EXERCISE 4.3

Solve the following equations:

1. $3x^2+2x-\sqrt{3x^2+2x-1}=7$

2. $x^2-\dfrac{x}{2}-7=x-3\sqrt{2x^2-3x+2}$

3. $\sqrt{2x+8}+\sqrt{x+5}=7$

4. $\sqrt{3x+4}=2+\sqrt{x-4}$

5. $\sqrt{x+7}+\sqrt{x+2}=\sqrt{6x+13}$

6. $\sqrt{x^2+x+1}-\sqrt{x^2+x-1}=1$

7. $\sqrt{x^2+2x-3x}+\sqrt{x^2+7x-8}=\sqrt{5(x^2+3x-4)}$

8. $\sqrt{2x^2-5x-3}+\sqrt{2x+1}=\sqrt{2x^2+25x+12}$

9. $\sqrt{3x^2-5x+2}+3\sqrt{6x^2-11x+5}\sqrt{5x^2-9x+4}$

10. $(x+4)(x+1)=\sqrt{x^2+2x-15}+3x+31$

11. $\sqrt{3x^2-2x+9}+\sqrt{3x^2-2x-4}=13$

12. $\sqrt{5x^2+7x+2}-\sqrt{4x^2+7x+18}=x-4$

1. $3x^2+2x-\sqrt{3x^2 2x-1}=7$

Solution:

Let $3x^2+2x-1=t^2$

$3x^2+2x=t^2+1$

Put the value

$$t^2+1-\sqrt{t^2}=7$$
$$t^2+1-t=7$$
$$t^2+1-t-7=0$$
$$t^2-t-6=0$$
$$t^2-3t+2t-6=0$$
$$t(t-3)+2(t-3)=0$$
$$(t-3)(t+2)=0$$

Either t+2=0 or t-3=0

Either t=-2 & t=3

So,

$$3x^2+2x-1=(-2)^2$$
$$3x^2+2x-1=4$$
$$3x^2+2x-1-4=0$$
$$3x^2+2x-5=0$$
$$3x^2+5x-3x-5=0$$
$$x(3x+5)-1(3x+5)=0$$
$$(x-1)(3x+5)=0$$

Either x-1=0 or 3x+5=0

X=1 & x= $\frac{-5}{3}$

And

$$3x^2+2x-1=(3)^2$$
$$3x^2+2x-1=9$$
$$3x^2+2x-1=9-0$$
$$3x^2+2x-10=0$$
$$x=\frac{-2\pm\sqrt{4-4(3)(-10)}}{2(3)}$$
$$x=\frac{-2\pm\sqrt{4+120}}{6}$$
$$x=\frac{-2\pm2\sqrt{31}}{6}$$

$$x=\frac{-1\pm\sqrt{31}}{3}$$

Checked the roots of x

Let x=1

$$3(1)^2+(1)-\sqrt{3(1)^2+2(1)-1}=7$$

$$3+2-\sqrt{3+1-1}=7$$

$$3+2-\sqrt{4}=7$$

5+2=7

7=7

Hence ; x =1 is the root of the equation

And let x= $\frac{-5}{3}$

$$3\left(\frac{-5}{3}\right)+2\left(\frac{-5}{3}\right)-\sqrt{3\left(\frac{-5}{3}\right)^2+2\left(\frac{-5}{3}\right)-1}=7$$

$$3\left(\frac{25}{9}\right)+\frac{-10}{3}-\sqrt{3\left(\frac{25}{9}\right)-\frac{10}{3}-1}=7$$

$$\frac{25}{3}-\frac{10}{3}-\sqrt{\frac{25}{3}-\frac{10}{3}-1}=7$$

$$\frac{15}{3}-\sqrt{\frac{15}{3}-1}=7$$

$$5+\sqrt{5-1}=7$$

$$5+2=7$$

$$7=7$$

Hence x = $\frac{-5}{3}$; is the root of the equation

$$x=\frac{-1\pm\sqrt{31}}{3}$$

Let

$$3\left(\frac{-1\pm\sqrt{31}}{3}\right)^2+2\left(\frac{-1\pm\sqrt{31}}{3}\right)-\sqrt{\left(\frac{-1\pm\sqrt{31}}{3}\right)^2+2\left(\frac{-1\pm\sqrt{31}}{3}\right)-1}=7$$

$$\left(\frac{-1\pm\sqrt{31}}{3}\right)^2+\frac{2}{3}(-1+\sqrt{31})-\sqrt{\left(\frac{-1\pm\sqrt{31}}{3}\right)^2+\frac{2}{3}(-1\pm\sqrt{31})-1}=7$$

$$\frac{1+31-2\sqrt{31}}{3}+\frac{-2}{3}\pm\frac{2\sqrt{31}}{3}-\sqrt{\frac{32-2\sqrt{31}}{3}\pm\frac{-2}{3}\pm\frac{2}{3}\sqrt{31}-1}=7$$

$$\frac{30-2\sqrt{31}\pm2\sqrt{31}}{3}-\sqrt{\frac{27-2\sqrt{31}\pm2\sqrt{31}}{3}}=7$$

