

(c) Variation:

The word variation is frequently used in all sciences. There are two types of variations:

- (i) Direct variation (ii) Inverse variation.

(i) Direct Variation

If two quantities are related in such a way that increase (decrease) in one quantity causes increase (decrease) in the other quantity, then this variation is called direct variation.

In other words, if a quantity y varies directly with regard to a quantity x . We say that y is directly proportional to x and is written as $y \propto x$ or $y = kx$.

$$\text{i.e., } \frac{y}{x} = k, k \neq 0.$$

The sign \propto read as "varies as" is called the sign of proportionality or variation, while $k \neq 0$ is known as constant of variation.

e.g.,

- (i) Faster the speed of a car, longer the distance it covers.
(ii) The smaller the radius of the circle, smaller the circumference is.

(ii) inverse Variation

If two quantities are related in such a way that when one quantity increases, the other decreases, then this variation is called inverse variation.

In other words, if a quantity y varies inversely with regard to quantity x . We say that y is inversely proportional to x or y varies inversely as x and is written as

$$y \propto \frac{1}{x} \quad \text{or} \quad y = \frac{k}{x}$$

i.e., $xy = k$, where $k \neq 0$ is the constant of variation.

Exercise 3.2

1. If y varies directly as x , and $y = 8$ when $x = 1$, find

(i) y in terms of x

Solution:

Given that y varies directly as x .

Therefore $y \propto x$

$$\Rightarrow y = kx \text{ _____ (i)}$$

Where k is constant of variation.

Put $x = 2$ and $y = 8$ in eq. (i), we have

$$8 = k (2)$$

$$\text{or } 2k = 8$$

$$\Rightarrow k = 4$$

Put $k = 4$ in eq. (i), we get

$$y = 4x.$$

(ii) y when $x = 5$

Solution:

Given that y varies directly as x .

Therefore $y \propto x$

$$\Rightarrow y = kx \text{ _____ (i)}$$

Where k is constant of variation.

$$8 = k (2)$$

$$\text{or } 2k = 8$$

$$\Rightarrow k = 4$$

Put $k = 4$ and $x = 5$ in eq. (i), we get

$$y = (4)(5) = 20$$

(iii) x when y = 28

Solution:

Given that y varies directly as x.

Therefore $y \propto x$

$$\Rightarrow y = kx \text{ _____ (i)}$$

Where k is constant of variation.

Put $x = 2$ and $y = 8$ in eq. (i), we have

$$8 = k (2)$$

$$\text{or } 2k = 8$$

$$\Rightarrow k = 4$$

Put $k = 4$ and $y = 28$ in eq. (i), we get

$$28 = 4x$$

$$\text{or } 4x = 28$$

$$\Rightarrow x = 7$$

2. If $y \propto x$, and $y = 7$ when $x = 3$ find

(i) y in terms of x

Solution:

$$\Rightarrow y = kx \text{ _____ (i)}$$

Where k is constant of variation.

Put $x = 3$ and $y = 7$ in eq. (i), we have

$$7 = k (3)$$

$$\text{or } 3k = 7$$

$$\Rightarrow k = \frac{7}{3}$$

Put $k = \frac{7}{3}$ in eq. (i), we get

$$y = \frac{7}{3}x$$

(ii) x when y = 35 and y when x = 18

Solution:

Given that y varies directly as x.

Therefore $y \propto x$

$$\Rightarrow y = kx \text{ _____ (i)}$$

Where k is constant of variation.

Put $x = 3$ and $y = 7$ in eq. (i), we have

$$7 = k (3)$$

$$\text{or } 3k = 7$$

$$\Rightarrow k = \frac{7}{3}$$

Put $k = \frac{7}{3}$ and $y = 35$ in eq. (i), we get

$$35 = \frac{7}{3}x$$

$$x = 35 \times \frac{3}{7}$$

$$x = 5 \times 3 = 15$$

Put $k = \frac{7}{3}$ and $x = 18$ in eq. (i), we get

$$y = \left(\frac{7}{3}\right)(18)$$

$$y = 7 \times 6 = 42$$

3. If $R \propto T$ and $R = 5$ when $r = 8$, find the equation connecting R and T . Also find R when $T = 64$ and F when $R = 20$.

