Exercise 2.8

1. The product of two positive consecutive numbers is 182. Find the numbers.

Solution:

Let the numbers be $x, x+1$

Then, $(x)(x+1)=182$

$$x^2+x-182=0$$

$$x^2+14x-13x-182=0$$

$$x(x+14)-13(x+14)=0$$

$$(x-13)(x+14)=0$$

$x-13=0$ *gives* $x=13$

Numbers are: $x=13$

$x+1=13+1=14$

13,14

Now, $x+14=0$

gives $x=-14$

we ignore this value because it is negative

2. The sum of squares of three positive consecutive numbers is 77. Find the numbers.

Solution:

Let the numbers be $x, x+1, x+2$

Applying the given conditions

$$(x)^2+(x+1)^2+(x+2)^2=77$$

$$x^2+x^2+2x+1+x^2+4x+4=77$$

$$3x^2+6x+5=77$$

$3x^2 + 6x - 72 = 0$

$x^2 + 2x - 24 = 0$ (Dividing by 3)

$x^2 + 6x - 4x - 24 = 0$

$x(x+6) - 4(x+6) = 0$

$(x-4)(x+6) = 0$

$x - 4 = 0$ *gives* $x = 4$

Numbers are: $x = 4$

$x + 1 = 4 + 1 = 5$

$x + 2 = 4 + 2 = 6$

4, 5, 6

$x + 6 = 0$

$x = -6$, we ignore the negative number.

3. The sum of five times a number and the square of the number is 204. Find the number.

Solution:

Let the numbers be x

According to the given conditions,

$x^2 + 5x = 204$

$x^2 + 5x - 204 = 0$

$x^2 + 17x - 12x - 204 = 0$

$x(x+17) - 12(x+17) = 0$

$(x+17)(x-12) = 0$

$x - 12 = 0$ *gives* $x = 12$

or $x + 17$ *gives* $x = -17$

Number is 12 or -17.

4. The product of five less than three times a certain number and one less

Solution:

Let the numbers be x

According to the given conditions,

$(3x-5)(4x-1)=7$

$12x^2-3x-20x+5=7$

$12x^2-23x+5=7$

$12x^2-23x+5-7=0$

$12x^2-23x-2=0$

$12x^2-24x+x-2=0$

$12x(x-2)+1(x-2)=0$

$(12x+1)(x-2)=0$

$x-2=0$ *gives* $x=2$

and $12x+1=0$ *gives* $x=-\frac{1}{12}$

Number is 2 or $-\frac{1}{12}$.

5. The difference of a number and its reciprocal is $\frac{15}{4}$. Find the number.

Solution:

Let the numbers be x

Then, According to the given conditions,

$$x - \frac{1}{x} = \frac{15}{4}$$

$$4x^2 - 4 = 15x \quad (\text{Multiplying by } 4x)$$

$$4x^2 - 15x - 4 = 0$$

$$4x^2 - 16x + x - 4 = 0$$

$$4x(x-4) + 1(x-4) = 0$$

$$(x-4)(4x+1) = 0$$

$x - 4 = 0$ *gives* $x = 4$

and $4x + 1 = 0$ *gives* $x = -\frac{1}{4}$

Number is 4 and $-\frac{1}{4}$.

6. The sum of squares of two digits of a positive integral number is 65 and the number is 9 times the sum of its digits. Find the number.

Solution:

Let xy be the number, where unit digit is y and tens digit is x.

According the given condition,

$x^2 + y^2 = 65$ .....(A)

Number $= y + 10x$

$y + 10x = 9(x + y)$

$y + 10x = 9x + 9y$

$10x - 9x = 9y - y$

$x = 8y$ .....(B)

Putting $x = 8y$ *in* (A)

$(8y)^2 + y^2 = 65$

$64y^2 + y^2 = 65$

$65y^2 = 65$

$y^2 = 1$

$y = \pm 1$

$y = 1$ (taking +ve value)

Putting $y = 1$ *in* (B)

$x = 8(1) = 8$

Now,

Number $= y + 10x$

$= 1 + 10(8)$

$= 81$

Therefore, number is 81.

7. The sum of the co-ordinates of a point is 9 and sum of their squares is 45. Find the co-ordinates of the point.

Solution:

Let $P(x, y)$ be the point.

$$x+y=9 \quad \text{.....}(A)$$

and $x^2+y^2=45 \quad \text{.....}(B)$

From A $x=9-y \quad \text{.....}(C)$

Putting $x=9-y$ in (B)

$(9-y)^2+y^2=45$

$81-18y+y^2+y^2=45$

$2y^2-18y+81-45=0$

$2y^2-18y+36=0$

$y^2-9y+18=0 \qquad (\text{Dividing by } 2)$

$y^2-6y-3y+18=0$

$y(y-6)-3(y-6)=0$

$(y-6)(y-3)=0$

$y-6=0 \qquad \text{gives } y=6$

Then from (C)

$x=9-6=3$

Point is $P(3,6)$

When y-3=0 then y=3

From(C)

$x=9-3=6$

Point is $P(6,3)$

8. Find two integers whose sum is 9 and the difference of their squares is also 9.

Solution:

Let the integers be x,y.

Then,

$$x+y=9 \quad \text{.....}(A)$$

and $x^2-y^2=9 \quad \text{.....}(B)$

From A $x=9-y$

Putting $x=9-y$ in (B)

$$(9-y)^2 - y^2 = 9$$
$$81 - 18y + y^2 - y^2 = 9$$
$$-18y = 9 - 81$$
$$-18y = -72$$
$$y = -\frac{72}{18}$$
$$y = 4$$

Putting $y = 4$ in (A)

$x + 4 = 9$

$x = 9 - 4 = 5$

Integers are 5,4

9. Find two integers whose difference is 4 and whose squares differ by 72.

Solution:

Let the integers be x,y.

Then,

$x - y = 4$ .....(A)

and $x^2 - y^2 = 72$ .....(B)

From A $x - y = 4$

$x = 4 + y$

Putting $x = 4 + y$ in (B), we get

$(4+y)^2 - y^2 = 72$

$16 + y^2 + 8y - y^2 = 72$

$8y = 72 - 16$

$8y = 56$

$y = \frac{56}{8}$

$y = 7$

Putting $y = 7$ in (A)

$x - 7 = 4$

10. Find the dimensions of a rectangle, whose perimeter is 80cm and its area is 375cm².

Solution:

Let x and y be the length and width respectiely of the rectangle.

According to the given conditions,

Perimeter $2x + 2y = 80$

or $x + y = 40$ .....(A)

Area $xy = 375$ .....(B)

From A $x = 40 - y$

Putting $x = 40 - y$ in (B), we get

$(40 - y)y = 375$

$40y - y^2 = 375$

$\Rightarrow y^2 - 40y + 375 = 0$

$y^2 - 25y - 15y + 375 = 0$

$y(y - 25) - 15(y - 25) = 0$

$(y - 15)(y - 25) = 0$

$y - 15 = 0$ *gives* $y = 15$

Putting $y = 15$ in (A)

$x + 15 = 40$

$x = 40 - 15 = 25$

Length = $25cm$. Breadth = $15cm$.