Hence $S.S=\left\{1,\frac{-5}{3}\right\}$

2 $x^2-\frac{x}{2}-7=x-3\sqrt{2x^2-3x+2}$

Solution

$$\left\{\frac{-1}{2},2\right\}$$

Put the value in equation

$$t^2-2-14+6\sqrt{t^2}=0$$
$$t^2-16+6t=0$$
$$t^2+6t-16=0$$
$$t^2+8t-2t-16=0$$
$$t(t+8)-2(t+8)=0$$

Either t-2=0 or t+8=0

T=2 & t=-8

Put the value of 't'

$$2x^2-3x+2=2^2$$
$$2x^2-3x+2=4$$
$$2x^2-3x+2-4=0$$
$$2x^2-3x-2=0$$
$$2x^2-4x-2=0$$
$$2x(x-2)+1(x-2)=0$$
$$(2x+1)(x-2)=0$$

Either 2x+1=0 or x-2=0

$x = \frac{-1}{2}$ or x=2

And

$$2x^2 - 3x + 2 = (-8)^2$$
$$2x^2 - 3x + 2 - 64 = 0$$
$$2x^2 - 3x - 62 = 0$$
$$x = \frac{3 \pm \sqrt{9 - 4(2)(-62)}}{2(2)}$$
$$x = \frac{3 \pm \sqrt{9 + 496}}{4}$$
$$x = \frac{3 \pm \sqrt{505}}{4}$$

Check the possible roots of x

$$\frac{-1}{2}, 2, \frac{3 \pm \sqrt{505}}{4}$$

Let $x = \frac{-1}{2}$

$$\left(\frac{-1}{2}\right)^2 \frac{\left(\frac{-1}{2}\right)}{2} - 7 = \left(\frac{-1}{2}\right) - 3\sqrt{2\left(\frac{-1}{2}\right) - 3\left(\frac{-1}{2}\right) + 2}$$
$$\frac{1}{4} + \frac{1}{4} - 7 = -\frac{1}{2} - 3\sqrt{\frac{4}{2} + 2}$$
$$\frac{1-14}{2} = \frac{-1}{2} - 3\sqrt{\frac{4}{2} + 2}$$
$$\frac{-13}{2} = \frac{-1}{2} - 3(2)$$
$$\frac{-13}{2} = \frac{-1-12}{2}$$

It is true for $x = \frac{1}{2}$

And x=2

$$2^2-\frac{2}{2}-7=2-3\sqrt{2(2)^2-3(2)+2}$$
$$4-1-7=2-3\sqrt{8-6+2}$$
$$4-8=2-3\sqrt{10-6}$$
$$-4=2-3(2)$$
$$-4=-4$$
$$x=\frac{3\pm\sqrt{505}}{4}$$
$$\left(\frac{3\pm\sqrt{505}}{4}\right)^2-\left(\frac{3\pm\sqrt{505}}{4}\right)-7=\frac{3\pm\sqrt{505}}{4}$$
$$-3\sqrt{\left(\frac{23\pm\sqrt{505}}{4}\right)-3\left(\frac{3\pm\sqrt{505}}{4}\right)+2}$$

It is not equal , so

$r=\frac{3\pm\sqrt{505}}{4}$ is not a root

Hence S.S = $\left\{\frac{-1}{2},2\right\}$

3. $\sqrt{2x+8}+\sqrt{x+5}=7$

Solution:

$$\sqrt{2x+8}+\sqrt{x+5}=7$$
$$\left(\sqrt{2x+8}\right)^2=\left(7-\sqrt{x+5}\right)^2$$
$$2x+8=49+x+5-14\sqrt{x+5}$$
$$x-46=-14\sqrt{x+5}$$
$$\left(x-46\right)^2=\left(-14\sqrt{x+5}\right)^2$$
$$x^2+2116-92x=96\left(x+5\right)$$
$$x^2+2116-92x=196x+980$$
$$x^2-288+1136=0$$

$$x^2 - 4x - 284x + 1136 = 0$$
$$x(x-4) - 284(x-4) = 0$$
$$(x-284)(x-4) = 0$$

Either

$x - 284 = 0$ or $x - 4 = 0$

$x = 284$ $x = 4$

Check the possible roots of equations 284 &4

Let x=4

$$\sqrt{2(4)+8} + \sqrt{4+5} = 7$$
$$\sqrt{8+8} + \sqrt{9} = 7$$
$$\sqrt{16} + 3 = 7$$
$$\sqrt{4} + 3 = 7$$
$$7 = 7$$