Solution:

Given that $R \propto T$

$$\Rightarrow R = kT \text{ _____ (i)}$$

Put $R = 5$ and $T = 8$ in eq. (i), we get

$$5 = k (8)$$

$$\text{or } 8k = 5$$

$$\Rightarrow k = \frac{5}{8}$$

Put $k = \frac{5}{8}$ in eq. (i), we get

$$R = \frac{5}{8}T \text{ _____ (ii)}$$

Put $T = 64$ and in eq. (ii), we get

$$R = \frac{5}{8}(64)$$

$$= 5 \times 8 = 40$$

$$20 = \frac{5}{8}T$$

or $\frac{5}{8}T = 20$

$$T = 20 \times \frac{8}{5}$$

$$T = 4 \times 8 = 32$$

4. If $R \propto T^2$ and $R = 8$ when $T = 3$, find R when $T = 6$.

Solution:

Given that $R \propto T^2$

$$\Rightarrow R = kT^2 \quad \text{_____ (i)}$$

Put $R = 8$ and $T = 3$ in eq. (i), we get

$$8 = k(3)^2$$

$$8 = 9k$$

or $9k = 8$

$$\Rightarrow k = \frac{8}{9}$$

Put $k = \frac{8}{9}$, $T = 6$ in eq. (i), we get

$$R = \frac{8}{9}(6)^2$$

$$R = \frac{8}{9}(36)$$

$$R = 8 \times 4 = 32$$

Given that $V \propto R^3$

$$\Rightarrow V = kR^3 \quad \text{_____ (i)}$$

Put $V = 5$ and $R = 3$ in eq. (i), we get

$$5 = k (3)^3$$

$$5 = 27k$$

or $27k = 5$

$$\Rightarrow k = \frac{5}{27}$$

Put $k = \frac{5}{27}$, $V = 625$ in eq. (i), we get

$$625 = \frac{5}{27} R^3$$

or $\frac{5}{27} R^3 = 625$

$$R^3 = 625 \times \frac{27}{5}$$

$$R^3 = 125 \times 27$$

$$R^3 = 5^3 \times 3^3$$

$$R^3 = (5 \times 3)^3$$

$$\Rightarrow R = 5 \times 3 = 15$$

6. If w varies directly as u^3 and $w = 81$ when $u = 3$. Find w , when $u = 5$.

Solution:

Given that $w \propto u^3$

$$\Rightarrow w = ku^3 \quad \text{_____ (i)}$$

Put $w = 81$ and $u = 3$ in eq. (i), we get

$$81 = 27k$$

or $27k = 81$

$$\Rightarrow k = 3$$

Put $k = 3$ and $u = 5$ in eq. (i), we get

$$w = (3) (5)^3$$

$$w = 3 \times 125 = 375$$

7. If y varies inversely as x and $y = 7$ when $x = 2$, find y when $x = 126$.

Solution:

Given that $y \propto \frac{1}{x}$

$$\Rightarrow y = \frac{k}{x} \text{ _____ (i)}$$

Put $y = 7$ and $x = 2$ in eq. (i), we get

$$7 = \frac{k}{2}$$

$$k = 14$$

Put $k = 14$ and $x = 126$ in eq. (i), we get

$$y = \frac{14}{126}$$

$$y = \frac{1}{9}$$

8. If $y \propto \frac{1}{x}$ and $y = 4$ when $x = 3$, find x when $y = 24$.

Solution:

$$\Rightarrow y = \frac{k}{x} \text{ ————— (i)}$$

Put $y = 4$ and $x = 3$ in eq. (i), we get

$$4 = \frac{k}{3}$$

$$k = 12$$

Put $k = 12$ and $y = 24$ in eq. (i), we get

$$24 = \frac{12}{x}$$

$$x = \frac{12}{24} = \frac{1}{2}$$

9. If $w \propto \frac{1}{z}$ and $w = 5$ when $z = 7$, find w when $z = \frac{175}{4}$.

Solution:

