \frac{1}{yz^3}$

In joint variation, we can write

$$u \propto \frac{x^2}{yz^3}$$

$$\Rightarrow \quad u = k\frac{x^2}{yz^3} \quad ....(i)$$

Put u = 2, $x = 8$ and y = 7 and z = 2, we get

$$2 = k\frac{(8)^2}{(7)(2)^3}$$

$$2 = \frac{64}{56}k$$

$$\frac{64}{56}k = 2$$

$$k = 2 \times \frac{56}{64} = \frac{7}{4}$$

Put $k = \frac{7}{4}$ in eq.(i), we get

$$u = \frac{7x^2}{4yz^3} \quad ....(ii)$$

Put x = 6 and y = 3 and z = 2 in eq.(ii), we get

$$u = \frac{7(6)^2}{4(3)(2)^3}$$

$$= \frac{7 \times 36}{4 \times 3 \times 8} = \frac{252}{96} = \frac{21}{8}$$

5. If v varies directly as the product xy^3 and inversely as z^2 and v = 27 when x = 7, y = 6, z = 7. Find the value of v when x = 6, y = 2, z = 3.

Solution:

Given that v varies directly as xy^3.

Therefore $v \propto xy^3$

Therefore $v \propto \frac{1}{z^2}$

In joint variation, we can write

$$v \propto \frac{xy^3}{z^2}$$

$$\Rightarrow \quad v = k\frac{xy^3}{z^2} \quad ....(i)$$

Put v = 27, $x = 7$ and y = 6 and z = 7, we get

$$27 = \frac{k(7)(6)^3}{(7)^2}$$

$$27 = \frac{k(7)(216)}{49}$$

$$27 = \frac{1512k}{49}$$

$$or\ k = 27 \times \frac{49}{1512}$$

$$k = \frac{1323}{1512} = \frac{7}{8}$$

Put k $= \frac{7}{8}$ in eq.(i), we get

$$v = \frac{7xy^3}{8z^2} \quad ....(ii)$$

Put x = 6 and y = 2 and z = 3 in eq.(ii), we get

$$v = \frac{7(6)(2)^3}{8(3)^2}$$

$$= \frac{336}{72} = \frac{14}{3}$$

6. If w varies inversely as the cube of u, and w = 5 when u = 3. Find w, when u = 6.

Given that w varies inversely as u^3.

Therefore $w \propto \frac{1}{u^3}$

$\Rightarrow \quad w = \frac{k}{u^3}$ ....(i)

Put $w = 5, u = 3$ in eq.(i), we get

$5 = \frac{k}{(3)^3}$

$k = 27 \times 5 = 135$

Put $k = 135$ in eq.(i), we get

$w = \frac{135}{u^3}$ ....(ii)

Put $u = 6$ in eq.(ii), we get

$w = \frac{135}{(6)^3}$

$= \frac{135}{216} = \frac{5}{8}$