Its true for x=4

And let x=284

$$\sqrt{2(284)} + 8 + \sqrt{284+5} = 7$$
$$\sqrt{\sqrt{568+8} + \sqrt{284+5}} = 7$$
$$\sqrt{586+8} + \sqrt{289} = 7$$
$$\sqrt{574} + 17 = 7$$
$$24 + 17 = 7$$
$$31 \neq 7$$

So, it is not true for x=284

Hence $S.S. = \{4\}$

4. $\sqrt{3x+4} = 2 + \sqrt{2x-4}$

Solution:

$$(\sqrt{3x+4})^2 = (2+\sqrt{2x-4})^2$$
$$3x + 4 = 4 + 2x - 4 + 4\sqrt{2x-4}$$
$$3x + 4 - 4 - 2x + 4 = 4\sqrt{2x-4}$$

$x+4=4\sqrt{2x-4}$

$(x+4)^2=\left(4\sqrt{2x-4}\right)^2$

$x^2+16+8x=16(2x-4)$

$x^2+16+8x=32x-64$

$x^2+8x-32x+16+64=0$

$x^2-24x+80=0$

$x^2-4x-20x+80=0$

$x(x-4)-20(x-4)=0$

$(x-20)(x-4)=0$

either $x-20=0$ *or* $x-4=0$

$x=20$ & $x=4$

Check possible roots of the equation 20 and 4

Let x=20

$\sqrt{3(20)+4}=2+\sqrt{2(20)-4}$

$\sqrt{60+4}=2+\sqrt{40-4}$

$\sqrt{64}=2+\sqrt{36}$

$8=2+6$

$8=8$

It is true for x=20

And let x=4

$\sqrt{3(4)+4}=2+\sqrt{2(4)-4}$

$\sqrt{12+4}=2\sqrt{8-4}$

$\sqrt{16}=2+\sqrt{4}$

$4=2+2$

$4=4$

It is true for x=4

Hence $S.S=\{4,20\}$

5. $\sqrt{x+7}+\sqrt{x+2}=\sqrt{6x+13}$

Solution:

$$\left(\sqrt{x+7}+\sqrt{x+2}\right)^2=\left(\sqrt{6x+13}\right)^2$$
$$x+7+x+2+2\sqrt{(x+7)(x+2)}=6x+13$$
$$2x+9-6x-13=-2\sqrt{(x+7)(x+2)}$$
$$-2(2x+2)=-2\sqrt{(x+7)(x+2)}$$
$$2x+2=\sqrt{(x+7)(x+2)}$$
$$(2x+2)^2=\left(\sqrt{(x+7)(x+2)}\right)^2$$
$$4x^2+4+8x=(x+7)(x+2)$$
$$4x^2-x^2+8x-9x+4-14=0$$
$$3x-x-10=0$$
$$3x(x-2)+5(x-2)=0$$
$$(3x+5)(x-2)=0$$

Either

$3x+5=0$ or $x-2=0$

$x=\frac{-5}{3}$ $x=2$

Check the possible roots of the equation -5/3 and 2

Let x=2

$$\sqrt{2+7}+\sqrt{2+2}=\sqrt{6(2)+13}$$
$$\sqrt{9}+\sqrt{4}=\sqrt{12+13}$$
$$3+2=\sqrt{25}$$
$$5=5$$