Given that $w \propto \frac{1}{z}$

$$\Rightarrow w = \frac{k}{z} \text{ ————— (i)}$$

Put $w = 5$ and $z = 7$ in eq. (i), we get

$$5 = \frac{k}{7}$$

$$k = 35$$

Put $k = 35$ and $z = \frac{175}{4}$ in eq. (i), we get

$$w = \frac{35}{\frac{175}{4}}$$

$$w = 35 \times \frac{4}{175}$$

∴

10. $A \propto \frac{1}{r^2}$ and $A = 2$ when $r = 3$, find r when $A = 72$.

Solution:

Given that $A \propto \frac{1}{r^2}$

$$\Rightarrow A = \frac{k}{r^2} \quad \text{———— (i)}$$

Put $A = 2$ and $r = 3$ in eq. (i), we get

$$2 = \frac{k}{(3)^2}$$

$$2 = \frac{k}{9}$$

$$k = 18$$

Put $k = 18$ and $A = 72$ in eq. (i), we get

$$72 = \frac{18}{r^2}$$

$$r^2 = \frac{18}{72}$$

$$r^2 = \frac{1}{4}$$

$$r = \pm \frac{1}{2}$$

11. $a \propto \frac{1}{b^2}$ and $a = 3$ when $b = 4$, find a when $b = 8$.

Solution:

Given that $a \propto \frac{1}{b^2}$

$$\Rightarrow a = \frac{k}{b^2} \quad \text{———— (i)}$$

$$3 = \frac{k}{(4)^2}$$

$$3 = \frac{k}{16}$$

$$k = 48$$

Put $k = 48$ and $b = 8$ in eq. (i), we get

$$a = \frac{48}{(8)^2} = \frac{48}{64}$$

$$a = \frac{3}{4}$$

12. $V \propto \frac{1}{r^3}$ and $V = 5$ when $r = 3$, find V when $r = 6$ and r when $V = 320$.

Solution:

Given that $V \propto \frac{1}{r^3}$

$$\Rightarrow V = \frac{k}{r^3} \quad \text{—————(i)}$$

Put $V = 5$ and $r = 3$ in eq. (i), we get

$$5 = \frac{k}{(3)^3}$$

$$5 = \frac{k}{27}$$

$$k = 135$$

Put $k = 135$ and $r = 6$ in eq. (i), we get

$$V = \frac{135}{(6)^3}$$

$$V = 135$$

$$V = \frac{5}{8}$$

Put $k = 135$ and $V = 320$ in eq. (i), we get

$$320 = \frac{135}{r^3}$$

$$r^3 = \frac{135}{320}$$

$$r^3 = \frac{27}{64}$$

$$r^3 = \left(\frac{3}{4}\right)^3$$

$$\Rightarrow r = \frac{3}{4}$$

13. $m \propto \frac{1}{n^3}$ and $m = 2$ when $n = 4$, find m when $n = 6$ and n when $m = 432$.

Solution:

Given that $m \propto \frac{1}{n^3}$

$$\Rightarrow m = \frac{k}{n^3} \quad \text{—————(i)}$$

Put $m = 2$ and $n = 4$ in eq. (i), we get

$$2 = \frac{k}{(4)^3}$$

$$2 = \frac{k}{64}$$

$$k = 128$$

Put $k = 128$ and $n = 6$ in eq. (i), we get

$$m = \frac{128}{6^3}$$

$$m = \frac{128}{216}$$

$$m = \frac{16}{27}$$

Put $k = 128$ and $m = 432$ in eq. (i), we get

$$432 = \frac{128}{n^3}$$

$$n^3 = \frac{128}{432}$$

$$n^3 = \frac{8}{27}$$

$$n^3 = \left(\frac{2}{3}\right)^3$$

$$\Rightarrow n = \frac{2}{3}$$