It is true for x=2

$$Let\ x = -5/3$$

$$\sqrt{-5/3 + 7} + \sqrt{-5/3 + 2} = \sqrt{6\left(-5/3\right) + 13}$$

$$\sqrt{\frac{-5+21}{3}} + \sqrt{\frac{-5+6}{3}} = \sqrt{-10+13}$$

$$\sqrt{\frac{16}{3}} + \sqrt{\frac{1}{3}} = \sqrt{3}$$

$$\frac{4}{\sqrt{3}} + \frac{1}{\sqrt{3}} = \sqrt{3}$$

$$\frac{4+1}{\sqrt{3}} = \sqrt{3}$$

$$\frac{5}{\sqrt{3}} \neq \sqrt{3}$$

It is not equal to x=-5/3

Hence $S.S = \{2\}$

6. $\sqrt{x^2 + x + 1} - \sqrt{x^2 + x + 1} = 1$

Solution:

$$let$$

$$x^2 + x = t^2$$

$$\sqrt{t+1} - \sqrt{t-1} = 1$$

$$\sqrt{t+1} = 1 + \sqrt{t-1}$$

$$\left(\sqrt{t+1}\right)^2 = \left(1 + \sqrt{t-1}\right)^2$$

$$t + 1 = 1 + t - 1 + 2\sqrt{t-1}$$

$$t + 1 = t + 2\sqrt{t-1}$$

$$1 = 2\sqrt{t-1}$$

$$(1)^2 = \left(2\sqrt{t-1}\right)^2$$

$$1 = 4t - 4$$
$$4t = 1 + 4$$
$$4t = 5$$
$$t = \frac{5}{4}$$
$$x^2 + x = \frac{5}{4}$$
$$4x^2 + 4x = 5$$
$$4x^2 + 4x - 5 = 0$$
$$x = \frac{-4 \pm \sqrt{16 - 4(4)(-5)}}{2(4)}$$
$$= \frac{-4 \pm \sqrt{16 + 80}}{8}$$
$$= \frac{-4 \pm \sqrt{96}}{8}$$
$$= \frac{-4 \pm 4\sqrt{6}}{8}$$
$$x = \frac{-1 \pm \sqrt{6}}{2}$$

Check the possible roots

$$\frac{-1 \pm \sqrt{6}}{2}$$

Let $x = \frac{-1 \pm \sqrt{6}}{2}$

$$\sqrt{\left(\frac{-1 \pm \sqrt{6}}{2}\right)^2 + \left(\frac{-1 \pm \sqrt{6}}{2}\right)^2 + 1} - \sqrt{\left(\frac{-1 \pm \sqrt{6}}{2}\right)^2 + \left(\frac{-1 \pm \sqrt{6}}{2}\right)^2 - 1} = 1$$

$$\sqrt{\left(\frac{1 + \sqrt{6} \pm 2\sqrt{6}}{2}\right)^2 + \frac{-1 \pm \sqrt{6}}{2} + 1} - \sqrt{\left(\frac{1 + 6 \pm 2\sqrt{6}}{4}\right) + \frac{-1 \pm \sqrt{6}}{2} - 1} = 1$$

$$\sqrt{\frac{7 - 2 \pm 2\sqrt{6} \pm \sqrt{6} + 4}{4}} - \sqrt{\frac{7 - 2 \pm 2\sqrt{6} \pm \sqrt{6} - 4}{4}} = 1$$

$$\sqrt{\frac{9 + 3\sqrt{6}}{4}} - \sqrt{\frac{1 \pm 3\sqrt{6}}{4}} = 1$$

$$\sqrt{\frac{9\pm 3\sqrt{6}-1\pm 3\sqrt{6}+8}{4}}=1$$

$$1=1$$

It is true for $x=\dfrac{-1\pm\sqrt{6}}{2}$

Hence $S.S.=\left\{\dfrac{-1\pm\sqrt{6}}{2}\right\}$

7. $\sqrt{x^2+2x-3x}+\sqrt{x^2+7x-8}=\sqrt{5\left(x^2+3x-4\right)}$

Solution:

$$\sqrt{x^2+3x-x-3x}+\sqrt{x^2+8x-x-8}=\sqrt{5\left(x^2+4x-x-4\right)}$$

$$\sqrt{x(x+3)-1(x+3)}+\sqrt{x(x+8)-1(x+8)}=\sqrt{5\left(x(x+4)\right)-1(x+4)}$$

$$\sqrt{(x-1)(x+3)}+\sqrt{(x-1)(x+8)}=\sqrt{5(x-1)(x+4)}$$

$$\sqrt{x-1}\left[\sqrt{x+3}+\sqrt{x+8}\right]=\sqrt{x-1}\left[\sqrt{5(x+4)}\right]=0$$

$$\sqrt{x-1}\left[\sqrt{x+3}+\sqrt{x+8}\right]-\left[\sqrt{5(x+4)}\right]=0$$

$$\sqrt{x-1}=0$$

or

$$\sqrt{x+3}+\sqrt{x+8}-\sqrt{5(x+4)}=0$$

$$x-1=0$$

$$x=1$$

and

$$\sqrt{x+3}+\sqrt{x+8}-\sqrt{5(x+4)}=0$$

$$\sqrt{x+3}+\sqrt{x+8}=\sqrt{5(x+4)}$$

$$\left(\sqrt{x+3}+\sqrt{x+8}\right)^2=\left(\sqrt{5(x+4)}\right)^2$$

$$x+3+x+8+2\left(\sqrt{x+3}.\sqrt{x+8}\right)=5(x+4)$$

$$2x+11+2\sqrt{(x+3)(x+8)}=5x+20$$
$$5x-2x+20-11=\left[2\sqrt{(x+3)(x+8)}\right]$$
$$(3x+9)^2=\left[2\sqrt{(x+3)(x+8)}\right]^2$$
$$9x^2+81+54x=4(x+3)(x+8)$$
$$9x^2+81+54x=4\left(x^2+11x+24\right)$$
$$9x^2+81+54x=4x^2+44+96$$
$$9x^2+81+54x-4x^2-44-96=0$$
$$5x^2+10x-15=0$$
$$5\left(x^2+2x-3\right)=0$$
$$\left(x^2+2x-3\right)=0$$
$$\left(x^2+3x-x-3\right)=0$$
$$x(x+3)-1(x+3)=0$$
$$(x+3)(x-1)=0$$

Either

$x+3=0$ or $x-1=0$

$x=-3$ $x=1$

Check the possible roots of the equation is 1,-3

Let x=1

$$\sqrt{(1)^2+2(1)-3}+\sqrt{(1)^2+7(1)-8}=5(11)^2+3(1)-4$$
$$\sqrt{1+2-3}+\sqrt{1+7-8}=5(1+3-4)$$
$$\sqrt{6}+\sqrt{6}=\sqrt{5(0)}$$
$$\sqrt{-20}=\sqrt{-20}$$

It is true for x=1

Let x=-3

$$\sqrt{(3)^2+2(-3)-3}+\sqrt{(3)^2+7(-3)-8}=\sqrt{5(3)^2+3(-3)-4}$$
$$\sqrt{9-6}-3+\sqrt{9-21-8}=\sqrt{5(9-9-4)}$$
$$\sqrt{0}+\sqrt{0-20}=\sqrt{5(-4)}$$
$$\sqrt{-20}=\sqrt{-20}$$

It is true for x=-3

Hence $S.S=\{1,-3\}$

8. $\sqrt{2x^2-5x-3}+3\sqrt{2x+1}=\sqrt{2x^2+25x12}$

Solution:

$$\sqrt{2x^2-6x+x-3}+3\sqrt{2x+1}=\sqrt{2x^2+25x12}$$
$$\sqrt{(2x+1)(x-3)}+1\sqrt{(2x+1)}=\sqrt{2x(x+12)+1\sqrt{(x+12)}}$$
$$\sqrt{(2x+1)(x-3)}+3\sqrt{2x+1}=\sqrt{(2x+1)(x+12)}$$
$$\sqrt{(2x+1)(x-3)}+3\sqrt{2x+1}-\sqrt{(2x+1)(x+12)}=0$$
$$\sqrt{(2x+1)}\left[\sqrt{(x-3)}+3-\sqrt{x+12}\right]=0$$

Either $\sqrt{2x+1}=0$

Or $\sqrt{x-3}+3-\sqrt{x+12}=0$

$$\sqrt{2x+1}=0$$
$$2x+1=0$$
$$x={}^{-1}\!/_{2}$$

And

$$\sqrt{x-3}+3-\sqrt{x+12}=0$$
$$\sqrt{x-3}+3=\sqrt{x+12}$$
$$\left(\sqrt{x-3}+3\right)^2=\left(\sqrt{x+12}\right)^2$$
$$x-3+9+6\sqrt{x-3}=x+12$$
$$x-6-x-12+6\sqrt{x-3}=0$$

$-6+6\sqrt{x-3}=0$

$-6\left(1-\sqrt{x-3}\right)=0$

$-\sqrt{x-3}+1=0$

$-\sqrt{x-3}=-1$

$\left(\sqrt{x-3}\right)^2=(-1)^2$

$x-3=1$

$x=1+3$

$x=4$

Check it is possible for the root of equation 4 & -1/2

Let x=4

$\sqrt{2(4)^2-5(4)-3}+3\sqrt{2(4)+1}=\sqrt{2(4)^2+25(4)+12}$

$\sqrt{32-20-3}+3\sqrt{9}=\sqrt{32+100+12}$

$3+9=\sqrt{144}$

$12=12$

It is true for x=4

And x=-1/2

$\sqrt{2\left(-\frac{1}{2}\right)^2-5\left(-\frac{1}{2}\right)-3}+3\sqrt{2\left(-\frac{1}{2}\right)+1}=\sqrt{2\left(-\frac{1}{2}\right)^2+25\left(-\frac{1}{2}\right)+12}$

$\sqrt{\frac{1}{2}+\frac{5}{2}-3}+3\sqrt{0}=\sqrt{\frac{1}{2}-\frac{25}{2}+12}$

$\sqrt{0}+0=\sqrt{\frac{1-25+24}{12}}$

$0=0$

It is true for x=-1/2

Hence $S.S=\left\{4,-\frac{1}{2}\right\}$

9. $\sqrt{3x^2-5x+2}+3\sqrt{6x^2-11x+5}=\sqrt{5x^2-9x+4}$

Solution:

$$\sqrt{3x^2-3x-2x+2}+3\sqrt{6x^2-6x-5x+5}=\sqrt{5x^2-5x-4x+4}$$
$$\sqrt{3x(x-1)-2(x-1)}+\sqrt{6x(x-1)-5(x-1)}=\sqrt{(5x-4)(x-1)}$$
$$\sqrt{(3x-2)(x-1)}+\sqrt{(x-1)(6x-5)}=\sqrt{(5x-4)(x-1)}$$
$$\sqrt{(3x-2)(x-1)}+\sqrt{(x-1)(6x-5)}-\sqrt{(5x-4)(x-1)}=0$$
$$\sqrt{(x--1)}\left[\sqrt{3x-2}+\sqrt{6x-5}-\sqrt{5x-4}\right]=0$$

Either $\sqrt{x-1}=0$

Or $\sqrt{3x-2}+\sqrt{6x-5}-\sqrt{5x-4}=0$

$$\sqrt{x-1}=0$$
$$x-1=0$$
$$x=1$$

And

$$\sqrt{3x-2}+\sqrt{6x-5}-\sqrt{5x-4}=0$$
$$\sqrt{3x-2}+\sqrt{6x-5}=\sqrt{5x-4}$$
$$\left(\sqrt{3x-2}+\sqrt{6x-5}\right)^2=\left(\sqrt{5x-4}\right)^2$$
$$3x-2+6x-5+2\sqrt{(3x-2)(6x-5)}=5x-4$$
$$9x-5x-7+4+2\sqrt{(3x-2)(6x-5)}=0$$
$$4x-3=-2\sqrt{(3x-2)(6x-5)}$$
$$(4x-3)^2=\left[-2\sqrt{(3x-2)(6x-5)}\right]^2$$
$$16x^2+9-24x=4(4x-3)(6x-5)$$
$$16x^2+9-24x=4\left[18x^2-15x-12x+10\right]$$
$$16x^2+9-24x=72x^2-60x-48x+40$$
$$16x^2+9-24x=72x^2-108x+40$$
$$72x^2-16x-108x+24+40-9=0$$
$$56x^2-84x+31=0$$
$$\sqrt{3x-2}+\sqrt{6x-5}-\sqrt{5x-4}=0$$
$$\sqrt{3x-2}+\sqrt{6x-5}=\sqrt{5x-4}$$
$$\left(\sqrt{3x-2}+\sqrt{6x-5}\right)^2=\left(\sqrt{5x-4}\right)^2$$

$$3x-2+6x-5+2\sqrt{(3x-2)(6x-5)}=5x-4$$
$$9x-5x-7+4+2\sqrt{(3x-2)(6x-5)}=0$$
$$4x-3=-2\sqrt{(3x-2)(6x-5)}$$
$$(4x-3)^2=\left[-2\sqrt{(3x-2)(6x-5)}\right]^2$$
$$16x^2+9-24x=4(4x-3)(6x-5)$$
$$16x^2+9-24x=4\left[18x^2-15x-12x+10\right]$$
$$16x^2+9-24x=72x^2-60x-48x+40$$
$$16x^2+9-24x=72x^2-108x+40$$
$$72x^2-16x-108x+24+40-9=0$$
$$56x^2-84x+31=0$$
$$x=\frac{84\pm\sqrt{(84)^2-4(56)(31)}}{2(56)}$$
$$=\frac{84\pm\sqrt{7056-6944}}{112}$$
$$=\frac{84\pm\sqrt{112}}{112}$$
$$x=\frac{84\pm4\sqrt{7}}{112}$$
$$=\frac{21\pm\sqrt{71}}{28}$$

Check the possible roots of the equation 1, $=\frac{21\pm\sqrt{71}}{28}$

Let x=1

$$\sqrt{3x^2-5x+2}-\sqrt{6x^2-11x+5}=\sqrt{5x^2-9x+4}$$
$$\sqrt{3-5+2}-\sqrt{6-11+5}=\sqrt{5-9+4}$$
$$\sqrt{0}-\sqrt{0}=\sqrt{0}$$
$$0=0$$

It is true for x=1

And $x=\frac{21\pm\sqrt{7}}{28}$

$$\sqrt{\frac{3\left(\frac{21\pm\sqrt{7}}{28}\right)^2+5\left(\frac{21\pm\sqrt{7}}{28}\right)+2}{28}}+\sqrt{6\left(\frac{21\pm\sqrt{7}}{28}\right)^2-11\left(\frac{21\pm\sqrt{7}}{28}\right)+5}$$

i.e it is not real no so it is not the root of the equation

Hence $S.S=\{1\}$

10. $(x+4)(x+1)=\sqrt{x^2+2x-15}+3x+31$

Solution:

$$(x+4)(x+1)-\sqrt{x^2+2x-15}-(3x+31)=0$$
$$(x+4)(x+1)-(3x+31)-\sqrt{x^2+2x-15}=0$$
$$\left(x^2+5x+4\right)-(3x+31)-\sqrt{x^2+2x-15}=0$$
$$\left(x^2+5x-3x+4-31\right)-\sqrt{x^2+2x-15}=0$$
$$\left(x^2+2x-27\right)-\sqrt{x^2+2x-15}=0$$

Let

$\sqrt{x^2+2x-15}=t^2$

and ______ $x^2+2x=t^2+15$

therefore

$$t^2+15-27-t=0$$
$$t^2-t-12=0$$
$$t^2-4t+3t-12=0$$
$$t(t-4)+3(t-4)=0$$
$$(t-4)(t+3)=0$$

Either $t+3=0$, $t=-3$ **or** $t-4=0$, $t=4$

$$x^2+2x-15=(-3)^2$$
$$x^2+2x-15=9$$
$$x^2+2x-15-9=0$$
$$x^2+2x-24=0$$
$$x^2+6x-4x-24=0$$
$$x(x-4)+6(x-4)=0$$
$$(x-4)(x+6)=0$$

Either $x-4=0$, $x=4$ **or** $x+6=0$, $x=-6$

$$x^2+2x-15=(4)^2$$
$$x^2+2x-15-16=0$$
$$x^2+2x-31=0$$

$$x=\frac{-2\pm\sqrt{4-4(1)(-31)}}{2(1)}$$
$$=\frac{-2\pm\sqrt{4+124}}{2}$$
$$=\frac{-2\pm\sqrt{128}}{2}$$
$$=\frac{-2\pm8\sqrt{2}}{2}=-1\pm4\sqrt{2}$$

Check the possible solution roots for the equation, $4,-6,-1\pm4\sqrt{2}$

Let x=4

$$(4+4)(4+1)=\sqrt{(4)^2+2(4)-15}+3(4)+31$$
$$40=\sqrt{9}+43$$
$$40=3+43$$
$$40\neq46$$

It is not the root of the equation and let

$x = -3$
$(-3+4)(-3+1) = \sqrt{(-3)^2 + 2(-3) - 15 + 3(-3) + 31}$
$(1)(-2) = \sqrt{9 - 6 - 15 - 9 + 31}$
$-2 = \sqrt{12} + 22$
$-2 \neq \sqrt{12} + 22$

It is not the root of equation

Hence $S.S = \{1, \pm 4\sqrt{2}\}$

11. $\sqrt{3x^2 - 2x + 9} + \sqrt{3x^2 - 2x - 4} = 13$

Solution:

Let $\sqrt{3x^2 - 2x + 9} + \sqrt{3x^2 - 2x - 4} = a$

And

$\sqrt{3x^2 - 2x - 4} = b$
$a + b = 13$
and
$a^2 - b^2 = \left(\sqrt{3x^2 - 2x + 9}\right)^2 + \left(\sqrt{3x^2 - 2x - 4}\right)^2$
$= 3x^2 - 2x + 9 - 3x^2 + 2x + 4$
$a^2 - b^2 = 13$
$(a+b)(a-b) = 13$
$(13)(a-b) = 13$
$a - b = \frac{13}{13} = 1$
$a + b = 13$
Therefore $\frac{a - b = 1}{2a = 14}$
$a = 7$
and $b + 7 = 13$

$b = 13 - 7 = 6$

so

$\sqrt{3x^2 - 2x + 9} = 7$

and

$\sqrt{3x^2 - 2x + 9} = 6$

$\left(\sqrt{3x^2 - 2x + 9}\right)^2 = (7)^2$

$3x^2 - 2x + 9 = 49$

$3x^2 - 2x + 9 - 49 = 0$

$3x^2 - 2x - 40 = 0$

$3x(x-4) + 10(x-4) = 0$

$(3x+10)(x-4) = 0$

Either

$3x + 10 = 0$

$x = -\frac{10}{3}$ **or** $x - 4 = 0$

$x = 4$

And $\sqrt{3x^2 - 2x - 4} = 6$

$\left(\sqrt{3x^2 - 2x - 4}\right)^2 = (6)^2$

$3x^2 - 2x - 4 = 36$

$3x^2 - 2x - 4 - 36 = 0$

$3x^2 - 2x - 40 = 0$

$3x(x-4) + 10(x-4) = 0$

$(x-4)(3x+10) = 0$

$x - 4 = 0$

$x = 4$ **or** $3x + 10 = 0$

$x = -\frac{10}{3}$

Check the possible root equations of $4, 6, -\frac{10}{3}$

Let x=4

$$\sqrt{3(4)^2-2(4)-4}+\sqrt{3(4)^2-2(-6)-4}=13$$
$$\sqrt{108+12+9}+\sqrt{108+12-4}=13$$
$$\sqrt{129}+\sqrt{125}=13$$

It is not true for x=4

And let x=-6

$$\sqrt{3(-6)^2-2(-6)+9}+\sqrt{3(-6)^2-2(-6)-4}=13$$
$$\sqrt{108+12+9}+\sqrt{108+12-4}=13$$
$$\sqrt{49}+\sqrt{36}=13$$
$$7+6=13 \Rightarrow 13=13$$

It is true for $x=-6$

And let $x=\frac{-10}{3}$

$$\sqrt{3\left(\frac{-10}{3}\right)^2-2\left(\frac{-10}{3}\right)+9}+\sqrt{3\left(\frac{-10}{3}\right)^2-2\left(\frac{-10}{3}\right)-4}=13$$

$$\sqrt{\frac{100}{3}+\frac{20}{3}+9}+\sqrt{\frac{100}{3}-\frac{10}{3}-4}=13$$
$$\sqrt{49}+\sqrt{36}=13$$
$$7+6=13 \Rightarrow 13=13$$

It is true for $x=\frac{-10}{3}$

Hence $S.S=\left\{4,\frac{-10}{3}\right\}$

12. $\sqrt{5x^2+7x+2}-\sqrt{4x^2+7x+18}=x-4$

Solution:

Let $\sqrt{5x^2+7x+2}=a$

And $\sqrt{4x^2+7x+18}=b$

$$a^2-b^2=x-4$$
$$a^2-b^2=\left(\sqrt{5x^2+7x+2}\right)^2-\left(\sqrt{4x^2+7x+18}\right)^2$$
$$=\left(5x^2+7x+2\right)-\left(4x^2+7x+18\right)$$
$$=5x^2+7x+2-4x^2-7x-18$$

Then $=x^2-16$

$$a^2-b^2=x^2-16$$
$$(a+b)(a-b)=(x-4)(x+4)$$
$$(a+b)(a-4)=(x-4)(x+4)$$
$$a+b=x+4$$

$a+b=x+4$ **(by adding)**
$a-b=x-4$

$2a=2x$ $\qquad a=x$

$a+b=x+4$ **(by subtraction)**
$a-b=x-4$

$2b=8$

Put the value of a&b

$$\sqrt{5x^2+7x+2}=x$$
$$5x^2+7x+2=x^2$$
$$5x^2-x^2+7x+2=0$$
$$4x^2+7x+2=0$$
$$x=\frac{-7\pm\sqrt{49-4(4)(2)}}{2(4)}$$

$$= \frac{-7 \pm \sqrt{49-32}}{8} = \frac{-7 \pm \sqrt{17}}{8}$$

and

$$\sqrt{4x^2 + 7x + 18} = 4$$

$$4x^2 + 7x + 18 = 16$$

$$4x^2 + 7x + 2 = 0$$

$$x = \frac{-7 \pm \sqrt{49 - 4(4)}(2)}{2(4)}$$

$$= \frac{-7 \pm \sqrt{49-32}}{8}$$

$$= \frac{-7 \pm \sqrt{17}}{8}$$

Hence, $S.S = \left\{ \frac{-7 \pm \sqrt{17}}{8} \right\}$